INTER-BUNCH COMMUNICATION THROUGH CSR IN WHISPERING GALLERY MODES*

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Abstract

Theory predicts that coherent synchrotron radiation (CSR) in electron storage rings should appear in whispering gallery modes, which are resonant modes of the vacuum chamber, characterized by their high frequencies and concentration near the outer wall of the chamber. Such modes produce an extended wake field behind a particle bunch, which can influence the CSR from a succeeding bunch in a train. We review experimental evidence for the resonances and for the inter-bunch communication. We then present a calculation based on coupled Vlasov-Fokker-Planck equations which confirms an effect seen experimentally, namely that increasing the charge in a leading bunch increases the radiation from a following bunch.

RESONANCES: THEORY AND EVIDENCE

Synchrotron radiation in electron accelerators is strongly affected by the metallic vacuum chamber surrounding the beam, as was already shown theoretically by Schwinger in 1945 [1]. In the parallel plate model of the chamber [1] there is an exponential suppression of radiation at wavelengths greater than $\lambda_0 \approx 2h(h/R)^{1/2}$, where h is the separation of infinite conducting plates, and R is the radius of the particle's circular trajectory lying in the midplane. The model is made more realistic by adding conducting side walls perpendicular to the plates, so that the beam moves in a circular torus of rectangular cross section, with width w and height h. For w of order h, the cutoff λ_0 is roughly the same as before, but a markedly different physical picture emerges.

Radiation to infinity is now forbidden, and radiation synchronous with the beam appears in resonant modes, sharply concentrated in frequency [2]. We call these *whispering gallery modes*, because of a close analog in acoustics [3]; similar phenomena are widely studied in RF engineering and optics. .

A theoretical spectrum of resonant modes is shown in Fig. 1, which is a plot of the real part of the radiation impedance $Z(k)$ for a beam centered in a circular torus with rectangular cross-section and resistive walls of conductivity σ [2]. The parameters correspond to those of the ANKA light source in Karlsruhe: $R = 5.559$m, $w =$ 6.8cm, $h = 3.2$cm, $\sigma = 1.33 \cdot 10^6$ (SI units). The power spectrum of CSR is proportional to $ReZ(k)|\hat{\lambda}(k)|^2$, where $\hat{\lambda}(k)$ is the Fourier transform of the bunch profile.

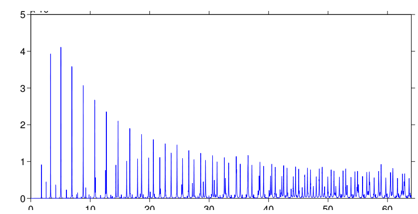

Figure 1: Re $Z(k)$ in ohms for ANKA parameters, vs. $k = 1/\lambda$ in cm^{-1}.

At ANKA and other light sources (CLS, BESSY, MLS, *et al.*) there are large flared vacuum chambers where the IR radiation is extracted, and these may alter the observed spectrum in comparison to that of the smooth toroidal model. The longitudinal electric field $E(s,t)$ may always be regarded as a function of the two independent variables $z = s - \beta ct$ and s. That leads to the concept of position dependent wake $\mathcal{W}(z,s)$. We define

$$\mathcal{W}(z,s) = E(s,t)\,,\ \bar{\mathcal{W}}(z) = \frac{1}{C}\int_0^C \mathcal{W}(z,s)ds\,. \quad (1)$$

The average $\bar{\mathcal{W}}(z)$ over the circumference C is probably adequate to describe the bunch dynamics, since the synchrotron period is large compared to the revolution time. We suppose that $\bar{\mathcal{W}}$ can be approximated as the wake from the impedance of the smooth toroidal model, with central radius R equal to the bending radius of the dipoles.

Note that an interferometer at the position $s = s_{IR}$ of a flared IR chamber will see a field with time dependence given by $\mathcal{W}(z - \beta ct, s_{IR})$, in general different from the time dependence of $\bar{\mathcal{W}}(z - \beta ct)$. Specific theoretical modeling of fields in the flared chamber will be necessary to compare interferometer data with theory.

In an older ring, the NSLS-VUV at Brookhaven, the IR chamber is the same as in other dipoles, so the aforementioned difficulty is not so severe. Moreover, a power spectrum was measured at VUV in the bursting CSR mode, using a Michelson interferometer and also microwave techniques. In Ref.[4] a good fit of the model to experimental positions of spectral lines was shown. Zhou [5] did a similar fit including straight sections, but with less detailed discussion of low frequency lines. The straights induce a width of the lines, which could be part if not all of the explanation of experimental widths.

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Spectra of CSR have been observed with high precision at the Canadian Light Source (CLS), by using an interferometer with resolution down to 0.001 cm^{-1}. The spectra show a remarkable stability with respect to changes in the machine setup and the structure of the IR beam line. Figure 2 compares a power spectrum (red) taken in the bursting mode of CSR with one bunch at 2.9 GeV, on 18/5/2010, with another (blue) taken in the continuous mode with 210 bunches at 1.5 GeV, on 30/1/2012. The latter was multiplied by a factor of 8 to aid comparison. Because of the large change in the beam one expects the positions of peaks to agree better than the relative heights, as is found, but even the relative heights show a lot of similarity between the two runs. We take this stability as a strong indication that the spectrum is determined primarily by the vacuum chamber and bends.

The form of the IR chamber at CLS is shown in Fig. 3. Equations to model fields in this structure are under study. As the crudest approximation we can try the simple toroidal model with an increased distance d from the beam to the outer wall. With $d = 33$cm the spacing of the dominant TE modes [2] is $\Delta k = \Delta(1/\lambda) = 0.073\mathrm{cm}^{-1}$, fitting the fine structure of the spectral data.

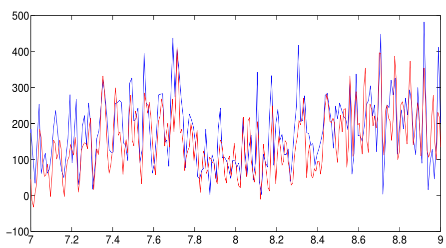

Figure 2: CLS spectra from two very different runs, power (a.u.) vs. k in cm^{-1}.

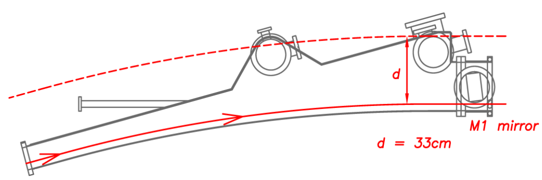

Figure 3: Vacuum chamber of CLS at dipole where IR is extracted

LONG RANGE WAKE FIELD: THEORY AND EVIDENCE

An impedance like that of Fig. 1 implies an appreciable wake field at long distances from the bunch, much greater than the bunch length. Fig. 4 shows the wake potential corresponding to Fig. 1, as a function of distance in centimeters, with the head of the bunch on the right. A feature of the model, if not of the real system, is that the wake wraps all the way around the ring, resulting in a precursor field in front of the bunch. If we try to model the wake field expected at the IR chamber of the CLS, using the effective beam-to-wall distance d as mentioned above, we get the rather different long-range wake pattern of Fig. 5.

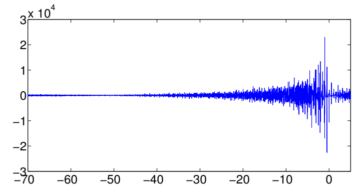

Figure 4: Wake potential $W(z)$ in V/pC from impedance of Fig. 1., vs. z in cm.

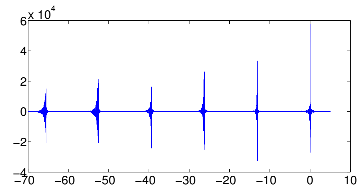

Figure 5: Wake potential for CLS model with $d = 33$cm.

Experiments which aspire to observe the wake field directly were carried out at the CLS, following an idea of S. Kramer to put a microwave horn and diode detector at a backward port (in the horizontal pipe seen in Fig. 3) in one of the two IR dipole chambers. The diode has a bandwidth of roughly 50-75 GHz, and receives signals traveling opposite to the direction of the beam, which might come from backward reflections off structures present in the chamber. Fig. 6 shows a typical oscilloscope trace of the diode signal. Labeling the prominent *downward* peaks from left to right as 1 to 4, we have a plausible explanation as follows: 1 and 2 are reflections from the first downstream obstacle in the flared chamber, a copper photon absorber (in second circle from right in Fig. 3), whereas 3 and 4 are from the next obstacle, a structure supporting the M1 pick-off mirror. The 1-3 separation corresponds closely to the separation of these obstacles. Peak 1 is seen as the prompt wake field from the bunch, while peak 2 is a delayed pulse in the wake field, about 13.5 cm behind the bunch; similarly for 3 and 4, coming from the later reflection. The point to emphasize is that the distance from bunch to delayed wake pulse is very close to the reciprocal of the average spacing of peaks in the power spectrum shown in Fig. 2, namely $\Delta k \approx 0.073\mathrm{cm}^{-1}$. Correspondingly, the distance between the center burst and the first side peak in the interferogram is 13.5cm, as is the spacing of peaks in the naive theory of Fig. 5.

The interpretation of the peaks in terms of reflections is given added weight by a second experiment in which the diode was moved to a "normal" dipole chamber. That resembles the special IR chamber, but lacks the pick-off mirror and has a slotted wall (centered slot of width 1 cm) between the beam and the large flared box. The analog of peaks 1-2 is seen, but that of 3-4 goes away, in accord with the absence of the mirror support structure.

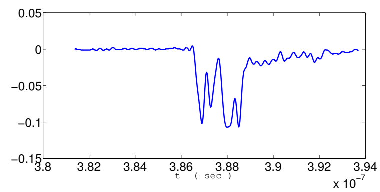

Figure 6: Signal (a.u.) from backward viewing diode at CLS vs. time in seconds.

Further evidence of a long range wake field comes from an experiment at ANKA by V. Judin [6] and collaborators. This observed THz radiation with a fast bolometer having sufficient time resolution to distinguish radiation from individual bunches. A large number of buckets were filled with known but varying amounts of charge. Bolometer signals from the various bunches were sorted into two groups: the blue group in which the preceding bunch in the fill has at least 10% less charge, and the red group in which the preceding bunch has at least 10% greater charge. The power signals were preponderantly greater for the red group, as is seen in the histogram of Fig. 7. The latter gives the distribution of red and blue signals relative to a curve which is a global fit to all the signals.

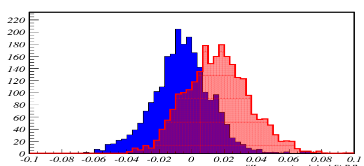

Figure 7: Radiation enhancement by higher charge in preceding bunch.

VLASOV-FOKKER-PLANCK (VFP) SIMULATION WITH TWO BUNCHES

We demonstrate the effect of the long range wake field from whispering gallery modes by solving a nonlinear VFP system for two bunches in two adjacent buckets, taking ANKA parameters for which the interbunch spacing is 60cm. The impedance is that of Fig. 1. We have two coupled VFP equations, each referring to the longitudinal phase space distribution for a bunch in its beam frame, but with a term in the wake field defined by the distribution of the other bunch. The equations are solved by the method of Ref.[7] , but with bi-cubic rather than bi-quadratic interpolation to update the distribution. The initial distributions are Haïssinski equilibria, which are highly unstable at the currents considered.

We plot the total coherent power (a.u.) radiated by each of the two bunches vs. time in synchrotron periods. There are $N_a = 1.14 \cdot 10^9$ particles (0.49mA)in bunch

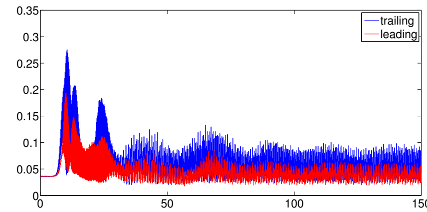

Figure 8: Power from two bunches with equal charges.

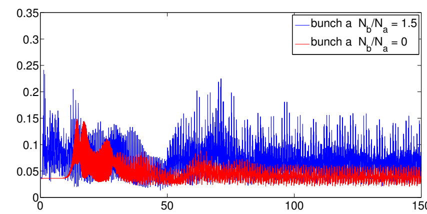

Figure 9: Power from trailing bunch (a), with and without leading bunch, which has 50% more charge.

(a), and an unperturbed bunch length for both bunches of $\sigma_z = 1.92$mm, typical for a low-α setup of ANKA used in CSR runs. For Fig. 8 we have $N_a = N_b$, but the power from the trailing bunch (blue) is consistently greater than that from the leading bunch (red). Fig. 9 shows the power from trailing bunch (a), without a leading bunch (red) and with a leading bunch having 50% more charge (blue). The strong enhancement due to the leading bunch is perhaps surprising in view of the seemingly small wake potential at 60cm shown in Fig. fig:wake, but is believed to be an authentic consequence of the model, evidently a feature of the unstable bunch dynamics at the large (but realistic) currents considered. Of course, the corresponding calculation with the parallel plate impedance shows no inter-bunch communication.

ACKNOWLEDGMENT

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