SIMULATIONS AND CALCULATIONS OF CAVITY-TO-CAVITY COUPLING FOR ELLIPTICAL SCRF CAVITIES IN ESS

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Abstract

The proton linac of the European Spallation Source (ESS) will rely on two families of superconducting cavities for the medium and high beta regions. Presented here are simulations of various cavity designs for different betas. The simulations are performed using the ACE3P codes developed at SLAC National Accelerator Laboratory, and the simulated eigenmode and R/Q spectrum will be shown for each design. Dangerous modes are identified. Of particular importance is the investigations of multiple cavity (cryomodule) configurations. From this, the simulated cavity-to-cavity coupling within a cryomodule is extracted. A theoretical model of this coupling based on coupled potential wells in quantum mechanics has also been developed, and a comparison made with the results of the simulation.

INTRODUCTION

The European Spallation Source (ESS) [1] is a high current proton Linac to be built in Lund, Sweden. The Linac is designed to deliver 5 MW of power to the target at an energy of 2500 MeV, with a nominal current of 50 mA. The Linac will consist of two families of elliptical superconducting cavities, a low and high beta design whose provisional parameters are shown in Table 1. Both are designed to resonate at 704MHz.

Table 1: Provisional RF parameters of elliptical cavities [2]

Structure	Energy MeV	Geometric beta β_G	Length m
Low β	590	0.65	57
High β	2500	0.86	215

The high beam power of the machine and requirement of very small losses mean that the distribution of Higher Order Modes (HOMs) in the cavities need to be considered in their design.

CAVITY DESIGN

The models for the elliptical RF cavities are still in the conceptual design phase however preliminary designs for 704MHz high power cavities from [3] are used in order to investigate cavity-to-cavity coupling. Two geometric betas (0.65, 0.83) were considered for these studies. Although these are designs are not identical to the parameters given in Table 1, they will provide a good approximation for the cavity-to-cavity coupling. An example model of the $\beta_G = 0.83$ cavity used for simulations is shown in Figure 1. For simplicity, the fundamental power and HOM couplers have been omitted from simulations. R/Q spectra are shown in Figure 2.

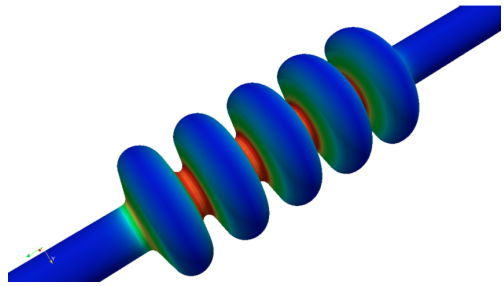

Figure 1: Example model of the $\beta_G = 0.83$ cavity used for simulations.

COUPLED MODES

For a single cavity, each cell may be considered as a resonant cavity which is weakly coupled to neighbouring cells. Therefore, the system is well described by a coupled oscillators model which predicts that each mode will split into a passband of modes with different resonant frequencies. Each mode inside this passband is distinguished by a phase advance, which for $N+1$ cells will result in $N+1$ modes with a phase difference $i.\pi/N$, where i is an integer running from 0 to N. Each mode may be plotted on a dispersion diagram as described by the dispersion relation,

$$\omega_\theta^2 = \omega_{\frac{\pi}{2}}^2 (1 - k\cos\theta) \quad (1)$$

where θ is the phase advance, $\omega_{\frac{\pi}{2}}$ is the frequency of the

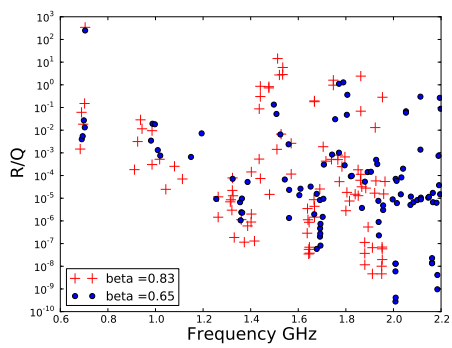

Figure 2: Calculated R/Q spectrum for the $\beta_G = 0.65$ and $\beta_G = 0.83$ cavity designs.

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mode in the centre of the dispersion plot with a phase advance of $pi/2$ and k is the coupling constant. Thus, the coupling is defined by the frequency spread of the modes and can represented by,

$$k = \frac{\omega_\pi^2 - \omega_0^2}{\omega_\pi^2 + \omega_0^2} \quad (2)$$

It is useful to define the coupling in a terms of the frequency of the 'zero' and 'pi' mode as they have the lowest and highest frequency in a passband.

COUPLED CAVITIES

For the case of cavities in a cryomodule, the interconnect region between the cavities has a cut-off frequency too high to allow the fundamental modes and other high R/Q modes to propagate. These modes decay exponentially as an evanescent field and may possibly couple to an adjacent cavity.

In a similar fashion to the previous section, cavities in a cryomodule may also be described by that of coupled oscillators where each cavity mode splits into a passband corresponding to the number of coupled cavities in the cryomodule. The coupling between these cavities can once again be described by (2).

SIMULATIONS

The simulations were performed on the Hopper machine at NERSC [4] using Omega3P [5]. These simulations were performed on cryomodules containing four cavities to minimise the computing resources needed.

The mesh used for a four cavity cryomodule consisted of ∼1.5 million second order mesh points. The magnitude of coupling between the cavities is expected to be relatively small compared to that of cell to cell coupling inside a cavity, therefore, this very high number of mesh points are needed despite the second order nature of the mesh.

The interconnect regions will have the strongest effect on the coupling due to their small radius. The length and radius of the interconnect regions are shown in Table 2 along with the cut-off frequencies for the TM_{01} and TE_{11} modes.

The results of the simulations are shown in Figure 3. As expected, coupling of the modes increase as their frequency become closer to cut-off for TE like modes. The sudden drop in coupling around 1.3GHz is thought to be due to these modes being TM like however it is uncertain why these show less coupling then the fundamental passband which is much further from cut-off.

Table 2: Interconnet region parameters

Structure	Length cm	Radius cm	TM_{01} f_c GHz	TE_{11} f_c GHz
Low β	32	5.65	2.03	1.56
High β	32	7.0	1.64	1.26

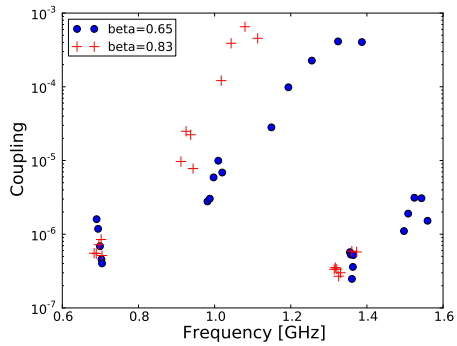

Figure 3: Coupling for the $\beta_G = 0.65$ and $\beta_G = 0.83$ cavity designs.

THEORETICAL MODEL

The coupling calculations shown in the previous section provide good accuracy however are very expensive in terms of computing resource given than one simulation requires $\sim$ 4000 CPU hours. A theoretical model has been developed in an attempt to provide a good approximation to the coupling using much less resources.

Inside the cavity, a particular mode will oscillate at its resonant frequency, however once this mode propagates into an interconnect region it will decay exponentially as an evanescent field away from the cavity. This formalism is mathematically identical to the situation of a Quantum Mechanical particle in a finite potential well. In the case of multiple finite wells, the energy levels of a system split when the wells become coupled which is analogous to coupled oscillator model as described earlier. Therefore, in order to predict the cavity-to-cavity coupling, each cavity was modelled as a potential well. From this a model was built based upon [6] which was originally used to calculate the eigenvalues for bound states of particles in multiple square well potentials.

Consider a single cavity. The mode oscillation in the cavity and in an interconnect can be described by,

$$\psi_j(z) = A_j e^{(k_j z)} + B_j e^{(-k_j z)} \quad (3)$$

where

$$k_j = \left[(p_{nm}/a)^2 - (\omega/c)^2\right]^{1/2} \quad (4)$$

where the index j represents each region and p_{nm} is the m_{th} root of the n_{th} order Bessel function. When the mode frequency is below cut-off, k_j is complex resulting in a exponential decay. For a single cavity, there is one region for the cavity and two regions for the interconnect either side of the cavity. Across the three regions, both $\psi(z)$ and $\psi'(z)$ must be continuous, therefore at each boundary between two regions the following can be used

$$^jM_j \begin{pmatrix} A_j \\ B_j \end{pmatrix} = ^j M_{j+1} \begin{pmatrix} A_{j+1} \\ B_{j+1} \end{pmatrix} \quad (5)$$

where

$$ {}^{m}M_j = \begin{pmatrix} e^{k_j z_m} & e^{-k_j z_m} \\ k_j e^{k_j z_m} & -k_j e^{-k_j z_m} \end{pmatrix} \quad (6) $$

The coordinate z_m represents the position of the interface between a cavity and an interconnect region. Therefore, for one cavity, using (5), two equations can be formed which can be used to find the resonant frequency depending on the boundary conditions used i.e. (A_0, B_0, A_2, B_2). If it is assumed that the mode is bound inside the cavity, B_0 can be set to zero to represent no outgoing wave and B_0 is set to one for simplicity. Using these initial conditions, to find the resonant frequency, one must find at what frequency there is no outgoing wave at the end of the system i.e. the roots of B_{2N}.

Using the following expression

$$ \left(\prod_{2N-1}^{0} [({}^{j}M_{j+1})^{-1} * {}^{j}M_j] \right) \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \begin{pmatrix} A_{2N} \\ B_{2N} \end{pmatrix} \quad (7) $$

it is possible to compute the resonant frequencies for coupled cavities by setting $A_0 = 0, B_0 = 1$ and then search for the roots of B_{2N}. A comparison of this method against simulations explained earlier using Omega3P are shown in Figure 4. In order to set a well to resonate at the desired frequency before it is coupled, the following is used to set the length of the well

$$ z_{cav} = \frac{2\tan^{-1}(\frac{k_1}{k_0})}{k_0} \quad (8) $$

where k_0 is the defined in the region of the interconnect and k_1 is the region of cavity. The theoretical model shows good agreement to an order of magnitude with Omega3P up until $\sim$ 1.1GHz where the models begin to differ. This is probably due to the over simplistic nature of the theoretical model used in that each cavity is treated as just one well when it is more physical to treat each cell of a cavity as a well. In this model, the resonant frequency is defined by the well length due to its two dimensional nature however in reality, a cavities frequency is determined by its transverse dimensions. In its current formalism, the model does not allow the introduction of power couplers or HOM couplers due to their complex nature.

CONCLUSIONS

Simulations of the cavity-to-cavity coupling can be used to help optimise the length and radius of interconnect. While Omega3P provides great accuracy, it is relatively expensive in terms of computing resources taking $\sim$ 4000 CPU hours and so a theoretical model has been presented to approximate the coupling which takes $\sim$ 5 minutes on a standard laptop.

The model gave a good order of magnitude approximation at low frequencies however, it still requires some modifications in order to improve its accuracy. Current ideas are to model each cell of a cavity as a potential well rather than just the whole cavity.

It may be possible to modify the code to incorporate more complex components in the future such as power couplers or HOM couplers however, as the complexity increases so will the computing time which becomes counter productive given that Omega3P can provide excellent accuracy with enough time.

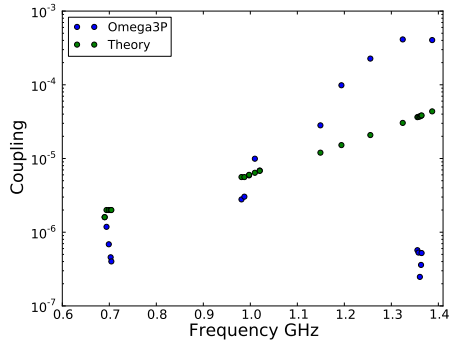

Figure 4: Coupling comparison between Omega3P and theoretical model for the $\beta_G = 0.65$.

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