# DEVELOPMENT OF A STEPWISE RAY-TRACING BASED ON-LINE MODEL AT AGS*

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*Abstract*

A model of the Alternating Gradient Synchrotron is being developed based on stepwise ray-tracing numerical tools. It provides a realistic representation of the lattice, and accounts for the two helical partial Siberian snake insertions. The aim is to make this stepwise ray-tracing based model an aid for the understanding of the AGS, in matter of both beam dynamics and polarization transmission.

## INTRODUCTION

A model of the Alternating Gradient Synchrotron lattice including the two siberian snakes [**?**] is being developed, based on the use of the stepwise ray-tracing code Zgoubi [**?**]. There are several reasons for opting for stepwise integration method, amongst the many possible ways of tracking in an accelerator: - a lattice based on combined function dipoles featuring quadrupole and sextupole components, fringe fields possibly, - presence of the snakes, with helical magnetic structures requiring dedicated modelling means, - interest of the method for its inherent accuracy in the modelling of magnetic fields, - including possible extensive (if not exclusive) use of measured or OPERA-type field maps in representing the main dipoles. On the other hand, the method has proven to be effective in difficult exercises, including highly non-linear ring optics [**?**] and spin dynamics in presence of snakes [**?**, **?**]. The ultimate goal in modelling the AGS is to work at getting the best understanding of the machine optics, which will help maximize the preservation of the polarization during the acceleration of the polarized protons for injection into RHIC. It is believed in particular that stepwise integration methods offer best possible accuracy on computation of spin motion in magnetic field models. A first Section presents the ingredients on which modelling of AGS lattice in the ray-tracing code Zgoubi is based. A second Section shows the effectiveness of the method by illustrating it via beam and spin dynamics. Comparisons with the MAD model of AGS are performed wherever useful.

## MODELLING OF THE AGS

### *Reference Line (OCO)*

A reference line, "OCO" (Optimum Closed Orbit), has been defined in the AGS, Fig. **??** [**?**]. The OCO coincides with the closed orbit in the straight sections between the

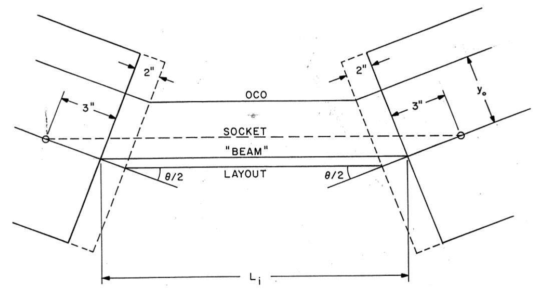


Figure 1: OCO line in the AGS, the reference optical axis.

Table 1: Angles, Equivalent Quadrupole Shift and Typical Strengths (Values at Injection, Here) Entering in the Definition of an "OCO Line" in Zgoubi.

| | $\theta/2$ (mrad) | $\Delta x$ (cm) | $K_1$ ($10^{-2}\mathrm{m}^{-2}$) | $K_2$ ($10^{-3}\mathrm{m}^{-3}$) | arc length (m) |
|---|---|---|---|---|---|
| BF | 11.751 | 23.17 | 5.0828 | 5.0563 | 2.006646 |
| CD | 13.982 | -24.07 | -5.0760 | 4.3783 | 2.387677 |
| AF | " | 23.00 | 5.0689 | 4.3617 | 2.387678 |
| BD | 11.751 | -23.93 | -5.0793 | 5.0017 | 2.006595 |
| CF | 13.982 | 23.00 | 5.0757 | 4.4226 | 2.387678 |
| AD | " | -24.07 | -5.0702 | 4.3048 | 2.387677 |

240 main dipoles. In the dipoles themselves, the OCO coincides with the chord of the arc of trajectory, it is localized by its distance to the socket line (the survey line). OCO coincides with the optical axis of all elements placed in the drifts, as tuning quads, sextupoles, control instrumentation, etc. The model of AGS dipoles and lattice in Zgoubi sticks to these principles, as shown in the next Section: the closed orbit in Zgoubi coincides with the OCO line, all lenses introduced further in the lattice are by default (i.e., in the absence of explicit request for a different alignment) centered on that line.

### *AGS Dipoles*

There are 6 different types of straight-axis combined function dipoles in the AGS ring (Tab. **??**), for a total of 240, 20 per superperiod, 12 superperiods. The quadrupole and sextupole strengths in these 6 magnets are represented by as many momentum-dependent polynomial, $K_1(p)$, $K_2(p)$, taken from the MAD model of AGS, installed in Zgoubi source code as part of the definition of the AGS dipole. $K_1(p)$, $K_2(p)$ account for the inhomogeneous variation of the field in the gap during the ramp. A classical multipole scalar potential model is used to derive the field components, namely

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- dipole:
  $V_1(s,x,y) = D(s)y - \frac{D''(s)}{8}(x^2+y^2)y + \frac{D''''(s)}{192}(x^2+y^2)^2y - ..., \quad D(0) = B_0$
- quadrupole:
  $V_2(s,x,y) = G(s)xy - \frac{G''(s)}{12}(x^2+y^2)xy + \frac{G''''(s)}{384}(x^2+y^2)^2xy - ..., \quad G(0) = B_1/R$
- sextupole:
  $V_3(s,x,y) = \frac{H(s)}{3}(3x^2-y^2)y - \frac{H''(s)}{48}(3x^4+2x^2y^2-y^4)y + ..., \quad H(0) = B_2/R^2$

As long as fringe fields are not involved, the model simplifies to

$$V_1 = Dy, \;\; V_2 = Gxy, \;\; V_3 = \frac{H}{3}(3x^2-y^2)y$$

The model assumes the dipole component $B_0$ to be due to beam positioning (coinciding with the OCO) at distance $\Delta x \neq 0$ from the multipole axis (Fig. **??**), namely, $B_0 = B_1\Delta x/R + B_2(\Delta x)^2/R^2$.

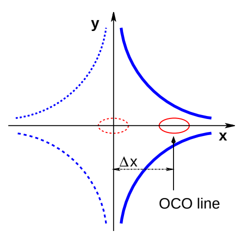


Figure 2: Off-centering of the beam (onto the OCO) in the multipole model.

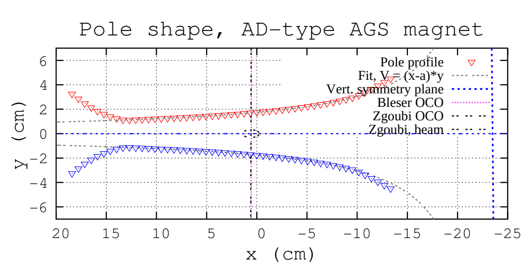


Figure 3: Pole profile in the OPERA model of AGS AF-type dipole. Machine center is to the left, virtual multipole vertical symmetry plane is to the right at $x \approx -23$ cm. (A second profile is of D-type, reversed gap shape wrt. the beam axis, it has been submitted to similar treatment.)

**OCO off-centering** Multipole shifts $\Delta x$ in Tab. **??** have been determined from pole profile geometry (Fig. **??**): the virtual multipole axis is obtained by matching that profile by a potential of the form $V(x,y) = (x - \Delta x)\,y$ (neglecting the sextupole component, of little effect). The origin of the referential for $V(x,y)$ is taken at the socket line which is located at $x = 0$ in Fig. **??**. This results in good agreement between the OCO as calculated by Zgoubi, and as calculated by Bleser [**?**]. The difference between Zgoubi's OCO and Bleser's is -0.145 mm, outward, in F-type dipoles (Fig. 3), and -0.031 mm, outward, in D-type dipoles.

## RING PARAMETERS

Ring data resulting from this model are shown in Table **??**. Beam is rotating counterclockwise, yielding negative dispersion ($x$ axis is oriented toward ring center in Zgoubi frame). Super-period geometry is sketched in Fig. **??**.

Table 2: AGS Parameters, Bare Lattice, No Snakes, $G\gamma = 45.5$, from MAD and Zgoubi Runs, Hard-Edged Magnets

| | MAD | Zgoubi |
|---|---|---|
| Orbit length (m) | 807.0913 | 807.0914 |
| Qx | 8.68727 | [8].67421 |
| Qy | 8.73440 | [8].71674 |
| Q'x | -23.287 | -23.305 |
| Q'y | 3.9347 | 4.0107 |
| $\alpha$ | 0.01408 | 0.01412 |
| $\sqrt{1/\alpha}$ | 8.42863 | 8.4168 |
| *Periodic functions at "Begin SuperA":* | | |
| $\beta_x,\ \beta_y$ (m) | 19.77, 11.78 | 19.80, 11.79 |
| $\alpha_x,\ \alpha_y$ | -1.57, 1.03 | -1.57, 1.03 |
| $D_x, D'_x$ (m,rad) | -2.06, -0.147 | -2.06, -0.148 |
| $x_{co}, x'_{co}$ (mm, mrad) | 0, 0 | 0, 0 |

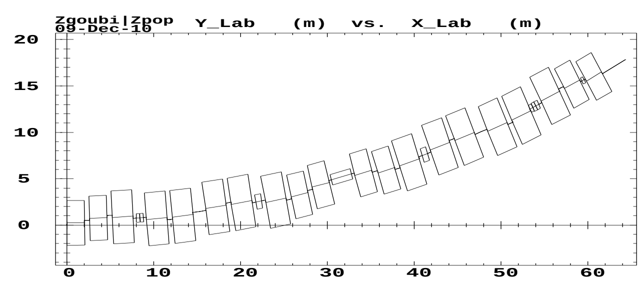


Figure 4: AGS super-period in Zgoubi. The origin of the super-period is at (0,0) on the "Zgoubi OCO line".

### *Including Snakes*

Mid-plane field maps of the snakes are used, they are treated using the "MAP2D" procedure in Zgoubi [**?**], Fig. **??** illustrates their typical behavior, note the off-centered orbit, necessary for the snake to fulfill its role as spin rotator. Machine parameters obtained from ray-tracing are given in Tab. **??** and Fig. **??**. Here, lattice is tuned using the 12 QHFH and 12 QHFV quadrupoles, magnet settings are taken from the AGS operation "ramp snap" records "1553, 31 Jan. 2011, timing 900 ms". Agreement with MAD is not as tight as it was in the bare lattice case, Tab. **??**. Snakes introduce some horizontal and vertical

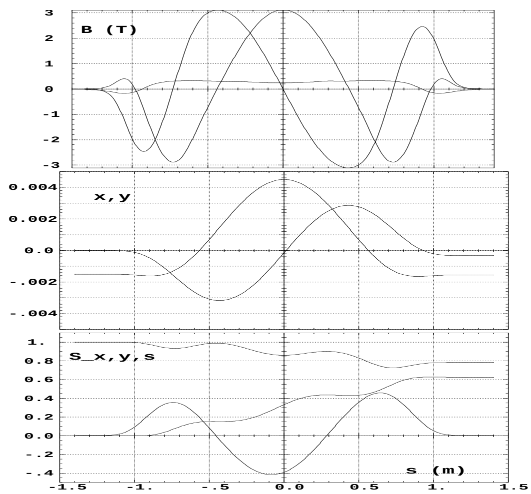

Figure 5: From top to bottom: field component, x and y coordinates, and spin components, at traversal of the cryogenic snake.

Table 3: AGS Parameters, Lattice Includes Snakes, $G\gamma = 45.5$. MAD Data, Based on the Same Magnet Settings, as Drawn from the AGS Operation "Ramp Snap", Are Given for Comparison.

| | MAD | Zgoubi |
|---|---|---|
| Orbit length (m) | 807.0913 | 807.0898 |
| Qx | 8.6890 | [8].6199 |
| Qy | 8.9142 | [8].9482 |
| Q'x | -8.69 | -8.60 |
| Q'y | 4.66 | 4.31 |
| $\alpha$ | 0.01408 | 0.01431 |
| $\sqrt{1/\alpha}$ | 8.4281 | 8.3661 |
| *Periodic functions at "Begin SuperA":* | | |
| $\beta_x,\ \beta_y$ (m) | 19.9, 10.6 | 20.3, 10.7 |
| $\alpha_x,\ \alpha_y$ | -1.56, 0.91 | -1.58, 0.91 |
| $D_x, D'_x$ (m,rad) | -2.02, -0.13 | -2.05, -0.13 |
| $D_y, D'_y$ (mm,mrad) | 1.6, 0.03 | -15, -14 |

closed orbit whereas MADs' are null by definition, as well as about half a meter vertical dispersion excursion whereas MAD model's $D_{y,max} \approx 5$ mm. Note that (i) the MAD model uses matrices to represent the snakes, (ii) the orbit bump that the cold snake introduces (cf. Fig. **??**-middle) is not compensated in Zgoubi (nor is it in reality on the high energy plateau).

## OBJECTIVES

The model so developed and tested is now being installed in the AGS command-control environment for further on-line modelling developments. It will take magnet data from the "ramp snap" (snaps of magnet settings during the cycle). A series of software tools are being developed to allow functionalities as fast producing tunes along the ramp, optical functions at arbitrary timings, etc., as well as for interfacing to a model viewer.

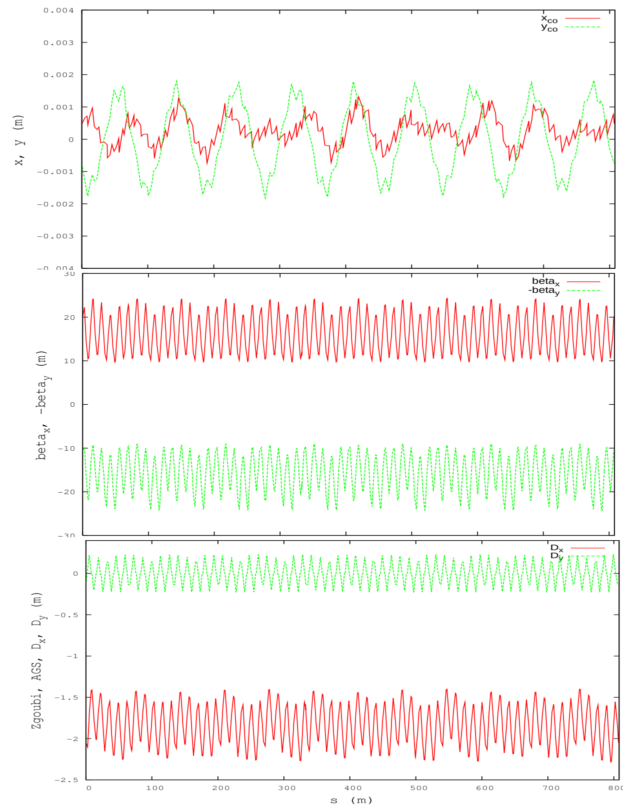

Figure 6: From top to bottom: H and V closed orbits, optical functions and dispersion.

## COMMENTS

Worth stressing, experience has long shown that relative lengthiness of stepwise integration compared to other possible methods (not necessarily the case, though), is not a concern: most of the time single (or few) particle tracking is needed, whereas relevant long term or multiparticle tracking runs can use multi-processor systems and in most problems result in CPU minutes time scales.

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