CURRENT DEPENDENT TUNE SHIFTS IN THE UNIVERSITY OF MARYLAND ELECTRON RING UMER*

D. F. Sutter, M.Cornacchia, S. Bernal, B. Beaudoin, R. A. Kishek, T. Koeth and P.G. O'Shea
Institute for Research in Electronics and Applied Physics,
University of Maryland, College Park, MD 20742, USA

Abstract

The shift in betatron tunes as a function of space charge has been studied in many accelerators and storage rings. Because of its low energy (10 keV, $\gamma = 1.02$) and wide range of operating currents (0.6 to 100 mA, corresponding to predicted incoherent tune shifts of ~ 1.0 to 5.8), the University of Maryland Electron Ring (UMER) provides a unique opportunity to study space charge driven tune shifts over a wide parameter space. Comparisons of predictions and measurements are presented, including a discussion of special factors such as the magnetic penetration of the vacuum chamber walls.

INTRODUCTION

The space charge induced shift in oscillation frequency of charged particle beams falls into two categories: one associated with the balance of internal space charge self forces of the beam with the external focusing forces of the optical lattice and one involving the coherent oscillation of the beam structure as a whole due to image forces in the beam enclosure [2-5]. The self field - or direct - space charge model describes an incoherent tune shift for the motion of individual particles. It is this tune shift that is commonly used to determine the Laslett tune shift limit in most circular accelerators and storage rings and to describe the extreme range of space charge effects in UMER noted above in the Abstract. Since the usual Laslett tune shift limit for safe operation without beam loss is taken as ≤ ½, even the lowest operating current of 0.6 mA in UMER exceeds it! The coherent tune motion of the whole beam, governed by the image forces, includes a rigid dipole mode and higher order modes. The coherent tune shifts are much smaller than the incoherent, typically ~ 1/25th as large. In UMER we are not yet able to measure the incoherent tunes or the coherent motion of the higher order modes; so only the tune shift of the coherent rigid dipole mode as a function of beam current is treated in the present study.

EXPERIMENTAL CONFIGURATION

UMER has been described in detail elsewhere [1]. It is based on a simple FODO lattice with 72 quadrupoles and 36 dipoles. From the standpoint of the internal environment seen by the beam, the machine design is relatively simple. The vacuum chamber is round over most of the 11.52 m circumference and is made of very low magnetization 316LN stainless steel tubing with an internal diameter of 49.8 mm. This is essential because the low operating kinetic energy of 10 keV makes the beam highly susceptible to the effect of residual magnetic fields – the earth's field coupled with the residual magnetic field of the building iron does 22% of the bending in the horizontal plane. The design of the BPM's preserves the beam pipe ID and the metal parts are also 316LN stainless. The most serious deviations in smoothness are in the 18 installed bellows, of the same ID as the beam pipe and each about 4 cm long, and in the 64 cm long injection section where about 24 cm of the beam pipe has an ID of ~ 80 cm, with tapered entrance and exit sections. Since normal beam operation, which is used for this study, is with a bunching factor of ½, the beam fills half the ring and is about 5.7 m long (~ 100ns), much longer than any of the above described discontinuities. A list of parameters of direct relevance to the tune versus beam current measurements is given in Table 1.

In order to obtain a measure of the coherent tune shift as a function of beam current only, it is essential to use exactly the same ring optical parameters - steering and focusing strengths - for all of the beam currents used (0.6, 6, 20, 40 and 80 mA). This has proved somewhat difficult to do in the past, and for this reason the quadrupoles are operated at only 83% of the design focusing strength for a 100mA beam current. As a consequence, the space charge forces seen with the 100mA beam are too strong at the operating quad strength, and so the tune measurements for this beam are not included.

Table 1: Relevant UMER parameters for the coherent tune versus beam current studies. The characteristic current, I_0, is related to the classical radius of the electron, r_0, through $I_0 r_0 = ec$, where e is the electronic charge and c is the velocity of light.

Circumference	1152 cm
Average Radius, R	183.3 cm
Kinetic Energy, T	10 keV
Relativistic β	0.1950
Relativistic γ	1.020
Beam Pipe Radius, b	2.489 cm
Beam Pipe Wall, d	0.0508 cm
Wall resistivity, ρ	7.4×10^{7} Ω-cm
Wall magnetization, μ	1.0
Characteristic current, I_0	17.05×10^{6} mA

COMPUTING COHERENT TUNE VS BEAM CURRENT

The analysis is based on the theoretical description of the effect of transverse images and space charge forces in

---

*Work supported by US Department of Energy Office of High Energy Physics, and by the US Department of Defence, Office of Naval Research and Joint Technology Office.

chapter 8 of reference 5 which in turn relies heavily on the classical paper by Laslett [4] and the work of Zotter, particularly [5]. The physical simplicity of the UMER vacuum chamber noted above and the very long bunch justifies neglecting the regions that are not smooth and treating the entire circumference as an identical cylindrical boundary structure. The basic equations for coherent tune shift address two cases, one for penetrating electromagnetic fields and one nonpenetrating. If the skin depth, $\delta_{wall} < \sqrt{bd}$, where b is the ID of the vacuum pipe and d is the wall thickness, then there is no penetration [5]. From the revolution frequency (5.07 MHz), taken as the worst case, and the numbers in Table 1, $\delta_{wall} = 0.19$ mm and $\sqrt{bd} = 3.6$ mm. So the following nonpenetrating expression [5] is appropriate,

$$\Delta\nu_{coh}^{x,y} = -\frac{Nr_0}{\pi\beta^2\gamma}\bar{\beta}_{x,y}\left[\frac{1-\beta^2}{B_f}\frac{\xi_1}{b^2}+\frac{\varepsilon_1}{b^2}+\frac{\varepsilon_2}{g^2}\right], \qquad (1)$$

where beam neutralization is assumed to be zero, B_f is the bunching factor, $\bar{\beta}_{x,y}$ is the average betatron amplitude equal to $R/\nu_{x0,y,0}$ and ξ_1, ε_1 and ε_2 are the Laslett parameters for the cylindrical geometry. For a circular beam pipe, $\xi_1 = ½$, $\varepsilon_1 = 0$. ε_2 is not defined for cylindrical geometry, but it is a term to correct for the effect of iron pole faces which don't exist in UMER; so the term can be ignored. Collecting terms and noting that the number of particles divided by the bunching factor is proportional to the peak current, I_{bm}, equation 2) can be rewritten as follows:

$$\Delta\nu_{coh}^{x,y} = -\left[\frac{1}{(\beta\gamma)^3}\frac{1}{I_o}\left(\frac{R}{b}\right)^2\right]\frac{1}{\nu_{xo,yo}}I_{bm} \qquad (2)$$

The terms in brackets are constants of the experiment that when divided by the appropriate value of ν_{ox} or ν_{oy} gives the slope of the curve relating beam current to the coherent tune shift. These numerical values are given under "predicted m" in Table 2.

MEASUREMENTS

Data is taken at five nominal beam currents with five scans for tune measured at each current setting. Since the injected beam current is constant over the length of the beam, the current amplitude in the middle is the "peak" current. Tune is measured at each of the 14 installed BPM's by measuring displacement of a 70ns segment in the center of the beam bunch on four consecutive turns and computing the equilibrium orbit and tune using equations derived from Courant Snyder theory [3,6,7]. The four turn expression for determining fractional tune is

$$f\nu_x = \frac{1}{2\pi}\cos^{-1}\left[\frac{x_n - x_{n+1} + x_{n+2} - x_{n+3}}{2\left(x_{n+2} - x_{n+3}\right)}\right] \qquad (3)$$

and similarly for y.

Because there are 14 BPMs in use for each measurement, there should in principal be 14 identical values of tune, but as is clear from equation (3), the measurements are very sensitive to noise, particularly at low beam currents and when the second and third turn displacements are close in value. Consequently, the number of valid BPM measurements is only ~ 8 to 11 for each scan. The averages of tune and the standard deviations are computed for each scan and the averages of the results of the five scans are averaged again. This turns out to be a reasonably large statistical sample at each beam current and gives very consistent measurements, even with data taken weeks apart. The data is plotted in Figures 1 and 2, as are the predictions described in section 3 above. The lines in the figures labelled "LS Fit" are computed using the slopes and zero current tune values obtained from a least squares fit to the measured data.

The process for getting the predicted slope is to fit the measured data and obtain the experimentally measured values of the line slope and the horizontal and vertical

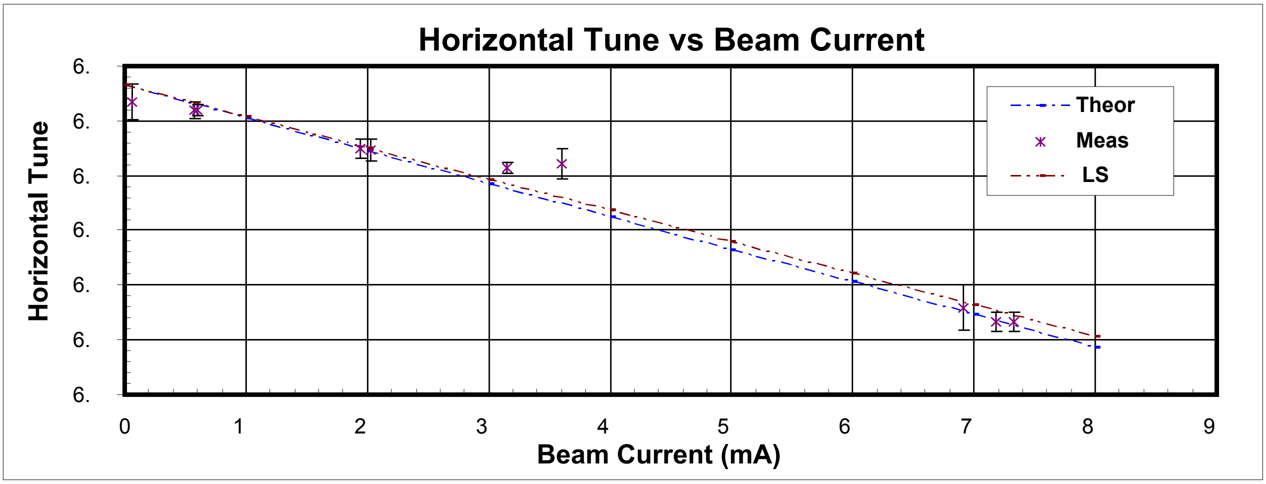

Figure 1: Plot of horizontal coherent tune versus beam current showing measured values, predictions using fitted slopes and zero current tune values, and theoretical predictions computed with equation 2.

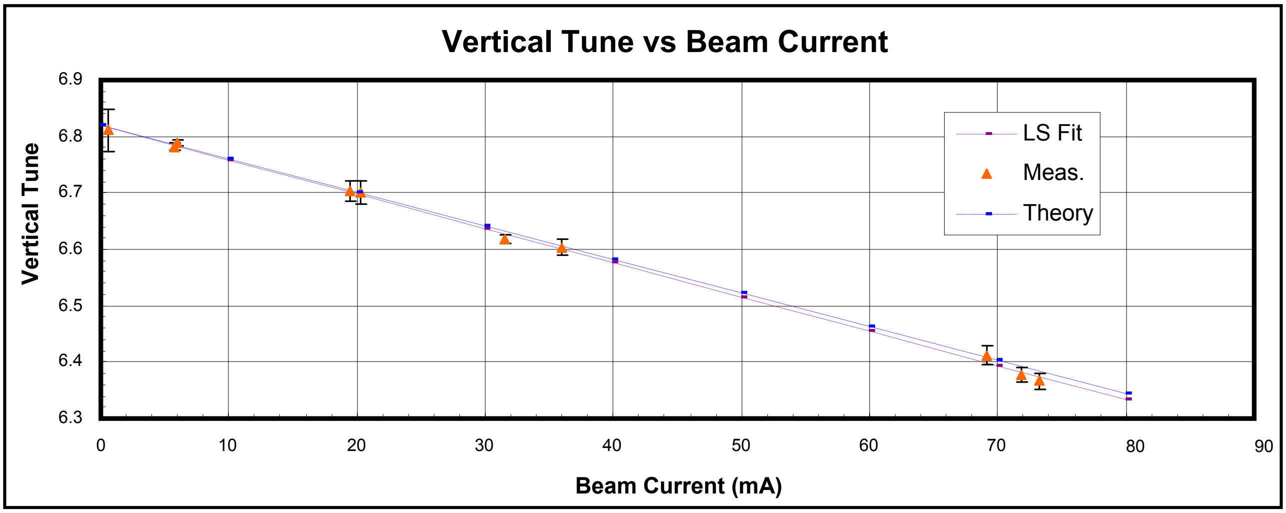

Figure 2: Plot of vertical coherent tune versus beam current showing measured values, predictions using fitted slopes and zero current tune values, and theoretical predictions computed with equation 2.

Table 2: Measured zero current horizontal tunes, measured slopes of coherent tune versus I_{mb}, and the predicted slopes from equation 2. The Δ(%) is the percent difference between measured and predicted slopes referred to the predicted slope.

	Measured ν_0	Measured m	Predicted m	Δ(%)
x	6.764 ± 0.014	-0.00572± 0.00033	-0.00599	4.5
y	6.819 ± 0.004	-0.00608 ± 0.00009	-0.00594	2.4

zero current tunes (Shown in Table 2). These are used in equation (2) to compute the horizontal and vertical predicted slopes which together with the measured zero point tunes are used to plot the "Theory" lines in Figures 1 and 2.

CONCLUSIONS

The coherent tune of rigid dipole motion has been measured at five beam currents: 0.6, 6, 20, 40, and 80 mA, a dynamic range of over 130 to 1 in beam current of highly space charge dominated beams. We have found that to the accuracy of the present measurements, the dependence of the coherent tune with beam current is linear. There were also concerns about the effect of the beams being off axis. The equilibrium orbits that were measured are surprisingly similar over all of the currents. But more relevant is the fact that the average of the equilibrium orbit displacements is less than 1mm both horizontally and vertically. This means that while some local excursions may be as larger as 4 or 5 mm, the very long beam averages out the effect of the image forces as though it is centred.

The work that has been presented is part of an ongoing program that will, as a next step extract the ring impedances, and then look for instability thresholds in the higher current beams with the longitudinal focusing activated [8].

REFERENCES

[1] R. Kishek et al, "The University of Maryland Electron Ring (UMER) Enters a New Regime of High Tune Shift Rings," Proceedings PAC 07, Albuquerque, NM, IEEE, NY, 2007, pp 820-824.

[2] M. Reiser, "Theory and Design of Charged Particle Beams," 2nd Edition; Wiley – VCH, 2008.

[3] Phillip J. Bryant and Kjell Johnson, "Circular Accelerators and Storage Rings," Cambridge University Press. 1993.

[4] L. J. Laslett, "On intensity limitations imposed by transverse space charge effects in circular particle accelerators, Proc. 1963 Summer Study on Storage Rings, Accelerators and Experimentation at Super - high Energies, BNL Report 7534, 324 – 367.

[5] B. W. Zotter, "Coherent Q – shift of a relativistic particle beam in a metallic vacuum chamber," CERN ISR – TH/72 – 8 (March 1972).

[6] Jean-Pierre Koutchuok, "Trajectory and Closed Orbit Correction;" Frontiers of Particle Beam Observations Diagnosis and Correction," edited by Mel Month and S. Turner, Lecture Notes in Physics 343, Springer-Verlag pp 46-64.

[7] D. F. Sutter, S. Bernal, et al, "Coherent phenomenon over a range of Beam intensities in the electron storage ring UMER;" Proceedings of PAC 09; Vancouver, B. C., IEEE, NY: 2009.

[8] B. Beaudoin et al, paper MOOD51, PAC 11.