MULTIPLE SCATTERING EFFECTS IN A STRONG MAGNETIC FIELD*

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Abstract

New computational tools are essential for accurate modeling and simulation of the next generation of muon-based accelerator experiments at the energy and intensity frontiers. There is a long list of crucial physics processes specific to muon accelerators that have not yet been implemented adequately. One of the long-standing entries on that list is multiple scattering in the presence of strong magnetic field. Earlier studies showed significant discrepancies depending on the models used and step sizes when stepping through the field. A liquid Hydrogen (LH2) absorber in a strong longitudinal magnetic field is analyzed using G4beamline to study and mitigate the effects of step size, and compared to a past study. The updated results and an algorithm to limit step size depending on the field strength are presented.

INTRODUCTION

Though accelerator simulation codes have been steadily improving over the years, there is still much room for improvement. Many single-particle processes and collective effects in vacuum and matter, such as space charge, beam-beam effects, plasma effects from ionized electrons and ions, etc. have not been studied thoroughly or implemented. In order to ensure proper accuracy of simulations, these effects have to be either deemed negligible or taken into account.

One of the processes that is a source of concern is multiple scattering in a strong magnetic field. In a paper by P. Lebrun [1] scattering of muons through a 32 cm length of LH2 is compared between a Moliere (GEANT3) model and a Discrete Scattering model for both a zero field and a several Tesla magnetic field arrangement, showing significant discrepancies, especially in the tails of the distributions. This paper attempts to recreate the simulations of Lebrun as best as possible in G4beamline [2] based off the GEANT4 package. Several problems presented themselves during the recreation, and a complete recreation was not possible, however we believe it to be accurate enough for our purposes. Since [1], the PDG value for the radiation length of LH2 has been updated. Attaining a perfect match to the magnetic field profile in [1] was also not possible. Overall, GEANT4 shows a much better agreement with a Discrete Scattering model studied by Lebrun.

Another issue is that scattering in a strong magnetic field as simulated in G4beamline will yield different results for different step sizes as shown in the first plot in Fig. 4. Consistent results are achieved at very small step sizes, so an algorithm is required that will reduce step size in matter according to the field strength, so that the performance of tracking of particles in vacuum or in a relatively weak magnetic field is not affected.

OVERVIEW OF ALGORITHMS

The main difference in multiple scattering algorithms between GEANT3 and GEANT4 (and as a result G4beamline) was the change from the Moliere formalism to Lewis theory. In GEANT3, Moliere theory was used. This has a few limitations [1, 3]:

- the angular deflection is small;
- the theory applies only in semi-infinite homogeneous media;
- the absorber is not a very low Z material, as the Thomas-Fermi model could be inaccurate here;
- there is no energy loss built into the theory.

GEANT4 is based on the more complete Lewis theory. Like Moliere theory, this simulates the scattering of a particle after a step, but Lewis Theory also computes the path length correction and the lateral displacement, along with the moments of spatial distribution as well [4].

WITHOUT MAGNETIC FIELD

The simulation presented in Lebrun's paper was recreated as close as possible using G4beamline 2.14. A pencil beam of 180 MeV/c muons are scattered through a 32 cm length of LH2 in and out of an intense magnetic field. The magnetic field comes from the 2 m long Alternate Solenoid cooling channel [5] which reaches a peak field of 15 T. The 32 cm length of LH2 begins where the field is at peak. Ref. [1] makes a comparison between a Discrete Scattering model for LH2 and the Moliere model used in GEANT3 at the time, while we compare these to the current G4beamline 2.14 based on the GEANT4 package. The step sizes of the simulations were discussed in [1] and are set to 4 mm in the Moliere results and G4beamline results, and to a micron in the Discrete Scattering results.

Table 1 shows the compared results for the case with no magnetic field. Extensive track cuts were made by insisting on an aperture cut of 2.5 cm and angle cut of 250 mrad and maximum $r \cdot \theta$ of 500 mm·mrad.

WITH MAGNETIC FIELD

In recreating the coils, the data from [5] was used and is summarized in Table 2. The Z offsets were adjusted to

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* Work supported by DOE.

ISBN 978-3-95450-138-0

Table 1: The Average and σ Values of the Distributions without Magnetic Field

Moments	Moliere (GEANT3)	Discrete Scatt.	G4beamline 2.14 (GEANT4)
$\langle r \rangle$ (mm)	3.43	2.87	3.02
σ_r (mm)	2.18	1.75	1.85
$\langle r' \rangle$ (mrad)	19.0	16.1	16.86
$\sigma_{r'}$ (mrad)	13.4	9.4	10.55
$\langle r \cdot r' \rangle$ (mm·mrad)	72.8	55.2	59.40
$\sigma_{r \cdot r'}$ (mm·mrad)	76.5	58.9	63.23

reflect the positions of the centers of the coils rather than their front faces. Actual current densities were calculated from the relative densities, scaled down by a constant factor. The constant factor was determined by trial and error, such that the maximum field in the configuration was 15 T, and found to be 1/1015.6. For symmetry and to alleviate fringe effects, five identical cells of coils were used. In Lebrun's work, the field decreases from its 15 T maximum to 10.9 T after 32 cm of LH2. It was found in our simulation that the field dropped to 11.2 T instead. That is consistent with a difference in radiation length of LH2 between the previous and current simulations. Figure 1 shows the arrangement of the simulation. Table 3 shows the comparison of the first and second moments in the case with magnetic field.

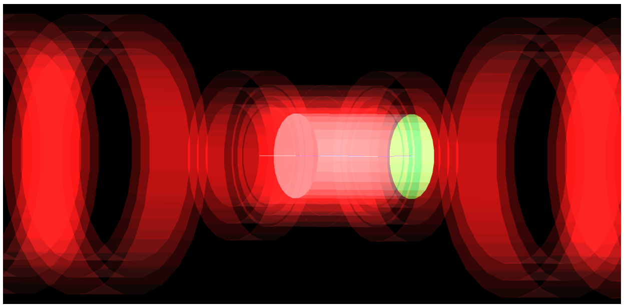

Figure 1: Arrangements of solenoids. The muon beam enters from the left. The 32 cm length of LH2 ends where the detector (green) is.

Figures 2 and 3 show the r and r' distributions, respectively, for the different models used. The data points were obtained using a digital graph analyzer for the points in [1]. It is argued in [1] that although the overall shape of the distributions is similar, the Moliere model did not simulate properly the tail end of the spectrum, while the Discrete Scattering model did. It is seen here that the current scattering model implemented in G4beamline 2.14 (based on GEANT4) follows the tail of the Discrete Scattering distributions much more closely than GEANT3.

EFFECTS OF STEP SIZE

As mentioned in the introduction, scattering in a strong magnetic field as simulated in G4beamline depends strongly on the tracking step size as per Fig. 4. Consistent results are achieved at very small step sizes, so an algorithm is required that will automatically control step size in matter according to the field strength.

Table 2: List of Current Sheets for the 15 T Solenoid, as adjusted by R. Palmer. (Actual Current Density adjusted by a Factor of 1015.6.)

Z offset (m)	Length (m)	Radius (m)	Curr. dens. (unnormalized)	Curr. dens. (A/mm^2)
.075	.15	.1133333	-3639588	-3583.68
.075	.15	.14	-3639588	-3583.68
.075	.15	.1666667	-3639588	-3583.68
.2	.1	.12	-6629837	-6528.00
.2	.1	.16	-6629837	-6528.00
.2	.1	.2	-6629837	-6528.00
.6	.16	.25	-7587805	-7471.25
.6	.16	.29	-7587805	-7471.25
.6	.16	.33	-7587805	-7471.25
.9	.16	.25	7747188	7628.19
.9	.16	.29	7747188	7628.19
.9	.16	.33	7747188	7628.19
1.3	.1	.12	6292018	6195.37
1.3	.1	.16	6292018	6195.37
1.3	.1	.2	6292018	6195.37
1.425	.15	.1133333	3990090	3928.80
1.425	.15	.14	3990090	3928.80
1.425	.15	.1666667	3990090	3928.80

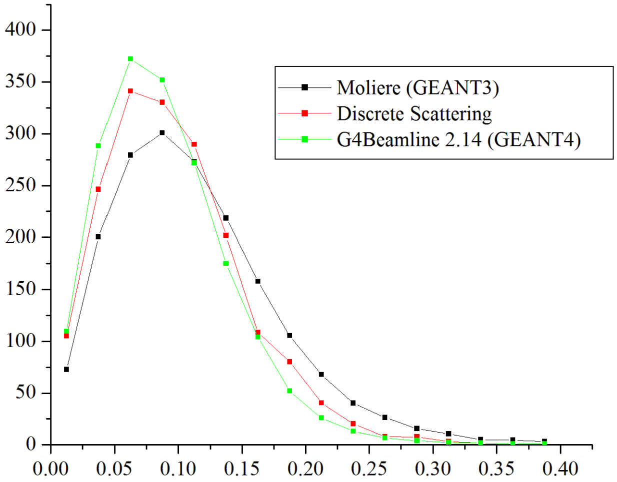

Figure 2: Comparison of the r distribution through different models. Data for the Moliere and Discrete Scattering models were taken from [1]. Our simulation matches the tail of the Discrete Scattering distribution much better than the older Moliere model, as suggested in [1] it should.

In a G4beamline simulation, a pencil muon beam with a momentum of 200 MeV was passed through 30 cm of LH2 in various uniform magnetic fields, and the beam widths were recorded. Ideally, there should be no variation in results from altering the step length used, but as the magnetic field was increased, it was found that a smaller step size was needed to maintain accuracy. The number of steps taken have a great impact on computing time, therefore it

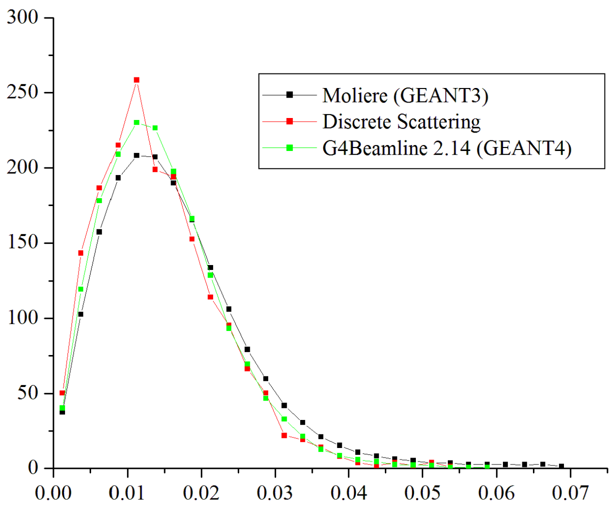

Figure 3: Comparison of the $r' = \theta$ distrubition through different models. Data for the Moliere and Discrete Scattering models were taken from [1]. Our simulation matches the tail of the Discrete Scattering distribution much better than the older Moliere model, as suggested in [1] it should.

Table 3: The Average and σ Values of the Distributions with Magnetic Field

Moments	Moliere (GEANT3)	Discrete Scatt.	G4beamline 2.14 (GEANT4)
$\langle r \rangle$ (mm)	1.18	0.99	0.927
σ_r (mm)	0.87	0.77	0.669
$\langle r' \rangle$ (mrad)	16.9	14.3	15.349
$\sigma_{r'}$ (mrad)	12.0	8.4	10.043
$\langle r \cdot r' \rangle$ (mm·mrad)	23.0	16.1	16.247
$\sigma_{r \cdot r'}$ (mm·mrad)	32.6	19.6	22.747

would be good to find an optimal way to balance accuracy and load.

To less than one percent difference of the value at a step size of one millimeter, it was found empirically that the step size should be scaled with magnetic field as $Step(\text{mm}) \leq 1/(0.004 * Field(\text{T}) + 0.01)$. The results before and after automatic step limiter is implemented are shown in Fig. 4. Similar simulations for Al, Be, LHe, Li and LiH show the same scaling. It was found that in addition, the step length should be shorter than one third the length of the material, to allow for at least 4 steps to take place. This allows for good accuracy with as little CPU time lost as possible.

Note that the step sizes obtained by using the limiting algorithm are consistent with those used for the purposes of comparison with Lebrun's work.

ACKNOWLEDGMENT

Authors would like to thank Paul Lebrun for fruitful discussions and for providing the relevant code decks of the original GEANT3 simulation.

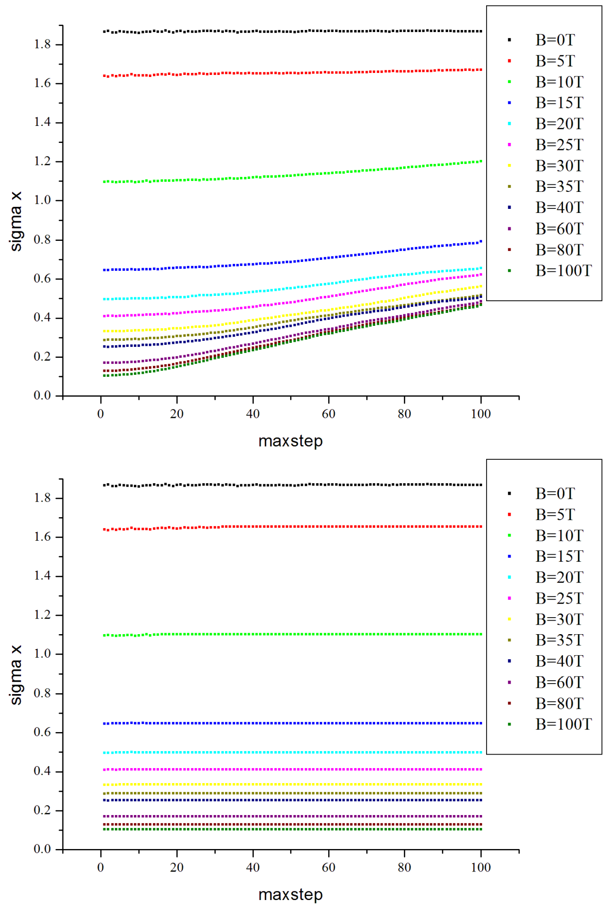

Figure 4: G4beamline results measuring the width of a 200 MeV muon pencil beam after passing through 30 cm of LH2 with 150k events before (top) and after (bottom) implementing a step limiter.

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