ACHIEVING A QUASI-THIRD-ORDER ACHROMAT IN AN APS UPGRADE LATTICE*

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Abstract

The next-generation of storage rings will require stronger quadrupole focusing to achieve very low emittance. Stronger sextupoles (usually at smaller dispersion locations) are then necessary to correct the natural chromaticity and suppress the head-tail instability. The geometric and chromatic optics aberrations introduced by these sextupoles have a large impact on the beam dynamics performance. In this paper, it is discussed how to achieve a quasi-third-order achromat in a possible Advanced Photon Source (APS) multi-bend achromat (MBA) lattice. Repetitive identical MBA arc cells with specified phase advance and mirror symmetry are employed. The beam dynamics performance of such an achromat design is compared with a normal scheme.

OVERVIEW

The development of optical achromats started in the 1700s when achromatic doublets and triplets were invented to bring light of different frequencies to the same focal point. First-order achromats have been applied in particle accelerators for a long time. In the 1970s, K. Brown developed a systematic matrix-based approach to designing second-order achromats [1]. It adopts at least four identical cells with dipoles, quadrupoles, and sextupoles and can eliminate all geometric and chromatic aberrations up to second order. Two strong points being proposed are the integer phase advance in the entire beamline and the -I phase separation between non-interleaved sextupoles to cancel even higher-order geometric aberrations. After that, similar third-order achromat design approaches were developed analytically and numerically [2] [3], where the concept of identical cells and integer phase advance is used again. Following these studies, W. Wan developed a general method with Lie algebra to design achromats to arbitrary order, taking advantage of mirror symmetry and using multipole magnets for each specified order (for example, octupoles for a third-order achromat) [4].

The latest PEPX design utilizes eight identical sevenbend achromat (7BA) cells in each arc section, where the total phase advance is an integer [5]. A third-order geometric achromat is achieved with the assistance of harmonic sextupoles [5]. In this paper, first a design similar to the APS storage ringe is presented. It employs more families of chromaticity sextupoles and may eliminate all geometric and chromatic aberrations up to third order. Then there is a discussion on how to achieve a quasi-third-order achro-



Figure 1: Twiss parameters in a 7BA arc cell with a length of 27.2 m. Natural emittance is 150 pm at a beam energy of 6 GeV. The black curve denotes horizontal beta function; red curve, vertical beta function; blue curve, horizontal dispersion function.

mat in the APS upgrade lattices where 40 identical MBA cells have to be used and no dedicated straight sections are allowed.

THIRD-ORDER ACHROMAT DESIGN

The current APS storage ring has a circumference of 1104 meters and is divided into 40 identical sectors. An APS-size ring is designed based on the third-orderachromat concept. Note that it may not fit in the current APS tunnel due to geometry, although the circumference is close to 1104 m. The ring consists of four arc sections plus short straight sections that are each 4 meters long. Each arc section has ten identical 7BA cells. The natural emittance is 150 pm at an electron beam energy of 6 GeV.

The linear and nonlinear optics optimization are performed with ELEGANT [6]. The Twiss parameters in a 7BA arc cell are shown in Figure 1. It is observed that in the central part there are five TME-like cells. Each cell consists of one combined function dipole (with defocusing quadrupole gradient), two focusing quadrupoles, and three sextupoles. The horizontal beta function and dispersion achieve a minimum in the center of the dipole magnet. The dispersion is matched to zero using two dedicated quadrupole magnets and an end dipole. A triplet is adopted outside of the end dipole to bring the beta functions down to $\beta_x = 1.8$ m and $\beta_y = 3$ m at the center of the insertion device (ID). The maximum dipole field is 0.45 Tesla and the maximum quadrupole gradient is 66 T/m.

The betatron phase advance of each 7BA cell is (2.1, 1.1) in units of 2π . The phase advance in each arc section is then (21, 11), i.e., multiples of 2π in both planes. In the 4-meter-

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Figure 2: Horizontal (black curve) and vertical (red curve) betatron tune as a function of momentum offset $\Delta E/E$, dominated by third-order chromaticity.



Figure 3: Dynamic acceptance with 50 ensembles of random, systematic, and tilt errors on quadrupoles and sextupoles. Tracking is performed at the center of the short straight section where $\beta_x = 4 \text{ m}, \beta_y = 1.2 \text{ m}.$

long straight section, which is between two arc sections, two triplets are placed with mirror symmetry. The linear optics is matched and the fractional tune of the storage ring is tunable in a range of (0.1, 0.4).

It is concluded that two families of sextupoles and five families of octupoles are enough to construct a third-order achromat [1] [2] [3]. In general, there are two independent linear chromaticity terms in the second-order matrix, and there are five independent terms in the third-order matrix (two second-order chromaticity terms, plus three transverse amplitude detuning terms). It is not realistic to use an octupoles plus sextupoles scheme due to limited space and maximum magnet strengths. Here only sextupoles are adopted to form a third-order achromat. Eight families of chromaticity sextupoles are placed in the dispersive region, while four families of harmonic sextupoles are placed between the end dipole and the ID.

Similar to the PEPX design [5], all third-order (h10200, h10020...) and most fourth-order (h20200, h40000...) betatron resonance driving terms are naturally cancelled between the ten identical cells [5]. These 12 families of sextupoles are tunable such that all the chromatic and geo-



Figure 4: Local momentum acceptance with 50 ensembles of errors (random, systematic magnetic errors, and tilt errors on quadrupoles and sextupoles). Tracking is performed at locations next to each magnet in one 7BA arc cell (28 m).

metric aberrations may be zero up to third order. In other words, the linear and second-order chromaticities $(d\nu/dp)$ and $d\nu/dp^2$ can be tuned to zero, and the betatron resonance driving terms (up to fourth order) plus amplitude detuning terms $(d\nu_x/dJ_x, d\nu_x/dJ_y)$ and $d\nu_y/dJ_y$) can be eliminated, too.

As shown in Figure 2, the off-momentum tune variation is relatively small for a momentum offset up to 3%. The linear chromaticities are tuned to be 1 and the second-order chromaticities are close to zero. One observes that the offmomentum tunes are dominated by third-order chromaticities. We evaluated the lattice with 50 error ensembles, with 1% optics beating plus 1% coupling from strength and tilt errors in the quadrupoles and sextupoles. The dynamic acceptance (DA) was calculated by tracking for 400 turns, giving the results shown in Figure 3. We see that the DA is 3 mm (x) by 1.3 mm (y) at the straight section center where $\beta_x = 4 \text{ m}, \beta_y = 1.2 \text{ m}.$ The shape of the DA curve is uniform and symmetric. The local momentum acceptance (LMA) was also evaluated at the exit of each magnet by tracking simulations. As observed in Figure 4, it is larger than 2%, which ensures a Touschek lifetime above four hours.

QUASI-THIRD-ORDER ACHROMAT AND BEAM DYNAMICS PERFORMANCE

"Quasi achromat" means that one wants to minimize/zero terms that are most relevant to the beam dynamics performance. For example, a chromatic achromat is essential in transporting a beam with large energy spread. As mentioned above, the lattice design described in the previous section may not fit the APS ring geometry. Alternatively, the following options are proposed and evaluated to achieve a quasi (or pure) third-order achromat in an APS MBA lattice. It is assumed to have 40 identical MBA cells.

True third-order achromat with octupoles, excluded due to space constraints

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Figure 5: Comparison of dynamic acceptance shows that the quasi-third-order achromat scheme (right figure) has better performance.



Figure 6: Comparison of local momentum acceptance shows that the quasi-third-order achromat scheme (right figure) has better performance.

- Add three additional quads at the two ends of every ten cells to change fractional tune, giving a superperiodicity of 4
- 40 identical cells, but each cell has phase advance like (2.104,1.103)

It is interesting to note that the third-order achromat condition is very sensitive to the phase advance. There will be large deviations of the resonance driving terms and detuning terms if the phase advance changes by 1×10^{-5} in each 7BA cell. Based on these considerations, option 3 is a good choice to achieve a quasi-third-order achromat in an APS MBA lattice. The 7BA cell phase advance is tuned to be something like (2.104,1.103), so that in ten repetitive cells the total phase advance is close to a multiple of 2π . The fractional tune is also naturally far away from integer resonance. Note that this scheme is also good for maintaining a large superperiodicity, which helps to suppress lower-order resonances.

A plain 7BA lattice was studied and optimized as one option for a future APS MBA upgrade [7]. It has a natural emittance of 150 pm at 6 GeV. The following two schemes are compared for their beam dynamics performance:

- Normal scheme with arbitrary phase advance in each 7BA cell
- Quasi-third-order achromat scheme, making a quasithird-order achromat in ten 7BA cells

The lattice is almost the same for these two schemes. The arc sextupoles and harmonic sextupoles arrangements are exactly the same. The major difference is the phase advance. Both schemes are optimized for nonlinear beam dy-

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Figure 7: Comparison of frequency map analysis shows that the quasi-third-order achromat scheme (right figure) has better performance.

namics performance, using a Multi-objective Genetic Algorithm [8]. As expected, the quasi-third-order achromat scheme has much smaller geometric and chromatic aberrations than the normal scheme. With the same error settings and tracking setup, the dynamic acceptance and local momentum acceptance are compared between these two schemes, as shown in Figure 5 and Figure 6, respectively. At a location with similar beta functions (roughly $\beta_x = \beta_y$ = 2 meters), the DA is $3.5 \times 3.5 \ mm^2$ for the achromat scheme, which is roughly a factor of two larger than the normal scheme. The LMA is also improved from 3% to 4%. As observed in Figure 7, there are fewer resonance lines showing up in the frequency map of the quasi- achromat scheme.

CONCLUSION

A true third-order achromat without octupoles was studied for an APS-size storage ring that has 150-pm emittance at 6 GeV. Quasi-third-order achromat schemes were also studied and implemented in APS MBA lattices. The effectiveness of the quasi-achromat scheme was confirmed for a plain 7BA lattice, as well as for a plain 5BA lattice [9] and a hybrid 7BA lattice.

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