

# PROPERTIES OF LONGITUDINALLY UNIFORM BEAM WAVEGUIDES \*

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## Abstract

Beam waveguide (BWG) geometry with two longitudinally uniform concave reflectors can support quasi-optical transverse resonances of electromagnetic waves and longitudinal power transmission. The quasi-optical resonance in BWG can be treated as a Gaussian beam. The BWG are often known to have high Q-factors while operating in higher order modes. The latest interests on these beam waveguides are the application for microwave or millimeter wave undulators for synchrotron radiation. The general properties of the BWG are discussed with the field solutions and dispersion properties derived with elliptical beam waveguides approximation. Potential applications of BWG for supporting circularly polarized wave are discussed.

## INTRODUCTION

The BWG can be useful for high frequency applications if the reflector diameter is sufficiently larger than the wavelength [1][2]. The BWGs are typically analyzed by using either physical optics or geometrical optics techniques. Physical optics can have high accuracy at the expense of complexity and computation time but useful for analyzing a large BWG with roughly  $> 20 \lambda$ . However, due to diffraction, the fields in a smaller reflector BWG will be different from the fields found with geometrical optics. The microwave signal propagating through the BWG can be seen as a Hermite-Gaussian beam where the Gaussian beam treatment is relatively easy to implement with light computation time. The zero-order mode is preferred and normally used in the BWG transmission design.

The BWGs were considered for application in microwave undulators for generating high photon energy synchrotron radiation with the wave travelling opposite to the direction of the charged particles [3][4]. With this approach, the synchrotron radiation can be generated without using the DC magnetic fields and the undulator period may be electrically adjustable. The BWG also provides a larger waveguide cross section compared to standard hollow waveguides such as a rectangular waveguide. The BWG can be excited in either a travelling wave or a standing wave mode.

## FIELD SOLUTIONS AND DISPERSION

Figure 1 shows the cross-section of a BWG consists of a pair of longitudinally uniform concave mirrors. The wave function presented by [5] was useful to derive the solutions of electric and the magnetic fields between the

concave mirrors of the BWG. The transmission line properties of the resonant modes are found by using elliptic cylindrical waveguide surfaces, as shown in [6], to approximate the boundaries for the BWG. This allows using a constant  $u=u_0$  surface as the reflector surfaces in an elliptic cylindrical coordinate system for performing the field integrations. In this discussion, only the Gaussian beam in  $TE_{m0}$  to  $z$  and the  $TM_{m0}$  to  $z$  modes are considered.

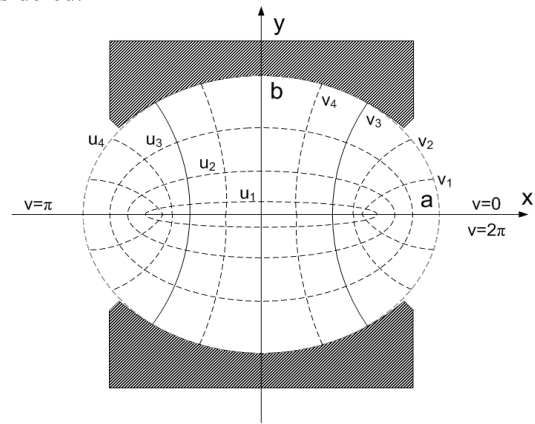


Figure 1: A pair of longitudinally uniform concave reflectors treated as an elliptic cylindrical beam waveguide. The distance between the reflectors  $d = 2b$ .

In applications such as microwave undulators that provide electron beam deflection, either a  $TE_{m0}$  (odd  $m$ ) or a  $TM_{m0}$  (even  $m$ ) mode can be used. These deflecting modes will have the transverse components of the electric and magnetic fields  $E_y$  and  $B_z$  at the center of the structure that result the net deflecting field of

$$B_u = \frac{E_y}{c} + B_z$$

The design parameters for the application would be the field strength, wavelength, beam waist size, etc. Waveguide losses are directly dependent on the field strength in a waveguide. The thermal and diffraction losses for BWGs were shown previously [5]. The maximum number of zero-order modes in a BWG can be found from

$$P = \left[ dk - \tan^{-1} \left( \frac{d}{\sqrt{2rd - d^2}} \right) \right] \frac{1}{\pi}$$

where  $d$  is the distance between the reflectors,  $r$  is the focal length of a concave reflector, and  $k$  is the free space wave number.

Figure 2 shows the deflecting field strengths of  $TE_{m0}$  modes at the beam axis center in a BWG with respect to the ratio  $a/d$  normalized to a power loss of 1W/m, where  $b$  is the major axis of the ellipse that approximates the concave mirror surfaces. The field strengths are shown for reflectors made of copper for room temperature operation.

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Figure 3 shows the strength of the deflecting fields of  $TM_{m0}$  modes for different ratios of  $b/d$  in a BWG with  $d = 10$  cm at 30 GHz operation. Compared to the TE modes, the total deflecting field intensity is lower in the TM modes. In TM modes, the field strength increases as the mode index  $m$  increases, while it decreases as the mode number increases in TE modes. Figure 4 shows the Q-factors and the waveguide wavelengths of  $TE_{m0}$  modes at the beam axis center in the BWG as a function of  $a/d$ . Figure 5 shows the Q-factors and the waveguide wavelengths of  $TM_{m0}$  modes with  $d = 10$  cm at 30 GHz.

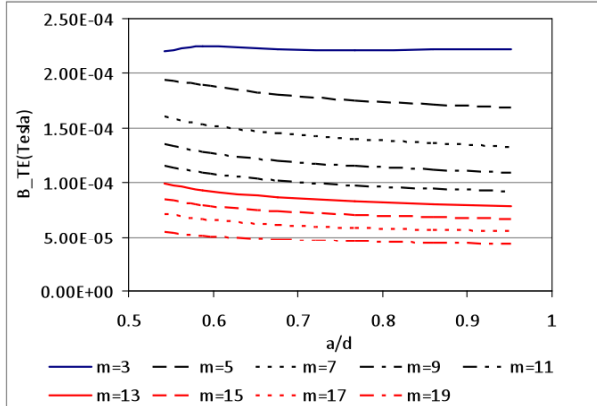


Figure 2: Total deflecting field strength of  $TE_{m0}$  modes as a function of  $a/d$  for  $d = 10$  cm,  $f = 30$ GHz.

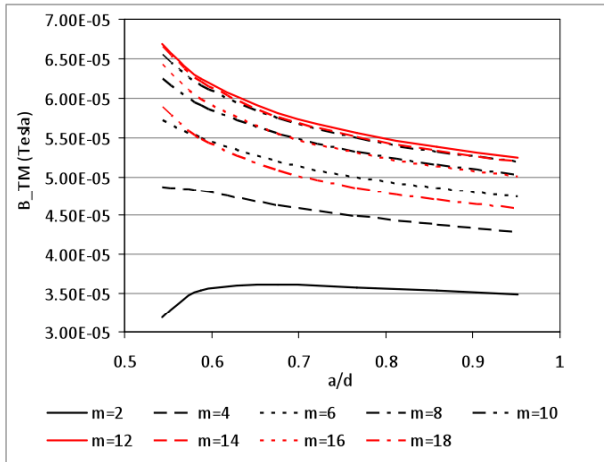


Figure 3: Total deflecting field strength of  $TM_{m0}$  modes as a function of  $b/d$  for  $d = 10$  cm,  $f = 30$ GHz.

Fig. 6 shows the Q-factors and field strength normalized to 1 watt/m of power loss for  $TE_{m0}$  modes in the BWG for the four lowest order modes as a function of  $a/d$ . It can be seen that the field strength increases as the reflector spacing  $d$  increases for a fixed mode index  $m$ . The field strength increases as the mode number decreases. The deflecting field has contributions from both the electric and the magnetic fields. In Figure 7, the beam waist sizes at the transversal center and the waveguide wavelengths of the BWG are shown. Note that the Q-factors of the beam waveguide modes are much greater than the hollow waveguide resonator operating with a lowest order dominant mode.

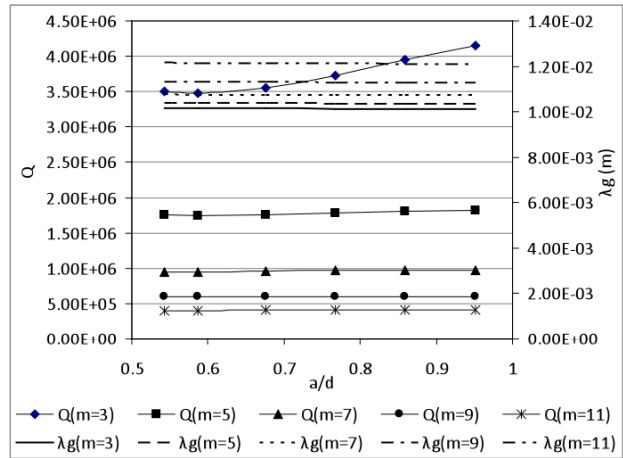


Figure 4: Q-factors and wavelengths as a function of  $a/d$  for representative BWG designs for  $TE_{m0}$  modes, shown only for 5 lower modes.  $f = 30$ GHz,  $d = 10$  cm.

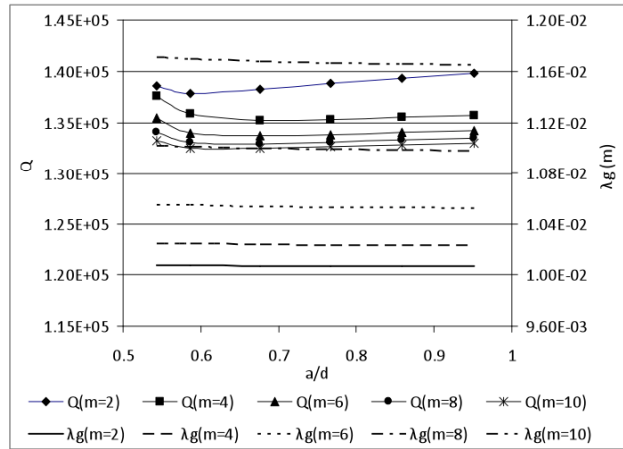


Figure 5: Q-factors and guide wavelengths as a function of  $a/d$  for representative BWG designs for  $TM_{m0}$  modes, shown only for 5 modes.  $f = 30$ GHz,  $d = 10$  cm.

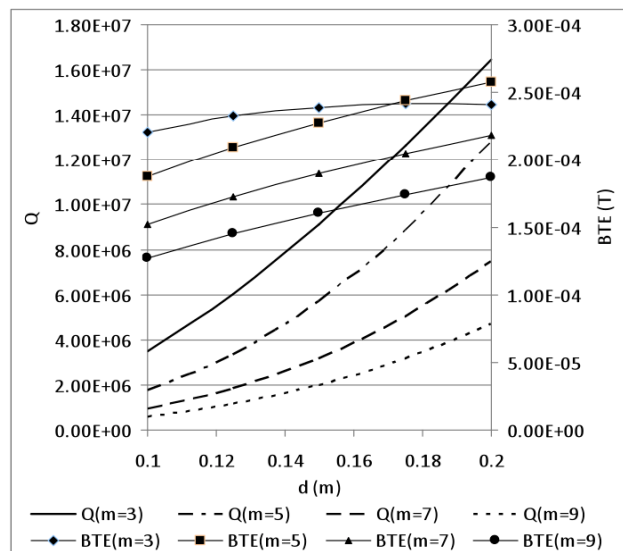


Figure 6: Q factors and deflecting field strengths for  $TE_{m0}$  modes as a function of  $d$ , shown only for 5 modes.  $f = 30$ GHz,  $a/d = 0.6$ .

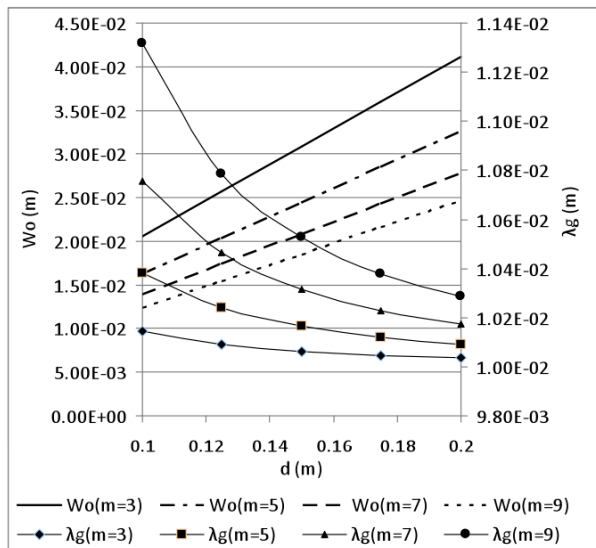


Figure 7: Beam waist sizes and guide wavelengths for  $TE_{m0}$  modes as a function of  $d$ ,  $f = 30\text{GHz}$ ,  $a/d=0.6$ .

## DISCUSSION

The TM modes in general exhibit higher conductor loss over the TE modes in hollow waveguides. The results shown in the BWG also show the lower Q and lower field strength with the TM modes. The ordinary hollow waveguide with completely closed conductor boundaries operates with a single dominant mode that is usually the lowest order mode. The BWG is naturally an overmoded structure that cannot operate with a single dominant mode like the ordinary hollow waveguide. Therefore, the selection and the excitation of the mode of operation need to be done carefully and precisely. Due to the low power loss of the waveguides, a longitudinal resonance needs to be established for efficient use of the generator power. Either a travelling wave resonant-ring or a standing wave resonance can be excited if the undulator wavelength, microwave frequency, and mode of operation are properly selected.

The quality factor is defined by  $Q = \omega W_t / P_d$  where  $W_t = W_e + W_m$  is the total stored energy and  $P_d$  is the dissipated power in the system. In a BWG, the integration of fields for the stored energy in the  $m$ -th order mode includes  $m$  harmonic nodes that are supported by the transverse resonance while the power dissipation exists only on the reflectors. Therefore, the Q-factor of the BWG becomes much greater than a cavity operating with a fundamental mode or the lowest order dominant mode. Unless the losses are uniformly distributed within the system, the Q factors of the BWG will not be directly comparable to the Q-factors of the resonators operating with their lowest order mode resonances (and eventually the comparison of field strengths). In other words, the factor  $R/Q$  of a BWG will be much smaller than a cavity operating with the lowest order resonant mode. However, the relationship between the Q and the frequency selectivity  $Q = f_o / \Delta f$  still stands in a resonator.

## Circular Polarization

Figure 8 shows a BWG system with two sets of beam waveguides that can be used to support a circularly polarized wave at the transverse center. If two pairs of reflectors are excited with two separate signals in the same mode but 90-degrees out of phase, the two orthogonal modes will excite circularly polarized wave fields at the center that could be used as a circularly polarizing undulator [7]. The circular polarization may be supported in either a standing wave operation with two short circuited ends or a travelling wave in a ring resonator system. In both cases, the total length of the resonant structure can determine the resonant frequency.

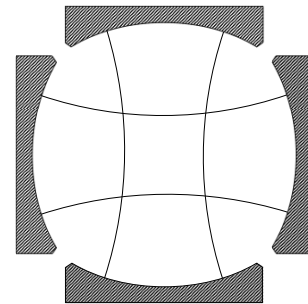


Figure 8: Two elliptical beam waveguides orthogonally arranged and excited 90-degrees out of phase to support circularly polarized wave.

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