BEAM-BEAM LIMIT IN AN INTEGRABLE SYSTEM*

A. Valishev[#], S. Nagaitsev, FNAL, Batavia, IL 60510, USA
D. Shatilov, BINP, Novosibirsk, 630090, Russia
V. Danilov, ORNL, Oak Ridge, TN 37831, USA

Abstract

Round colliding beams have been proposed as a way to push the attainable beam-beam tune shift limit, and recent successful experiments at the VEPP-2000 collider at BINP demonstrated the viability of the concept. In a round-beam system the dynamical stability is improved by introducing an additional integral of motion, which effectively reduces the system from a two and a half dimensional to one and a half dimensional. In this report we discuss the possible further improvement through adding the second integral of motion and thus making the system fully integrable. We explore the ultimate beam-beam limit in such a system using numerical simulations taking into account various imperfections.

INTRODUCTION

The strong nonlinearity of transverse focusing force is inherent to beam-beam interaction and affects the stability of particle motion in the present circular colliders. The beam-beam kick localized in a short interaction length along the accelerator circumference represents a nonlinear and time-dependent excitation of the otherwise linear betatron motion in the rest of the machine. In such systems, a multitude of resonances lead to particle diffusion and to chaotic-bounded motion, which limits the attainable beam brightness and collider performance. The typical maximum value of beam-beam parameter ξ in such 'conventional' machines is about 0.05÷0.06 for electron colliders, and up to 0.03 for hadron colliders due to the absence of synchrotron radiation damping.

In an attempt to mitigate the beam-beam effect, Krishnagopal and Siemann [1] analyzed the effect of the strong bunch length σ_z on the magnitude of resonances with the use of canonical perturbation theory. They concluded that '…the finite longitudinal extent of the beam-beam interaction results in averaging of the betatron phase over the collision, which mitigates the destructive effects of resonances'. Conversely, the synchrotron oscillations of the test particle lead to a greater depth of modulation and enhance the effect of resonances. Since the lengths of the two colliding bunches are usually equal, an optimal length should exist at which the two effects are compensating one another. The ratio of the bunch length to the beta-function of the order of one was determined as viable.

In further work along this direction Y. Alexahin [2] and T.Sen [3] studied the effect of phase averaging on the Tevatron performance, and also predicted the optimal $\sigma_z/b^* \approx 1$.

The growing interest to integrable systems in mathematical physics motivated Danilov and Perevedentsev to investigate the application of these systems to accelerators and colliding beams in particular [4, 5]. In a round colliding beams system, the beam-beam interactions as well as the transport through accelerator lattice possess axial symmetry. This results in the existence of an additional integral of motion, the longitudinal component of the angular momentum. An important consequence of the additional integral of motion is the elimination of transverse coupling resonances.

In Ref. [5], a round colliding beam system is proposed, which has two integrals of motion. The full integrability is achieved through proper shaping of the longitudinal bunch density, which makes the system Hamiltonian time-independent. The line charge density λ in this case must be inversely proportional to the beta-function β: $\lambda(2s)=C/\beta(s)$. In the case of no external focusing in the interaction region, the beta-function is a quadratic function of the azimuth s, $\beta(s)= \beta^*+s^2/\beta^*$, and the proper 'ideal' longitudinal distribution is

$$\lambda(s) = \frac{C}{1+(s/2\beta^*)^2}\ .$$

An important property of the fully integrable system is that the exact shape of transverse density distribution is irrelevant and does not affect the stability. However, the complete integrability is valid only for the center particles in the weak bunch, and synchrotron oscillations introduce modulation that disturbs the system. Also, the longitudinal distribution in a typical collider is Gaussian. Numerous numerical simulations have been performed to explore the stability of the round colliding beam systems to these imperfections (see e.g. [6, 7]). More importantly, the experimental implementation of the concept at the VEPP-2000 collider (BINP, Novosibirsk), where the record-high value of ξ=0.25 was achieved (at $\sigma_z/\beta^*\approx 1$), demonstrated its viability [8, 9].

This, and the recent progress in the development of nonlinear integrable systems [10] motivated us to re-visit the question of optimization of a round colliding beam system with the goal to achieve even higher beam-beam parameters. We use the Lifetrac weak-strong particle tracking code [11] to perform numerical simulations, and characterize the tracking data with the Frequency Map Analysis [12, 13].

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[#]valishev@fnal.gov

SIMULATION MODEL

The layout of the test lattice used in simulations is presented in Fig. 1. The machine consists of two main elements: a) a linear arc cell with axially symmetric focusing and phase advance of $\pi \times$n (n is integer) in both planes; b) interaction region (IR), which is a drift space with length L and phase advance $2\pi Q_0$. The horizontal and vertical beta-functions in the IR are equal, with the minimum at the center of the drift. The well known relation between the phase advance and beta-function in a drift determines the value of the beta-function minimum β^*: πQ_0=atan($L/2\beta^*$). The purpose of the arc cell is to match the beginning and the end of the IR lattice functions, otherwise being transparent – we consider dispersion and focusing chromaticity to be zero everywhere.

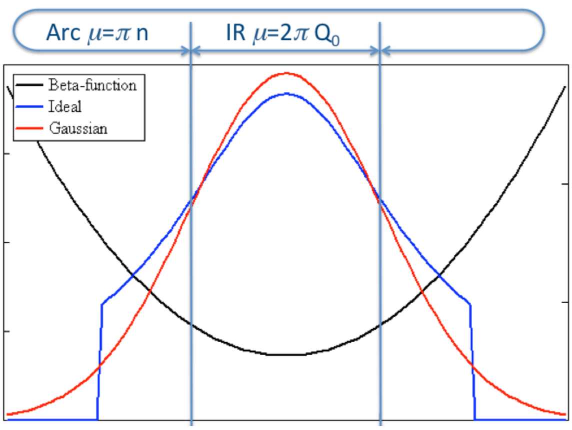

Figure 1: Schematic of the model lattice. Beta-function (black trace), ideal bunch density $\lambda=1/\beta$ (blue) and Gaussian density with $\sigma_z=\sqrt{2}\ \beta^*$ (red) as a function of azimuth. Q_0=0.3. Bunch length for the ideal case is $2\times L$ due to the counter-propagation of two beams.

Note that a significant difference between this model and the system proposed in [5] is that the betatron tune working point does not need to be close to integer or half-integer. Indeed, the betatron tune is n/2+Q_0, and Q_0 is determined by the ratio of β^* and L.

Next, we track a number of test particles with different initial conditions through the lattice with beam-beam interaction, and plot the tune variation along the trajectory as a color chart either in tune space (footprint) or in betatron amplitude space. The regions with significant tune variation represent either resonances or the chaotic motion.

RESULTS

For the synchronous particle in a weak bunch, in the case of ideal bunch density the system has two integrals of motion and the motion is infinitely stable for any value of beam-beam parameter. Figure 2 shows the tune footprint for ξ=1 (throughout this report the focusing beam-beam interaction with Gaussian transverse profile is used). The limited number of longitudinal slices of beam-beam interaction explains the existence of resonance lines. The motion also remains stable for ξ=5, 10.

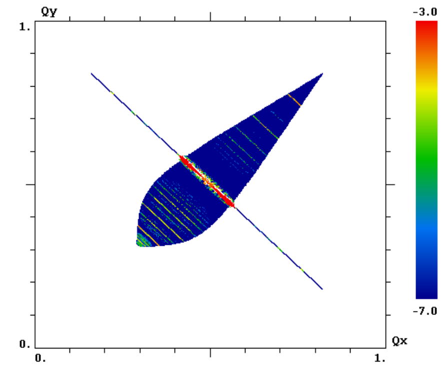

Figure 2: Tune footprint for the case of ideal bunch density $\lambda=1/\beta$, Q_0=0.3, and zero synchrotron amplitude. Beam-beam parameter ξ=1. Particles with betatron amplitudes up to 20 σ. The color represents tune jitter according to the scale on the right (log scale).

For the case of Gaussian longitudinal density, the integrability is lost and one can see the strong overlapping of resonances at ξ=1 (Fig. 3) but still no significant resonances at ξ=0.5 (Fig. 4). Note that in either case the large amplitude motion is stable, which could allow using such systems for purposes other than colliders, for example to create large betatron tune spread.

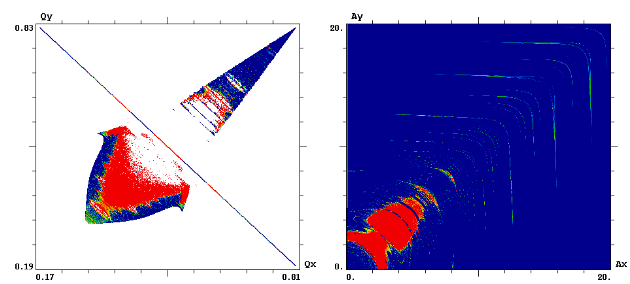

Figure 3: FMA plots for the case of Gaussian bunch density $\sigma_z=\sqrt{2}\ \beta^*$, Q_0=0.3, ξ=1, and zero synchrotron amplitude. Particles with betatron amplitudes up to 20 σ. Tune footprint (left), FMA in amplitude space (right).

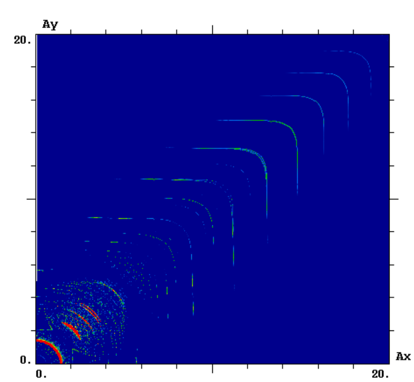

Figure 4: FMA for Gaussian bunch density $\sigma_z=\sqrt{2}\ \beta^*$, Q_0=0.3, and zero synchrotron amplitude. ξ=0.5.

In the simulations for particles with non-zero synchrotron amplitude, we investigated the effect of the bunch length ratio for weak and strong bunch, and of the synchrotron tune. The variation of synchrotron tune does change the resonance configuration, emphasizing some betatron amplitudes more than the others, but does not reduce resonances in a large enough area. As one could expect, the shortening of weak bunch length has a profound effect on the system stability. Simulations suggest that beam-beam parameter of 0.5 could be sustained for the bunch length ratio of about 1/10 (Fig. 5).

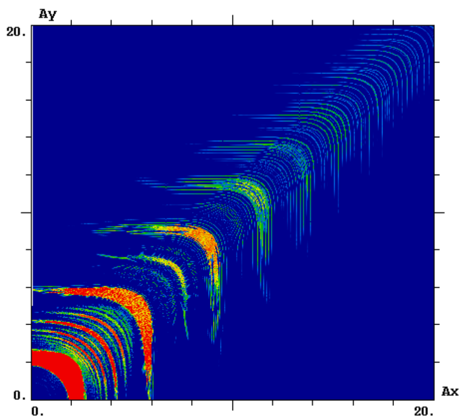

Figure 5: FMA for Gaussian bunch density σ_z=√2 β^*, Q_0=0.3, ξ=0.5. Synchrotron amplitude 0.1 σ_z.

The results of multi-particle simulations for the VEPP-2000 lattice, taking into account the dynamic beta-function and emittance, synchrotron radiation damping and quantum excitation, are not very encouraging. We observe a reduction of the luminosity with the increase of σ_z/β^* ratio exceeding that expected from the hour-glass effect.

CONCLUSION

We confirmed that in a specially designed accelerator lattice, and by a careful shaping of the longitudinal profile of strong bunch, the value of beam-beam parameter exceeding one could be achieved for particles with zero synchrotron amplitude. The approach described in this report is advantageous with respect to the previously considered schemes because the machine betatron tune does not need to be close to integer or half-integer. This makes the scheme less sensitive to imperfections of the accelerator lattice.

For particles with non-zero synchrotron amplitude, the integrability is lost but their motion remains stable even at large amplitudes, although a significant emittance growth is induced by the overlapping resonances. Simulations predict that a scheme with the bunch length ratio between the weak and the strong bunch of about 1/10 may be feasible, which could be useful for asymmetric machines such as the electron-ion colliders.

A more rigorous study of the feasibility of the scheme in a real accelerator is in progress.

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