CORRECTING ABERRATIONS IN COMPLEX MAGNET SYSTEMS FOR MUON COOLING CHANNELS*

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Abstract

Designing and simulating complex magnet systems needed for cooling channels in both neutrino factories and muon colliders requires innovative techniques to correct for both chromatic and spherical aberrations. Optimizing complex systems, such as helical magnets for example, is also difficult but essential. By using COSY INFINITY, a differential algebra based code, the transfer and aberration maps can be examined to discover what critical terms have the greatest influence on these aberrations.

INTRODUCTION

The challenging emittance size needed to implement a neutrino factory or muon collider has motiviates searches for innovating techniques in beam cooling [1, 2]. For example, the use of Parametric-resonance Ionization Cooling (PIC) has been proposed for the final stage of 6D cooling of a high-luminosity muon collider [3]. In this system, an induced resonance is used to cause periodic beam size reductions, and ionization cooling is then achieved via wedges of absorbing materials. An epicyclic twin helical channel offers to achieve the goals of PIC, correlating the dispersion and betatron functions of the beam [4]. The critical challenges of this system include correcting chromatic and spherical aberrations induced in the channel.

USING COSY INFINITY TO STUDY ABERRATIONS IN A SYSTEM

COSY INFINITY (COSY) is a DA-based code allowing simulation of beam transfer and aberration maps to arbitrary order [5]. With modification of the base code, the twin helix channel was implemented and simulated in COSY [6]. In these simulations, COSY takes a reference particle defined as:

$$\{r_k\} = (x_k, a_k, y_k, b_k, t_k, \delta_k) \quad (1)$$

Where a and b are the dimensionless horizontal and vertical momentum, t is time of flight and δ is the change in total energy of the particle. COSY calculates the function, $\mathcal{M}$, known the transfer map (or taylor map) for the system, which described the evolution of particles in the system. The linear terms of the transfer map function comprise the matrix M, often referred to as the linear map or transfer matrix of the system [5], that satisfies the relation:

$$\{r_f\} = M\{r_i\} = \begin{pmatrix} (x\,|\,x) & \cdots & (x\,|\,\delta) \\ \vdots & \ddots & \vdots \\ (\delta\,|\,x) & \cdots & (\delta\,|\,\delta) \end{pmatrix} \{r_i\} \quad (2)$$

The non-linear terms, N, remaining in the transfer map can be expressed separately:

$$\mathcal{M} = \mathrm{M} + \mathrm{N} \quad (3)$$

In terms of the component of the transfer map, each of the final vector components for a particle can be expressed in form:

$$x_f = \sum \left(x\,|\,x^{i_x} a^{i_a} y^{i_y} b^{i_b} t^{i_t} \delta^{i_\delta}\right) x^{i_x} a^{i_a} y^{i_y} b^{i_b} t^{i_t} \delta^{i_\delta} \quad (4)$$

Where the terms are summed over $(i_x,i_a,i_y,i_b,i_t,i_\delta)$ for each component. The terms in the transfer map of 2nd or higher order are commonly referred to as aberrations. COSY calculated these aberrations and generates output, referred to as an aberration map. The aberration map is in a format similar to the transfer map.

Examination of the transfer and aberration maps provides important clues in improving a beam system. If, for example, the input suffers from large variation in initial angle and final horizontal position needs minimization to fit a particular aperture in the beamline, then aberrations in (x_f) dependent on initial angle (a_i) may be particularly important to minimize. We would want to pay particular attention to terms in the aberration map involving higher orders of a, such as (x|aa) or (x|aaa). If those terms are not minimized, variations in initial angle threaten to blow up the horizontal position of the final beam. Similarly, if we know that initial position is small, we can put less emphasis on minimizing aberrations that depend on (x_i), particularly higher order terms involving 2 or more powers of x, such as (x|axx), where initial position may dominate initial angular spread.

To minimize these aberrations, it is also important to recognize how magnetic systems contribute to the transfer map. The linear terms of the transfer map are determined by the dipole and quadrupole moments of magnetic elements. Correction for higher order aberrations requires use of higher order multipoles. Thus, sextupoles are used to correct 2nd order aberrations, and octupoles are used for 3rd order aberrations.

Using various symmetries of the beam system can also be an effective technique for aberration correction [7].

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For example, systems are described as having midplane symmetry if particle motion is symmetric on each side of plane (called the miplane). If we let:

$$r_f = (x_f, a_f, y_f, b_f, t_f, \delta_f) = \mathcal{M}(x_i, a_i, y_i, b_i, t_i, \delta_i) \qquad (5)$$

then as a result of midplane symmetry,

$$(x_f, a_f, -y_f, -b_f, t_f, \delta_f) = \mathcal{M}(x_i, a_i, -y_i, -b_i, t_i, \delta_i) \qquad (6)$$

From this relation, the following coefficients of the transfer map can be determined to be zero:

$$(x \mid x^{i_x} a^{i_a} y^{i_y} b^{i_b} t^{i_t} \delta^{i_\delta}) = 0 \quad \text{if } i_y + i_b \text{ is odd} \qquad (7)$$

$$(a \mid x^{i_x} a^{i_a} y^{i_y} b^{i_b} t^{i_t} \delta^{i_\delta}) = 0 \quad \text{if } i_y + i_b \text{ is odd} \qquad (8)$$

$$(y \mid x^{i_x} a^{i_a} y^{i_y} b^{i_b} t^{i_t} \delta^{i_\delta}) = 0 \quad \text{if } i_y + i_b \text{ is even} \qquad (9)$$

$$(b \mid x^{i_x} a^{i_a} y^{i_y} b^{i_b} t^{i_t} \delta^{i_\delta}) = 0 \quad \text{if } i_y + i_b \text{ is even} \qquad (10)$$

$$(t \mid x^{i_x} a^{i_a} y^{i_y} b^{i_b} t^{i_t} \delta^{i_\delta}) = 0 \quad \text{if } i_y + i_b \text{ is odd} \qquad (11)$$

$$(\delta \mid x^{i_x} a^{i_a} y^{i_y} b^{i_b} t^{i_t} \delta^{i_\delta}) = 0 \quad \text{if } i_y + i_b \text{ is odd} \qquad (12)$$

This has the effect of eliminating nearly half of the terms, and allows us to substantially simplify the transfer map. For example, the linear evolution of the system reduces to:

$$\begin{pmatrix} x_f \\ a_f \\ y_f \\ b_f \\ t_f \\ \delta_f \end{pmatrix} = \begin{bmatrix} (x|x) & (x|a) & 0 & 0 & (x|t) & (x|\delta) \\ (a|x) & (a|a) & 0 & 0 & (a|t) & (a|\delta) \\ 0 & 0 & (y|y) & (y|b) & 0 & 0 \\ 0 & 0 & (b|y) & (b|b) & 0 & 0 \\ (t|x) & (t|a) & 0 & 0 & (t|t) & (t|\delta) \\ (\delta|x) & (\delta|a) & 0 & 0 & (\delta|t) & (\delta|\delta) \end{bmatrix} \begin{pmatrix} x_i \\ a_i \\ y_i \\ b_i \\ t_i \\ \delta_i \end{pmatrix}$$

ABERRATIONS IN THE TWIN HELIX

In the twin helix channel, several key factors contribute to aberrations in the system. The entire channel's dipole harmonic fields will create a continual dispersive effect. As a result, energy dependent (chromatic) aberrations have the potential to become a dominant problem in the channel. Figures 1 shows the evolution of the phase space of the beam in the horizontal planes, and strong skewing due to chromaticity in a twin helix cell without any absorbing wedge. These effects are increased after introduction of the absorbing wedge as shown in Figure 2.

Examination of the aberration map from of a single twin helix harmonic magnet cell shows that the only non-zero terms effecting the x and a vector components are the energy dependent aberrations. At second order, the $(a|x\delta)$ and $(a|\delta\delta)$ components are an order of magnitude greater than the other aberrations effecting horizontal phase space. A sampling of the aberration map terms effecting horizontal phase space is contained in Table 1. These show the all the aberrations effecting x_f and a_f at 2nd and the largest ones at 3rd order. Given their magnitude, the 3rd order aberrations look to pose the more challenging problem to solve.

Table 1: Selected Components for the Aberration Map for the Single Twin Helix Cell

Map Term	Value
2nd Order Horizontal Aberations	
$(x\|x\delta)$	.9189890 E-04
$(x\|a\delta)$	.7036055 E-05
$(x\|\delta\delta)$	.1887682 E-04
$(a\|x\delta)$	-.3331598 E-02
$(a\|a\delta)$	-.2550771 E-03
$(a\|\delta\delta)$	-.6843386 E-02
3rd Order Horizontal Aberrations	
$(x\|xxx)$	.3781061 E-02
$(x\|xyy)$	.1877792 E-02
$(x\|xx\delta)$	.2329986 E-02
$(a\|xxx)$	.1526752 E-01
$(a\|xyy)$	.7582325 E-02
$(a\|xx\delta)$	.9408237 E-02

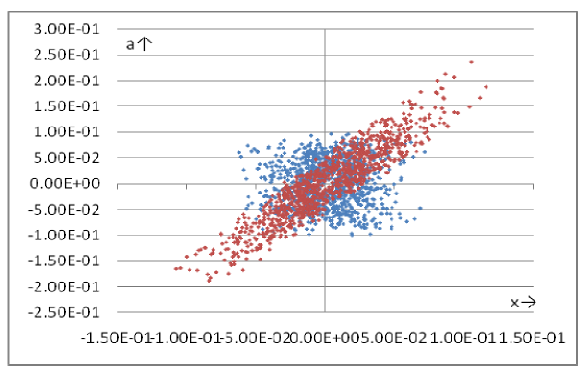

Figure 1: Initial (blue) and final (red) distribution in horizontal phase space for single Twin Helix cell without absorbing wedge.

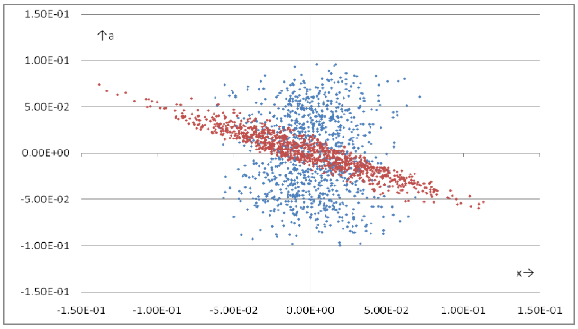

Figure 2: Initial (blue) and final (red) distribution in horizontal phase space for single Twin Helix cell with absorbing wedge.

Also, terms involving initial position (xi) are of concern. For beams that begin at a "point-like source," these aberrations tend to vanish, but in the case of a muon cooling channel, the large initial beam size can cause these terms to dominate in the evolution of the beam thru the channel.

METHODS FOR MINIMIZING ABERRATIONS

COSY's internal tools allow a user to optimize parameters in their simulations to meet particular objective functions. For example, helical sextupole harmonics can be added to the twin helix cell to attempt to minimize a second order aberration such as $(x|\delta\delta)$. These higher order magnetic moments will not alter the linear terms in the transfer map, since such terms are fixed by the dipole and quadrupole moments. Free parameters for the sextupole harmonic (such as field strength or phase) may be varied in a simulation in an attempt to minimize functions based upon specific terms in the transfer or aberration maps.

Due to the continuous dispersion occurring from the dipole harmonic in the helical channel, chromatic aberrations pose a considerable challenge. As an added complication, planar orbit of the reference particle will need to be maintained. One method is thru adjustment of the poletip field strength and the phase of the sextupole helical harmonic. It is also possible to introduce a series of sextupole harmonics with independent parameters to achieve the corrective effects of sextupole doublets utilized in linear beamlines.

The challenge in using these method is understanding the interdependence of the aberration and transfer map components. Attempts to minimize certain terms can easily create problems by increasing different aberrations in the system. COSY provides an excellent tool to help study beam aberrations and evaluate efforts to correct their effects.

CONCLUSIONS AND FUTURE WORK

With the twin helix channel simulated in COSY INFINITY, extensive testing can be done to study chromatic and spherical aberrations of the proposed system. Chromaticity poses a challenge in this channel that merits further study. Utilizing the aberration map, and understanding the symmetries in the beam line will be helpful in testing and optimizing aberrations. Further testing will include higher order magnet elements, including the sextupole helical harmonic field that has already been implemented in COSY.

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