

SIMULATION STUDY OF INTRABEAM SCATTERING IN LOW EMITTANCE RING*

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Abstract

HALS (Hefei Advanced Light Source) is under designing dedicated to good coherence and high brightness at 1.5GeV. Low emittance is required to reach the design request. Due to the low energy and emittance with relative high bunch charge, intrabeam scattering effect will be very strong. It is worth accurately calculating to check if the design goal can be reached. Theoretic calculation based on Gaussian beam distribution doesn't warrant in strong IBS regime. In this paper we present the results of particle simulation study of intrabeam scattering effect on a temporary design lattice of HALS ring.

INTRODUCTION

The purpose of Hefei Advanced Light Source (HALS) is aiming at synchrotron radiation with high brilliance and better coherence in the VUV and soft X-ray range to synchrotron users. Optimized design of lattice structure provide us natural emittance of about 0.07nm-rad. The main parameters of HALS are listed in Table 1.

Beam emittances in low-energy, ultra low-emittance and high-luminosity storage ring are dominantly affected by the small angle multiple intrabeam Coulomb scattering, which severely limits the minimum achievable emittance in HALS. The effect of IBS should be well discussed in physics design process of HALS.

Theory of IBS has been well discovered in many publications. We have already numerically estimate the emittance growth due to IBS for the temporary lattice design of HALS based on Piwinski and Bjorken-Mtingwa methods [1]. However, these analytical methods both assume that the distribution function is Gaussian for all degrees of freedom, which is adequately accurate in common situation, but not for some cases, for example the non-Gaussian beam tail caused by very intense beam. Simulation methods starting from kinetic mechanism may provide another way to estimate the emittance growth accurately. In this paper we describe the simulation method and present the result of its application to HALS design lattice.

Table 1: Main Parameters of HALS Storage Ring

Parameters	Values
Circumference	420m
Energy	1.5GeV
Transverse tunes	48.23/16.73
Natural chromaticities	-279.7/-39.9
Momentum Compaction	0.00016
Harmonic number	140
Emittance of bare lattice	0.068nm rad
Coupling factor	0.1
Energy spread	0.00022
Natural bunch length	0.0245cm

DESCRIPTION OF THE SIMULATION METHOD

The simulation is based on macroparticle method. The macroparticles which represent the electrons in the bunch are transported through the storage ring elements and encounter coulomb scattering simultaneously. The coulomb scattering process is simulated by the Binary Collision Model, which utilizes kinetic process precisely describing the diffusion phenomenon of particles interaction, in accordance with Fokker Planck equation [2].

BCM Treatment for IBS

Binary collision model (BCM) is first developed by Takizuka and Abe [3], to simulate the effects of collisions in plasma. The continuous multipole coulomb scattering is treated by discrete random process with Monte Carlo method. Particles grouped in the same grid cell according to their spatial position are paired randomly in cell and collided, the change of velocity due to the scattering process is calculated to add to the initial state of the particle. Figure 1 is an example of the random selection of scattering pairs.

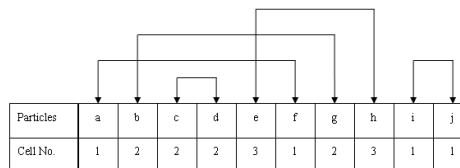


Figure 1: Random pairing of particles in same cell.

For two particles collide during the time interval Δt , the total momentum and total energy should be conserved

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since the magnitude of the relative velocity is unchanged, only the direction of the particles velocities is affected by scattering. The change is determined by scattering angle and the azimuthal angle. In the Monte Carlo simulation method, the azimuthal angle Φ is randomly chosen in uniform distribution from 0 to 2π . To determine scattering angle θ , we should choose the variable δ first, the relations between them shows:

$$\sin \theta = 2\delta / (1 + \delta^2) \quad (1)$$

$$1 - \cos \theta = 2\delta^2 / (1 + \delta^2)$$

The variable δ has a mean value of zero and variance given by

$$\langle \delta^2 \rangle = (e_\alpha^2 e_\beta^2 n_L \log \Lambda / 8\pi \epsilon_0^2 m_{\alpha\beta}^2 u^3) \Delta t, \quad (2)$$

in which e_0 is the permittivity of vacuum, n_L is the lower density between n_α and n_β of two kind of gas particles, and $\log \Lambda$ denotes the coulomb logarithm. u is the relative speed of the two colliding particles. m_{ab} is the reduced mass defined by

$$m_{\alpha\beta} = m_\alpha m_\beta / (m_\alpha + m_\beta) \quad (3)$$

Applying these equations to the condition of electron storage ring, we believe that:

$$m_\alpha = m_\beta = 0.5\gamma m_{e0}, e_\alpha = e_\beta = q_e, n_L = 1/2\rho_e \quad (4)$$

So we have

$$\langle \delta^2 \rangle = (q_e^4 \rho_e \log \Lambda / 4\pi \epsilon_0^2 \gamma^2 m_{e0}^2 u^3) \Delta t \\ = 4\pi r_e^2 c^4 \rho_e \log \Lambda \Delta t / (\gamma^2 u^3) \quad (5)$$

In which q_e and m_{e0} are the electron charge and rest mass, ρ_e is determined by number of the electron in cell. The Coulomb logarithm factor $\log \Lambda$ is defined $\log \Lambda(\Delta u) = \ln(\tilde{d}\beta^2 \gamma^2 |\Delta u|^2 / 2r_i)^{[2]}$, where \tilde{d} equals to the vertical beam size. With the equivalent scattering angle and azimuthal angle, the discreted random process will show agreement with the particle invariant change due to the IBS in the same time interval.

Based on the scattering and azimuthal angle defined, the postcollision velocities of the two particle are expressed as:

$$\Delta u_x = (u_x^t / u_\perp^t) u_z \sin \theta \cos \Phi - \\ (u_y^t / u_\perp^t) u \sin \theta \sin \Phi - u_x^t (1 - \cos \theta) \\ \Delta u_y = (u_y^t / u_\perp^t) u_z \sin \theta \cos \Phi + \\ (u_x^t / u_\perp^t) u \sin \theta \sin \Phi - u_y^t (1 - \cos \theta) \\ \Delta u_z = -u_\perp^t \sin \theta \cos \Phi - u_z^t (1 - \cos \theta) \quad (6)$$

In which $u_\perp = (u_x^2 + u_y^2)^{1/2}$, u_x, u_y, u_z are the horizontal, vertical and longitudinal components of the relative speed u .

Particle Transport in Storage Ring Lattice

Since the collision effect is considered as discrete process, correspondingly the lattice can be separated into pieces of segments. The particle pass through the lattice segment and change the momentum and position coordinates, then the scattering effect is calculated taking the small time interval as parameter. Obviously, the time

interval should be much less than the typical IBS time. The smaller time interval will provide higher accuracy.

A explicit transfer map derived from generation functions is applied in our simulation method [4, 5], which provides symplecticity and high speed in long term tracking. Symplectic tracking algorithm is suitable for long term calculation, preventing the truncation error of the non-symplectic methods. Complicated particle motions in storage ring such as betatron oscillation, synchrotron oscillation and dispersion are included in the full 6 dimensional tracking, the precision of which is first order to the tracking step length. For convenience to treat the tracking and scattering process, the 6-D particle coordinates are chosen $(x, y, \sigma, x', y', p_\sigma)$ taking s as independent variable, in which $\sigma = s - v_0 t$, $p_\sigma = \Delta E / (E_0 \beta_0^2)$. A transformation is needed when calculating the relative speed of the colliding particles.

Generally, the total steps of the simulation are: Divide the spatial region into cells; initial state of particles are set up at the beginning of the simulation randomly according to the natural beam emittance and size. As the time advance, the particles are transported forward through the small lattice segments, which is considered independent with the scattering process. Then particles are grouped at every cell according their position coordinates, and random pairs of particles in the same cell suffering the collision. The change of their velocities due to the binary collision is computed. The steps repeat as the particles move on. Emittance of particle bunch after each step is calculated and averaged in whole ring circumference.

RESULTS

Using the method described above we simulated the coulomb scattering effect of particles in HALS design lattice. To make computation reliable, particles are tracked more than 20000 turns of the ring, which is 10^{-2} second magnitude in time scale, longer than the typical IBS time of HALS. Different number of macroparticles is applied to see its influence to the simulation result. Number of spatial cells is set to 10^3 . Growth of horizontal emittance due to IBS is shown in Figure 2.

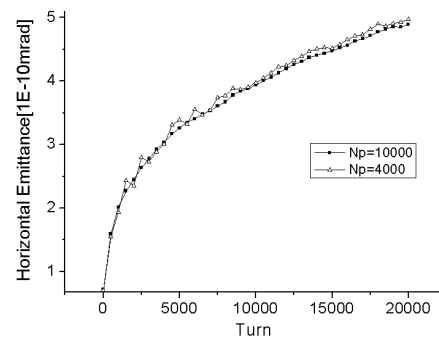


Figure 2: Horizontal beam emittance growth tracking with different number of macroparticles.

It's shown that the emittance growth due to IBS will be very strong. Result given by more macroparticles simulation appears more smooth. Since we haven't add the radiation damping to the simulation, final achievable emittance will be the balance of IBS emittance growth and damping effect, which need further investigation in our simulation. Typical Logarithm factor computed in simulation process is about 20, higher than the value adopted in other reports about 10~17 [6, 7].

CONCLUSION AND FUTURE WORK

Emittance growth due to intrabeam scattering in HALS is investigated by macroparticle simulation. The result shows the remarkable emittance growth due to IBS effect. To calculate the final steady state emittance, the radiation damping effect will be added into the simulation. Other random process in storage ring, for example Touscheck scattering and residual gas scattering, could also be looked into with this Monte Carlo simulation method.

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