TRANSVERSE BEAM DYNAMICS IN PLASMA GUIDED LASER DRIVEN ACCELERATION

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Abstract

An axially modulated plasma waveguide supports slow-wave laser modes suitable for direct laser acceleration. The channel studied here supports a gradient of 2 GeV/m driven by a 1 TW laser. The transverse beam dynamics are analyzed in the context of the envelope equation and simulated by particle tracing to demonstrate the feasibility of using a photogun as the electron source. External injection promises to take advantage of the precision offered by state-of-the-art RF photogun technology to increase shot-to-shot reproducibility.

INTRODUCTION

The acceleration of electrons directly by laser fields is an attractive design for a compact accelerator because of the large gradients lasers can provide. In particular, direct laser acceleration (DLA) requires less power than laser plasma wakefield acceleration (LWFA), which enables more compact designs. In order to take advantage of the high fields of the laser it is necessary to overcome the restrictions due to phase slippage implied by the Lawson-Woodward theorem [1]. The DLA scheme pursued here overcomes these limitations by guiding a high intensity pulse through a corrugated plasma channel [2]. Analogous to a conventional disk-loaded LINAC, the corrugations introduce slow wave components phase matched to the electron beam.

Corrugated plasma channels have been created in a laser-ionized plasma [3]. Gas clusters were ionized and then heated by a 100ps pulse focused to a line by an axicon. The corrugations considered for quasi-phase matching were created by using a ring diffraction grating to introduce radial modulations in the laser pulse. The axicon maps the radial intensity pattern to the axis, and uneven heating of the plasma leads to density modulations during the shock-wave expansion. The use of the heater pulse to shape the modulations offers control over the modulation parameters.

Here we propose fixing the modulation parameters to provide focusing for an emittance dominated beam. The transverse beam dynamics in an ideal channel are studied in the context of the envelope equation. A matched solution with asymptotically damped oscillations is calculated. The input beam to match the channel can be obtained from an RF photogun. This solution bypasses the problems LWFA experiments have faced with injecting bunches from the plasma into the accelerating field.

ANALYSIS

Following previous analysis [4], the density profile of the plasma channel used for Direct Laser Acceleration experiment can be approximately described as Eq.(1), with addition of a density ramp, $n_s(z)$ to be used for phase-matching throughout the accelerator [5].

Table 1: Channel Parameters

n_0	k_m	δ	w_{ch}	λ_0	a_0
$5 \cdot 10^{19}/\text{cm}^3$	232200	.9	37 μm	800nm	.2

$$n(z) = n_0\left[1 + \delta \sin\left(k_m z\right)\right] + n_0^{''} r^2/2 + n_s(z) \quad (1)$$

The parabolic density channel supports TM guided modes of radially polarized waves. The lowest order solution is expanded into slow-wave components and the transverse forces due to the resonant and non-resonant components are studied and used as the basis for an envelope calculation. Channel parameters used throughout the paper are listed in table 1 and are chosen to be consistent with experiment [3]. The channel is taken to be 1.8cm long so that the length of the accelerating pulse does not limit acceleration [4]. A laser power of $a_0 = .20$ is chosen to correspond with the TW laser in construction at UCLA's Pegasus beamline.

Guided Modes

Since we want a longitudinal field component for acceleration, we seek the radially polarized component of the laser vector potential in the slowly varying envelope approximation: $A_\perp = \hat{A}_r e^{i(k_0 z - \omega_0 t)}$, with $\hat{A}_r$ satisfying:

$$\left[2ik_0\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right) + \nabla_\perp^2\right]\hat{A}_r = \frac{\omega_p^2}{c^2}\hat{A}_r$$

where $\omega_p^2 = \frac{ne^2}{m\epsilon_0}$. This can be solved using separation of variables and expanded into harmonics using the Jacobi-Anger expansion. Palastro [4] includes discussions of the assumptions behind this model. Ignoring time dependence in the laser envelope, the lowest order solution is written in the Columb gauge as:

$$\hat{A}_r = A_0 \frac{r}{w_{ch}} e^{\frac{-r^2}{w_{ch}^2}} \sum_{q=-\infty}^{+\infty} i^q J_q(\Psi) e^{i\left(-\Psi + (\delta k + q k_m) z - \frac{\int k_{ps}^2 dz}{2k_0}\right)} \quad (2)$$

where:

- $w_{ch} = \left(\frac{8c^2}{\omega_{p0}^{''2}}\right)^{1/4}$
- $k_{ps}^2(z) = \frac{e^2 n_s(z)}{m\epsilon_0 c^2}$
- $\delta k = -\frac{1}{k_0}\left(\frac{\omega_{p0}}{2c^2} + \frac{4}{w_{ch}^2}\right)$
- $\Psi = \frac{\delta \omega_{p0}}{2c^2 k_0 k_m}$

The transverse potential peaks at $r = w_{ch}/\sqrt{2}$. For our parameters $\Psi \approx .5$ is small, so that the Bessel functions limit the relevant terms in Eq(2) to small q.

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The axial component of the potential is given by $\vec{\nabla} \cdot \vec{E} = 0 \Rightarrow \vec{\nabla} \cdot \vec{A} = 0$. The magnetic field is given by $\vec{B} = \nabla \times \vec{A}$ and the electric field by $\vec{E} = -\frac{\partial}{\partial t}\vec{A}$. The phase velocity of an individual harmonic is calculated by taking the derivative of the exponent of Eq.(2). The optical scale dominates $\left(k_0 >> \delta_k, nk_m, \int k_{ps}^2 dz/k_0\right)$ such that phase matched particles would have acceleration given by $\gamma = \gamma_0 + \gamma' z$. This determines v_p and allows phase matching of the largest subluminal harmonic (q=1) by using a density ramp. The density ramp asymptotically approaches $\frac{m\epsilon_0 c^2}{e^2}(2k0(\delta k + qk_m))$ from below and rises slowly compared to both the optical and modulation periods, but quickly compared to the length of the plasma channel. The condition to keep the total density positive is:

$$km > \frac{e^2}{m\epsilon_0 c^2 2k_0}\delta n_0 + \frac{4}{k_0 w_{ch}^2} - k_0\left(1 - \frac{1}{\sqrt{1-1/\gamma_0^2}}\right)$$

For the small values of ψ under consideration we have the relation $E_z \propto a_0 n_0 \delta / k_m w_{ch}$. This scaling helps determine acceptable values for the parameters used.

Transverse Force

The equations of motion for a particle in the channel described above are:

$$\ddot{x}_i = \frac{e}{m_e\gamma}\mathrm{Re}[\sum_{q=-\infty}^{+\infty} f_{i,q} e^{i(n-1)k_m v_p t}] + \text{c.c.}$$

where, for $r << w_{ch}$

$$f_{z,q} = \frac{m_e c\omega_0 a_0}{eK_q}\frac{1}{w_{ch}}J_q(\Psi)e^{i\phi + ik_m\xi(q-1) + i(\frac{\pi}{2}(q) - \Psi)}$$

$$f_{r,q} = f_{z,q}K_q r\left[1 - \frac{\beta_z}{\beta_q}\left(1 + \frac{8}{(K_q w_{ch})^2}\right)\right]e^{i\frac{3\pi}{2}}$$

where $K_q = \left(k_0 + \delta k + qk_m + k_{ps}^2/2k_0\right)$, $\xi \approx z - v_p t$, and ϕ is the phase. Since ξ and ϕ are constant due to the phase matching condition the f vary in time minimally, only through the K_q's dependence on $n_s(z(t))$. The $\frac{8}{(K_q w_{ch})^2}$ is due to the finite spot size of the laser.

Following the notation of [6] the motion is separated into a slowly varying secular motion and a fast (corrugation scale) motion. In solving for the rapid motion we have used $k_0 >> \delta_k, qk_m, \kappa(z)$ and neglected terms $\kappa(z)/k0$. Averaging over the fast motion yields the secular equation of motion:

$$R''(z) = \frac{-1}{\beta^2\gamma}\frac{\omega_0 a_0}{c}\frac{R}{w_{ch}}J_1(\Psi)\left(\frac{8}{(K_n w_{ch})^2}\right)e^{i\phi + i(\pi - \Psi)} \quad (3)$$

$$+\frac{a_0^2}{w_{ch}^2(n-1)^2}\left(\frac{K_1}{k_m}\right)^2 R\sum_{q\neq 1}$$

$$\frac{1}{\beta^2\gamma^2}J_q(\Psi)^2\left(-\frac{(q-1)k_m}{k_0} - \frac{8}{(K_q w_{ch})^2}\right)^2$$

$$+\mathcal{O}\left(\frac{k_0}{k_m\gamma^2}\right) + \mathcal{O}\left(J_q(\Psi)J_{2-q}(\Psi)\right)$$

The term outside the sum is due to the resonsant force. The terms in the sum are the ponderomotive force. For our parameters the terms in big O notation are small for $\gamma >> 6$ and q> 1. For $\gamma >> 30$ the resonant term dominates the pondermotive force. To study the propagation of an electron beam through the channel we consider the envelope equation in the emittiance dominated regime:

$$\sigma_r'' + \frac{(\beta\gamma)'}{\beta\gamma}\sigma_r' + k_r^2\sigma_r = \frac{\epsilon_{nr}^2}{(\beta\gamma)^2\sigma_r^3} \quad (4)$$

The pondermotive focusing term allows for a matched beam envelope:

$$\sigma_m \approx \left(\frac{\epsilon_{nr} w_{ch}}{\sqrt{2}a_0 J_0(\Psi)}\right)^{1/2}$$

Because $\sigma_r'(0) \neq 0$ the pondermotive force will cause spot size oscillations. In addition, the resonant term will add overfocusing as γ increases. The asymptotic behavoir of this system can be be easily interpreted as $\beta \to 1$ by writing $\sigma_r(z) = \sigma_m + \delta(z)$, where it is assumed $\delta(z) << \sigma_m$. Still approximating the particles' phases as constant, and changing variables to $\hat{z} = \gamma_0 + \gamma' z$ Eq.(4) becomes:

$$\delta'' + \frac{\delta'}{\hat{z}} + \frac{1}{\gamma'^2}\left(\frac{k_{\text{res}}^2}{\hat{z}} + \frac{4k_{\text{pond}}^2}{\hat{z}}\right) = -\frac{k_{\text{res}}^2\sigma_m}{\gamma'^2\hat{z}} \quad (5)$$

where k_{res}^2 is the focusing due to the resonant harmonic, k_{pond}^2 is the focusing due to the pondermotive forces, and primes denote derivatives with respect to $\hat{z}$. In the limit where $k_{\text{res}}^2\gamma >> k_{\text{pond}}^2$ the solution is a linear combination of $J_0(2k_{\text{res}}\sqrt{\hat{z}}/\gamma')$ and $Y_0(2k_{\text{res}}\sqrt{\hat{z}}/\gamma')$. For large z this represents damped oscillations with a growing period. When still in the regime $\beta \approx 1$, but $k_{\text{res}}^2\gamma \approx k_{\text{pond}}^2$ the solution can be shown to be oscillations formed from a combination of Bessel and Hypergeometric functions.

Table 2: Beam Parameters

	γ	Espread	ϵ_n	σ_r	Q
Initial	20	.2%	0.1 mm·mrad	2.6μm	1pC
Final	90	2.2%	0.3 mm·mrad	2.6μm	.64pC

Particle Tracking

The particle tracking code General Particle Tracer (GPT) was used to test the approximations made in determining the envelope behavior. The tracing is only as accurate as the input field, Eq.(2), as the tracer does not simulate the plasma. For simplicity the laser is still assumed to be much longer than the electron beam so that the overall pulse envelope can be ignored. The solver does calculate the small contribution from spacecharge fields, but it does not account for wakefields in the plasma.

Beam parameters before and after acceleration are listed in table 2. The input parameters are similar to those obtained in UCLA's Pegasus beamline, and the final parameters are for the accelerated electrons only. Further, the

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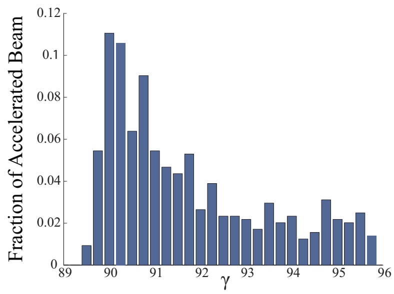

Figure 1: Energy spread of the accelerated bunch.

beam is assumed to have been previously microbunched into 20 optical length bins, with each box having a FWHM $\lambda_0/9$, as might be created by bunching in an undulator [7].

During the first millimeter of acceleration many of the particles far from the axis de-phase from the matched wave, slip into the defocusing region of the resonant harmonic, and are ejected from the channel. The remainder of the particles are accelerated with a linear gradient to $\gamma = 92$. A capture rate of as high as 64% was achieved by setting the initial phase at $\pi/6$. Because of the dephasing, the electrons fill the accelerating bucket and the final energy spread (Fig.1) is nearly 5%. The energy spread and capture rate are improved by reducing the dephasing. The capture rate is little improved by reducing the emmittance, as the dephasing is caused by betatron oscillations that are large enough to modulate γ and dephase the outer electrons. This effect would be reduced by increasing the channel width, but at the cost of the accelerating gradient. A more complex scheme may pursue a slowly changing channel width.

The envelope of a single bunch in the accelerated group of electrons is graphed in Fig.2. The beam has an input rms radius of 2.6μm, and then oscillates about the matched spot size. The oscillation magnitude is damped and the period is lengthening, consistent with the behavior predicted by the asymptotic solution to the envelope equation. The maximum spot size is 1/10 of the channel size, indicating that the forces remain linear, yet the normalized emittance triples. The increase in emittance is visually evident in the evolution of the transverse phase space (Fig.3) as an increase in the transverse momentum. Further increase in the transverse momentum is limited by dephasing due to coupling between the longitudinal and transverse momentum. Particles at the edge of the phase space are slowly dephase into the defocusing phase for arbitrarily large accelerators.

CONCLUSIONS

The transverse dynamics in an ideal corrugated waveguide have been studied and found to yield focusing forces suitable for matching a μm size beam. However, the large focusing and over-focusing forces contribute to the dephasing of electrons which leads to low capture rates, large energy spread, and emittance growth. Increasing the channel

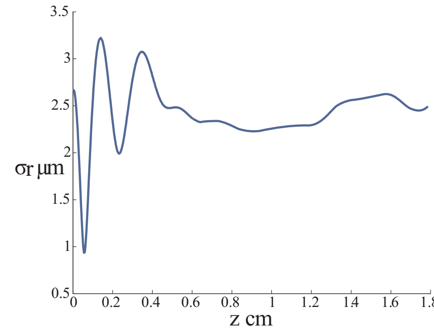

Figure 2: Envelope dynamics showing the spot size oscillations of the accelerated bunch.

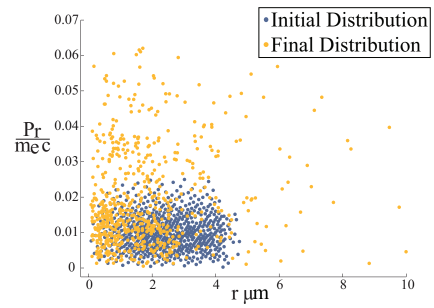

Figure 3: The inital and final phase space of the accelerated bunch. The growth in transverse momentum is due to betatron oscillations.

width decreases these effects and a tapered channel width may contribute to higher beam quality. Further analysis of the accelerator would benefit from modeling the plasma formation and beam-plasma interaction. Nonetheless, the analysis here suggests it would be possible to channel an externally injected beam in a proof of principle experiment.

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