

# ELECTROMAGNETIC FIELD MEASUREMENT OF FUNDAMENTAL AND HIGHER-ORDER MODES FOR 7-CELL CAVITY OF PETRA-II

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## Abstract

The booster synchrotron for NSLS-II will include a 7-cell PETRA cavity, which was manufactured for the PETRA-II project at DESY. The cavity fundamental frequency operates at 500 MHz. In order to verify the impedances of the fundamental and higher-order modes (HOM), which were calculated by computer code, we measured the magnitude of the electromagnetic field of the fundamental acceleration mode and HOM using the bead-pull method. To keep the cavity body temperature constant, we used a chiller system to supply cooling water at 20 degrees C. The bead-pull measurement was automated with a computer. We encountered some issues during the measurement process due to the difficulty in measuring the electromagnetic field magnitude in a multi-cell cavity. We describe the method and apparatus for the field measurement, and the obtained results.

## INTRODUCTION

In order to know shunt impedances for fundamental acceleration mode and higher-order modes (HOM), three dimensional computer simulation codes have been developed, and have become quite powerful and popular tools to know frequencies and shunt impedances. In NSLS-II project, to accelerate beam with 200 MeV from a linac up to 3 GeV, a 7-cell cavity is installed in booster synchrotron ring. The 7-cell cavity was manufactured at DESY for PETRA-II project. We obtained two 7-cell cavities from DESY, one of them will be placed in the booster ring. DESY offered us the data sheet of 7-cell cavity as to impedances, frequencies of HOM, which they calculated using the MAFIA code. Basing on the data given by DESY, we have investigated a method to suppress coupled-bunch instability problem due to 7-cell cavity [1]. In the process of investigation of HOM issue, we knew of the consistency between actual HOM frequencies and the data sheet. However, we had not checked the values of HOM impedance. In order to verify those of HOM, we have tried to carry out impedance measurement by using the bead-pull method. While the method is well known, it remains difficult to obtain correct values. In this article, we would like to review the bead-pull method and mention about some issues that we experienced to measure impedance.

## METHOD OF ELECTROMAGNETIC FIELD MEASUREMENT

First of all, let's review the bead-pull method to measure electromagnetic field strength. The method is well known, and we briefly review it here [2]. Electric

field  $E_0$  is detected by a bead with sphere shape and is presented by the following expression;

$$\frac{\omega_0^2 - \omega^2}{\omega_0^2} = \frac{2 \cdot \Delta f}{f_0} = 3E_0^2 \cdot \left(\frac{4\pi}{3} r_0^3\right) = 3E_0^2 \cdot V, \quad (1-1)$$

$$V \equiv \frac{4\pi}{3} r_0^3,$$

where  $\omega_0 = 2\pi f_0$ ;  $f_0$  is a targeted frequency,  $r_0$  is a radius of bead sphere. Thus we can get the following equation to obtain electric field strength;

$$E_0^2 = \frac{2}{3} \cdot \frac{1}{V} \cdot \left(\frac{\Delta f}{f_0}\right). \quad (1-2)$$

To know electric field strength, we use a bead made of an insulator. In order to obtain more correct data of the electric field, we use a correction factor  $(\epsilon+2)/(\epsilon-1)$  based on dielectric constant  $\epsilon$  [2]. In order to measure the magnetic fields, a metal bead is sensitive to both electric and magnetic fields. If only magnetic field exists, the magnitude of magnetic field  $H_0$  obtained by a metal sphere is presented by [2],

$$\frac{\omega_0^2 - \omega^2}{\omega_0^2} = -\frac{3}{2} H_0^2 \cdot \left(\frac{4\pi}{3} r_0^3\right), \quad (1-3)$$

$$H_0^2 = -\frac{4}{3} \cdot \frac{1}{V} \cdot \left(\frac{\Delta f}{f_0}\right). \quad (1-4)$$

If both electric and magnetic fields are presented, we can apply superposition principle for (1-2) and (1-4), and following formula is given by [2],

$$\frac{\omega_0^2 - \omega^2}{\omega_0^2} = 3V \left(E_0^2 - \frac{1}{2} H_0^2\right). \quad (1-5)$$

## TRANSVERSE COUPLING IMPEDANCE

Transverse shunt impedance  $R_{\perp}$  is expressed in the form [3]:

$$\frac{R_{\perp}}{Q_0} = Z_0 \cdot V_{\perp} \cdot \frac{V_{\parallel}}{k} = Z_0 \cdot (|I_{\perp}|)^2, \quad (1-6)$$

$Q_0$ : Q-value of a resonance

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}} = 376.73[\Omega]$$

$$k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}, \lambda: \text{wavelength.}$$

We used the relation of Panofsky-Wenzel theorem,

$$V_{\perp} = \frac{V_{\parallel}}{k}. \quad (1-7)$$

$V_{\perp}$  transverse voltage and  $V_{\parallel}$  longitudinal voltage are defined by following expressions;

$$V_{\perp} = j \int_{-d}^d E_0 \cdot e^{j kz} dz - \int_{-d}^d H_0 \cdot e^{j kz} dz, \quad (1-8)$$

$$V_{\parallel} = - \int_{-d}^d E_{0z} \cdot e^{j kz} dz. \quad (1-9)$$

We define the beam direction by z-coordinate, y: vertical direction, x: horizontal direction, respectively.

There are two methods to obtain transverse impedance.

**Method (1):**

A metal or ceramic bead is set on the beam axis. In case of metal bead, electric and magnetic fields are detected. Thus the equation (1-8) is useful for calculation. On the other hand, when ceramic bead is adopted, it detects only electric field. The equation (1-9) is available. To calculate shunt impedance, the equation of (1-6) is available.

**Method (2):**

Set a bead at a position away from beam axis; y. Only magnitude of electric field is measured by using a ceramic sphere bead. In this case,  $V_{\parallel}$  is given by

$$V_{\parallel} = \frac{1}{y} \cdot \int_{-d}^d E_{0z}(x=0, y, z) \cdot e^{j kz} dz. \quad (1-10)$$

Using the equation (1-10) and input it into (1-7), we get the following equation;

$$\begin{aligned} V_{\perp} &= \frac{V_{\parallel}}{k} = \left[ \frac{1}{y} \cdot \int_{-d}^d E_{az}(x=0, y, z) \cdot e^{j kz} dz \right] \cdot \frac{1}{k} \\ &= \frac{1}{y} \cdot \frac{1}{k} \cdot \int_{-d}^d E_{az}(x=0, y, z) \cdot e^{j kz} dz. \end{aligned} \quad (1-11)$$

In order to calculate transverse coupling impedance, we input the equation of (1-11) into (1-6),

$$\frac{R_{\perp}}{Q_0} = Z_0 \cdot \left( \frac{1}{y} \right)^2 = Z_0 \cdot \left[ \frac{1}{y} \cdot \frac{1}{k} \cdot \int_{-d}^d E_{az}(x=0, y, z) \cdot e^{j kz} dz \right]^2. \quad (1-12)$$

Electric field has already given in (1-2), and the equation of (1-12) is presented by using electric field,

$$\frac{R_{\perp}}{Q_0} = Z_0 \cdot \left( \frac{1}{y \cdot k} \right)^2 \cdot \frac{2}{3} \cdot \frac{1}{V} \cdot \left( \frac{\epsilon + 2}{\epsilon - 1} \right) \cdot \left| \int_{-d}^d \left( \frac{\Delta f}{f} \right)^{1/2} \cdot \cos kz \cdot dz \right|^2. \quad (1-13)$$

To make a correction, we added the term with dielectric constant  $\epsilon$  [2]. It should be noticed that we should add this correction term to detect electric field strength. Thus we obtained two methods to measure transverse impedance of a cavity. One is to adopt Method (1): taking data and analyze it using (1-8) and (1-6). The other is to use the Method (2). In this case, bead position is shifted from the beam axis; y. Ceramic bead is generally used for the measurement.

**LONGITUDINAL COUPLING IMPEDANCE**

Longitudinal shunt impedance is defined by [3]

$$\frac{R_{\parallel}}{Q_0} = 2Z_0 \cdot \frac{|V_{\parallel}|^2}{k}. \quad (1-14)$$

$V_{\parallel}$  is given by

$$V_{\parallel} = \int_{-d}^d E_{0z}(z) \cdot e^{j kz} dz. \quad (1-15)$$

**SET-UP SCHEME FOR ELECTROMAGNETIC FIELD MEASUREMENT**

We set up a bead-pull system to measure electromagnetic field of 7-cell cavity in a laboratory. The length of 7-cell cavity from a flange on the beam axis to the opposite side flange is 2220 mm long and the diameter of cross section is about 440 mm. To keep the body temperature of 7-cell cavity constant, we covered it with insulator and supplied cooling water at a constant temperature from a water chiller system. From equations of (1-2) and (1-4), the strength of electromagnetic field in each position is reflected by the change of frequency. If cavity body temperature is shifted depending on the circumference temperature change, it causes the frequency shift. Therefore obtained result does not correctly indicate the strength of electromagnetic field. For this measurement, body temperature of cavity should be constant. We use both ceramic and metal beads depending on the requirement of field measurements. The schematic view is drawn in Fig. 1, where we don't show the chiller system. In the figure, a computer automates bead position and taking data from a network analyzer. Computer first gives the number of steps to the motor controller to shift the bead position, then reads the center of frequency from the network analyzer. The computer records the number of steps and the center of frequency of the network analyzer to a file. Since the input coupler as shown in the Fig.1 is designed to pass through only the fundamental frequency of 500 MHz, we had to connect the network analyzer to transfer or detect signals of HOM by using pick-up ports and set up S11 or S21 modes. In case of the measurement of fundamental acceleration mode, we can use the input coupler to detect and also transfer the signal using S11 mode.

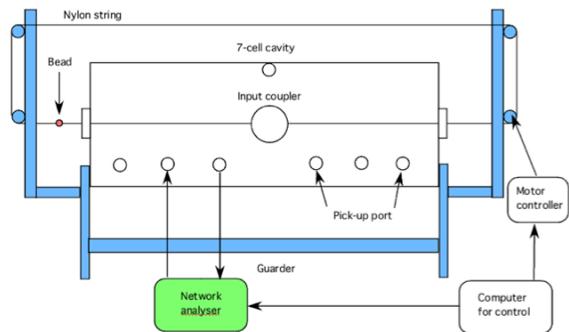


Figure 1: Electromagnetic field measurement using bead-pull method and network analyzer.

After installation of all tools, we first check the field strength distribution of the fundamental acceleration mode in each cell by using a bead. First of all, we transmitted the fundamental acceleration frequency from a network analyzer into a cavity and adjusted the cavity resonance frequency at 499.67 MHz by using two plunger type tuners, which are independently controlled. Then we checked the electric field strengths in each cell by using bead-pull method using the equation of (1-2). Since magnitudes of field strength in each cell were not equal, we made them almost equal levels by changing plunger length independently. After completion of all procedures, we started the electromagnetic field measurement.

### **FIELD MEASUREMENT OF TM010 MODE: FUNDAMENTAL ACCELERATION MODE**

In order to obtain the shunt impedance of the fundamental acceleration mode of TM010 (exact frequency: 499.67 MHz), we connected the network analyzer to the input coupler. A metal bead was set on the axis of beam and we carried out a frequency measurement of the fundamental mode. By using (1-15) and (1-14), we obtained the value of  $R/Q=1413$  by using S11 mode of network analyzer. A Q-value of 31826 was directly measured using the network analyzer. The data sheet provided by DESY gives  $R/Q=1278$  ( $R=46 \text{ M}\Omega$ ,  $Q=36000$ ). To obtain the value of shunt impedance, we can easily calculate and get  $R=45 \text{ M}\Omega$ . Both values of calculation and measurement show good consistency. To verify the method of measurement of S21 mode as shown in Fig.1, a ceramic bead with dielectric constant of 9.2 was set on the beam axis. The measured result was  $R/Q=1141 \times (\epsilon+2)/(\epsilon-1)=1558$  and obtained  $Q=29920$ . Thus we can deduce the  $R=47 \text{ M}\Omega$ . Those results show that different methods such as S11 or S21 as well as signal access of pick-up ports or input coupler port gave the same results. Thus it was proved that bead-pull method worked well to obtain R/Q for fundamental acceleration mode.

We tried the measurement of longitudinal HOM of TM011. This mode has the highest impedance value among HOM's [1]. However the signal level became too low to measure the frequency shift correctly, because this mode strongly couples with input coupler and results in a low impedance.

### **FIELD MEASUREMENT OF TM110 MODE: TRANSVERSE HIGHER-ORDER MODE**

We firstly identified the TM110 mode using a network analyzer. There were seven different modes. In order to detect electromagnetic field strength for transverse mode of TM110, we set a ceramic bead and located it away from the beam axis: we take the Method (II). For the measurement, we selected a frequency of 870.578 MHz because it indicated the highest Q-value. We tried the

measurements at different bead positions along the beam axis. Results obtained for R/Q were widely distributed between 751 and 4662. A Q-value of 28000 was obtained. The shunt impedance of TM110 obtained is between 21  $\text{M}\Omega/\text{m}$  and 130  $\text{M}\Omega/\text{m}$ . Data sheet provided by DESY says the value of  $R=28 \text{ M}\Omega/\text{m}$ . The actual obtained measurements gave quite large differences depending on the bead positions. In order to know the reason of large distribution of obtained data, we measured the magnitude of electric field distributions in each cell. As a result, we knew that different signal to noise (S/N) ratios for electric field distribution cause different values of R/Q. The better value of R/Q indicated the high S/N ratio of electric field distribution. Thus we changed ceramic bead with larger size or adopted an antenna to detect better signal. Those trials didn't improve the results drastically, but gave small improvements.

### **DISCUSSION AND SUMMARY**

For the fundamental acceleration mode TM010, the result obtained matched calculated values. And also the correction factor of  $(\epsilon+2)/(\epsilon-1)$  seems to be reasonable as long as we apply the formula to the acceleration mode. When we tried the measurement for transverse mode like TM110, results obtained were not consistent comparing with calculation. As we mentioned above, low S/N ratio causes large discrepancy from calculation results. To improve the S/N ratio, we tried to adopt a bead with larger size and also to use an antenna to get larger signal. We however could not improve the result remarkably. Only one trial, which changes bead position, gave better result. From our experiment, if one wants to get better result from experiment, we recommend taking field distribution maps in each cell first by changing bead positions away from the beam axis. If one would find clearer electromagnetic field map, one should fix the bead position away from beam axis and carry out the measurement of magnitude of field along beam axis. At the same time, if one had simulation codes to calculate electromagnetic field, one should do the simulation to know field strength in each cell. Basing on the simulation result, a bead should be set on the strongest electromagnetic field away from beam axis and measures magnitude of field along beam axis. We can say that it is not easy to obtain good result by using bead-pull method as to transverse mode of multi-cell cavity.

### **REFERENCES**

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