ESTIMATES OF THE NUMBER OF FOIL HITS FOR CHARGE EXCHANGE INJECTION*

D. Raparia, Brookhaven National Laboratory, Upton, NY 11973.

Abstract

For high intensity circular proton machines, one of the major limitations is the charge exchange injection foil. The number of foil hits due to circulating beam may cause the foil to fail and cause radiation due to multiple nuclear scattering and energy straggling. This paper will describe methods to estimate these quantities without going through lengthy simulations.

INTRODUCTION

In the charge exchange injection generally used for proton accelerators, one start with H- ions from the source after reaching an appropriate energy by means of linear accelerator or cyclotrons, H^- ions are injected into the circular machine (either synchrotrons or accumulator ring) with a thin foil which can stripped two electron from the H- ions[1]. With this technique, it is possible to inject many (thousands by turns [2]) into restricted acceptance by increasing space charge density to inject. This method is widely used in the accelerator chain all around the world.

This method is used for the half century successfully, with increase in demand for more intense beam now foils are becoming a limiting factor. For Higher intensities one has to accumulate large number of turns the ring. When charge particle go through the foil, they suffer from physical processes like elastics, un-elastic & nuclear scattering, and energy straggling. These processes are harmful for both foil and beam itself [3]. The most fundamental quantity to estimate this damage is average number of foil traversal per unit area of the foil.

.

TURN FACTOR

For given number of particles (N_P) per pulse, the number of injected turns (N_T) proportional to particle velocity (βc), and inversely proportional to linac current (I_L) and ring circumference C

$$N_T = \left(\frac{1}{I_L}\right)\left(\frac{\beta c}{C}\right) N_P e \qquad (1)$$

Where c is the velocity of light and e is the charge of the particle. Since the particles in the leading edge of the linac pulse make N_T turns by the last particles go through the foils. Therefore on the average every particle will go through the foil at least $N_T/2$ times. The betatron motion in the ring will follow a circular path, if we choose the coordinate normalizations as

$$\begin{pmatrix} X \\ X' \end{pmatrix} = \begin{pmatrix} \sqrt{\beta_x} & 0 \\ \frac{\alpha_x}{\sqrt{\beta_x}} & \sqrt{\beta_x} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix} \qquad (2)$$

where α_x and β_x are the Courant –Snyder parameters at the injection point.

In this coordinate system the matched linac emittance ε and ring acceptance A will be circles with radius $\sqrt{\varepsilon} = r$ and $\sqrt{A} = R$. To ensure a minimum number of proton traversal through the foil during the injection period, it is necessary to inject H- ions at maximum of the proton betatron amplitude i.e.

$$\beta_x \Delta x' + \alpha_x \Delta x = 0 \qquad => \alpha_x = 0 \qquad (3)$$

Figure 1 show a matched beam from an injector into the ring acceptance when the equilibrium orbit is displaced and rotated $2\pi\nu_x$, one turn later. Since the injection time is much higher than the betatron frequency and if $m\nu_x/n \neq 1$, where m and n are integer, injected turns will be smoothly spread around the annulus (r_1, r2). The Turn factor TF is define as the ratio of shaded area to the annulus in figure 1

$$TF = \frac{\theta r_2^2 - r_1^2 \tan\theta}{\pi(r_2^2 - r_1^2)} \qquad where\ \theta = \cos^{-1}\left(\frac{r_1}{r_2}\right) \qquad (4)$$

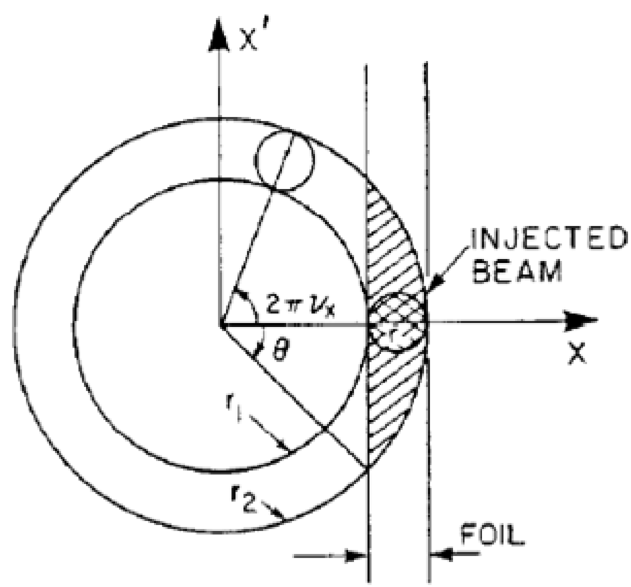

Figure 1: The injector beam stripped by a foil located at the edge of the ring acceptance precesses $2\pi\nu_x$ per turn. Continuous injection over many turns fills an annulus, the fraction interacting with foil in any turn is indicated by the shaded area. Normalized coordinate are used.

After N_T turns have been injected into the ring the first turn has traversed the foil 1+ (N_T -1)TF times while last turn has traversed the foil once. The average number of traversal being

$$\sim \frac{N_T\ TF}{2} \qquad (5)$$

For simplicity, if we have uniform space charge density after the painting processes turn factor will be the ratio of foil area which intercepts the acceptance of the ring to the acceptance of the ring, i.e. shaded area in figure 1 divided by the acceptance of the ring

$$TF = \frac{\theta\left(\frac{A}{\epsilon}\right) - \left(\sqrt{\frac{A}{\epsilon}} - 2\right)^2 tan\theta}{\pi \frac{A}{\epsilon}}$$

$$where\ \theta = \cos^{-1}\left(\frac{\sqrt{\frac{A}{\epsilon}}-2}{\sqrt{\frac{A}{\epsilon}}}\right) \qquad (6)$$

If the closed orbit collapsed rate equal to $(t/N_t\tau)^{1/2}$, the distribution will be uniform, if the rate is faster than distribution will be hallow and for slower rate, distribution will be peaked.

NUMBER OF FOIL TRAVERSAL

Let us consider at the injection point transverse cross-sectional area is Λ, and foil area which intersect Λ is a_f. Let us further assume the transverse density of injected at end of the injection is *uniform* over areaΛ. From the geometry the average number of foil traversal for N_T injected turn per particle is

$$h = \frac{1}{2}\frac{a_f}{\Lambda} N_T \qquad (7)$$

The factor of ½ is there because head of the linac pulse will make N_T turn and tail of the pulse only once.

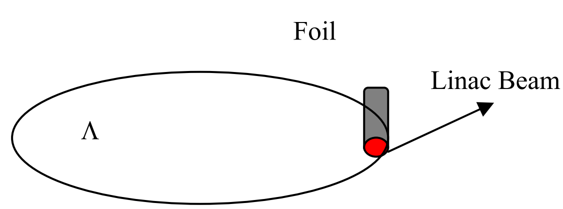

Figure 2: At foil, Λ area to be painted by linac beam in time $N_T\tau$ in N_T turn with linac beam of area a_b and intersection of foil area and ring area is a_f.

The number of foil traversal depends on the number of injected turns, acceptance of the ring and the foil size. To find the minimum number of foil traversal we will assume following

(1) Foil size ,a_f, is equal to the linac beam size,a_b, at the foil location (hence injection point) in the transverse plane, $a_f = a_b = a$.

(2) the H^- intensity of the linac (I_L) is constant over the injected period.

The injection time is $N_T\tau$, where τ is the revolution period of the ring. The number of particles in the ring at time t , $0 < t < N_T\tau$,

$$n(t) = I_L\, t \qquad 0 < t < N_T\tau \qquad (8)$$

$$I_R = \frac{N_p}{N_T\ \tau} \qquad (9)$$

Let P(t) be the area occupied (painted area) by the capture beam in the ring at injection point at time t , the instantaneous number of foil traversal is

$$h(t) = n(t)\frac{a\ dt}{P(t)\tau} = N_P\left(\frac{t}{N_T\tau}\right)\frac{a\ dt}{P(t)\tau} = \frac{N_P a t dt}{N_T\,\tau^2 P(t)} \qquad (10)$$

The average number of foil traversal is

$$h = \frac{1}{2N_p}\int_0^{N_T\tau} h(t)dt = \frac{a}{2N_t\tau^2}\int_0^{N_T\tau}\frac{tdt}{P(t)} \qquad (11)$$

Since the particle injected in the first turn will make N_T revolution and particle injected in the last turn will make only one traversal, therefore factor of 2 appear in denominator.

The transverse area for capture particles will depend on the painting scheme and will depend on the time. Let us assume that cross-sectional area varies with time as

$$P(t) = \Lambda\frac{t^{\frac{1}{\xi}}}{(N_T\tau)^{\frac{1}{\xi}}} \qquad (12)$$

Where ξ a parameter, when ξ=1, painted area will grow linearly with time and at end of injection ($t=N_t\tau$) , Painted area becomes equal to the ring transverse are t foil, Λ, and hence uniform density transverse charge density. Substituting this into the equation (11),

$$h = \frac{a\,(N_T)^{\frac{1}{\xi}-1}(\tau)^{\frac{1}{\xi}-2}}{2\Lambda}\int_0^{N_T\tau} t^{\left(1-\frac{1}{\xi}\right)}dt$$

$$= N_T\frac{a}{2\Lambda}\frac{1}{\left(2-\frac{1}{\xi}\right)} \qquad (13)$$

When parameter ξ=1 (uniform density) we get back the answer in the previous section i.e.

$$h = \frac{1}{2}N_T\frac{a}{\Lambda} \qquad (14)$$

The minimum number of foil traversal will when parameter ξ is very large number, in the limiting case the number of minimum foil traversal is given by

$$h_{min} = \frac{1}{4} N_T \frac{a}{\Lambda} \quad (15)$$

Where a is the transverse area of the linac beam.
Let us consider the matched case where linac beam is matched to the ring beta-function at the foil location.

$$a = \pi xy = \pi\sqrt{\beta_x \varepsilon_x \beta_y \varepsilon_y} \quad (16)$$

and

$$\Lambda = \pi XY = \pi\sqrt{\beta_x A_x \beta_y A_y} \quad (17)$$

Where β_x and β_y are the beta function at the foil and ε is the emittance of the linac beam and A is the acceptance if the beam. if $\varepsilon_x = \varepsilon_y$ and $A_x = A_y$ then the minimum number of traversal become

$$h_{min} = \frac{1}{4} N_T \frac{\varepsilon}{A} \quad (18)$$

Mismatch injection could be used to further reduce the number of foil traversal. The mismatched injection has to obey following conditions [3,4]

$$\frac{\alpha_i}{\beta_i} = \frac{\alpha_R}{\beta_R} = \frac{x_i'}{x_i} \ \ and \ \ \frac{\beta_i}{\beta_R} \geq \left(\frac{\epsilon}{A}\right)^{\frac{1}{3}} \quad (19)$$

Hence the minimum number of foil traversal for mismatched beam will be

$$h_{min} = \frac{1}{4} N_T \left(\frac{\varepsilon}{A}\right)^{\frac{4}{3}} \quad (20)$$

From equation (1), one gets

$$h_{min} = \frac{1}{4}\left(\frac{1}{I_L}\right)\left(\frac{\beta c}{C}\right) N_P e \left(\frac{\varepsilon}{A}\right)^{\frac{4}{3}} \quad (21)$$

This is an important result which estimates absolute minimum number of foil traversal for N_P particles in the ring, ring circumference, linac energy and linac current. If the number of traversal is unacceptable then one has to change a parameter which has least cost impact.

REFERENCES

[1] G. I. Budker and G. I. Dimov, Proc. Int. Conf. on High Energy accelerator, Dubna, 1963, p 1372

[2] D. Raparia et al, Proc. of. PAC 1985, Vancouver, Canada

[3] D. Raparia, M. Sc Report, University of Manitoba, 1984

[4] J. Beebe-Wang, Y.Y. Lee, D.Raparia and J.Wei, Pro. PAC 2001,