ELLIPTICAL MUON HELICAL COOLING CHANNEL COILS*

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Abstract

A helical cooling channel (HCC) consisting of a pressurized gas absorber imbedded in a magnetic channel that provides solenoid, helical dipole and helical quadrupole fields has shown considerable promise in providing six-dimensional phase space reduction for muon beams. The most effective approach to implementing the desired magnetic field is a helical solenoid (HS) channel composed of short solenoid coils arranged in a helical pattern. The HS channel along with an external solenoid allows the B_z and B_ϕ components along the reference orbit to be set to any desired values. To set dB_ϕ/dr to the desired value for optimum focusing requires an additional variable to tune. We shall show that using elliptical shaped coils in the HS channel allows the flexibility to achieve the desired dB_ϕ/dr on the reference orbit without significant change to B_z and B_ϕ.

INTRODUCTION

A muon beam cooling technique, using a continuous gaseous hydrogen absorber inside a helical solenoid channel has shown promise to reduce muon phase space. This goal of this technique is to create bright muon beams with small emittance for use in particle accelerators and storage rings. Of particular interest is the development of a muon collider, which requires the reduction of the beam phase space by at least a factor of 10^6. Simulations have shown that the HCC can provide a significant phase space reduction [1]. The HCC is composed of a magnetic channel filled with pressurized H_2 gas absorber to reduce the particle energy. The lost energy is replaced in the longitudinal direction by RF. The desired field in the magnetic channel is comprised of helical dipole, helical quadrupole and solenoid components [2]:

Helical Dipole:

$$b_\phi = 2b_d I_1(k\rho)\cos(\phi - kz)/k\rho$$
$$b_\rho = 2b_d I_1'(k\rho)\sin(\phi - kz)$$
$$b_z = -k\rho b_\phi$$

Helical Quadrupole:

$$b_\phi = \frac{2b_q}{k^2\rho} I_2(2k\rho)\cos 2(\phi - kz - \phi_2)$$
$$b_\rho = \frac{2}{k} b_q I_2'(2k\rho)\sin 2(\phi - kz - \phi_2)$$
$$b_z = -k\rho b_\phi$$

where b_d and b_q are the dipole and quadrupole strengths, respectively, at $\rho = 0$. Implementing this channel with helical harmonic coils is difficult since $I_n(nk\rho)$ grows exponentially at large radius and the channel has large apertures. The HCC magnetic channel will be implemented with short solenoid coils arranged along the helical reference orbit. Figure 1 is a sketch of the helical solenoid (HS) channel [3]. This approach minimizes the magnetic field at the coils. To be effective the B_ϕ, B_z and dB_ϕ/dr of the HS at the reference orbit must match values from the theoretical formulae. B_ϕ and B_z can be set by HS coil current and adjusting an external solenoid field $B_{solenoid}$ (shown in Fig. 1). dB_ϕ/dr is important to control the beam focusing in the channel. To set this variable to an arbitrary value desired to optimize the channel requires an additional degree of freedom. In this paper we explore deforming the circular solenoids into elliptical rings to control the field gradient.

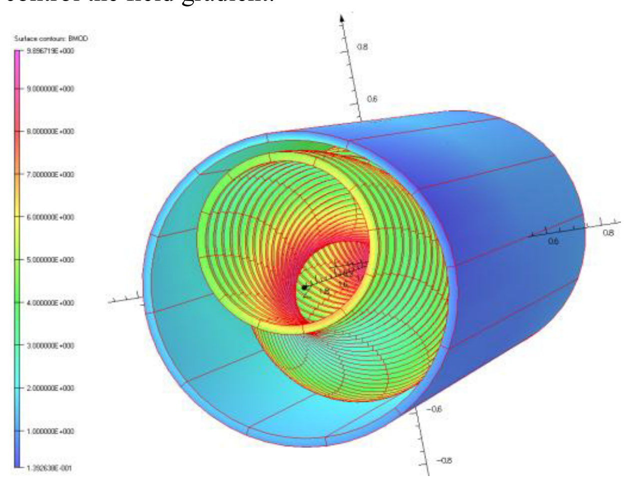

Figure 1: 3D layout of coils for the HCC channel. Note that the HS coils are placed in an external solenoid field.

Table 1: Field Properties of the Sections of the HCC Cooling Channel.

Section	λ	B_ϕ	dB_ϕ/dr	B_z	f
Units	m	T	T/m	T	MHz
1,2	1.0	1.29	-0.50	-4.25	325
3	0.9	1.43	-0.62	-4.73	325
4	0.8	1.61	-0.79	-5.32	325
5	0.5	2.58	-2.01	-8.51	650
6	0.4	3.22	-3.14	-10.63	650
7	0.3	4.30	-5.58	-14.18	650
8	0.3	4.30	-5.58	-14.18	1300

HCC CHANNEL DESIGN

The HCC channel that is proposed is divided into eight sections with each section providing a smaller final equilibrium emittance which requires progressively larger fields. The design field for each section is shown in

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Table 1 [4]. The study concentrates on section 2 at the beginning and section 6 near the end of the channel. Near the end of the channel the coils need to be thick to provide the necessary current to achieve the fields required. Table 2 summarizes the variables to describe sections 2 and 6. The RF cavities are assumed to be filled with a dielectric or made with a re-entrant design to reduce the cavity radius. This permits the use of the smaller radius coils shown in the table.

Table 2: Parameters Describing the HCC Sections Used in the Study

Parameter	Unit	Section 2	Section 6
Helical Period	m	1.0	0.4
Helical Orbit Radius	cm	15.9	6.366
RF Cavity Radius	cm	19	9.4
Coil Inner Radius	cm	25	10.5
Coil Outer Radius	cm	30	20.5
Coil Current Density	Amp/mm^2	256	340
Coils per Period		20	16

A Biot-Savart integration was used to calculate the field from the elliptical coils. The field at each point was determined by integrating over the current from all coils that were within one helical period from that point. Because of the large dimensions of the coils, they were represented by 40-60 filaments distributed over the coil cross section.

ELLIPTICAL COILS

We describe the elliptical shape by its asymmetry, $A = (R_{major} - R_{minor})/(R_{major} + R_{minor})$. R_{major} is aligned with the radial direction, while is in the azimuthal direction. This definition permits R_{minor} to be larger than R_{major} when the ellipse is oblate.

Section 2:

Figure 2 shows the field profiles on the radial axis for the section 2 configuration with different A. The coil current is adjusted to give the desired B_ϕ from Table 1 on the reference orbit. Since the external solenoid field can be set independent of the B_ϕ, setting the external solenoid field $B_{solenoid}$=2.0 T produces the desired B_z. Figures 2a, b, c show B_ϕ, dB_ϕ/dr, and B_z, respectively. Figure 3 shows the value of these fields on the reference orbit as a function of the ellipse asymmetry. The field magnitude at the coils is 6.4 T which is within the range for using NbTi at 4.3 K.

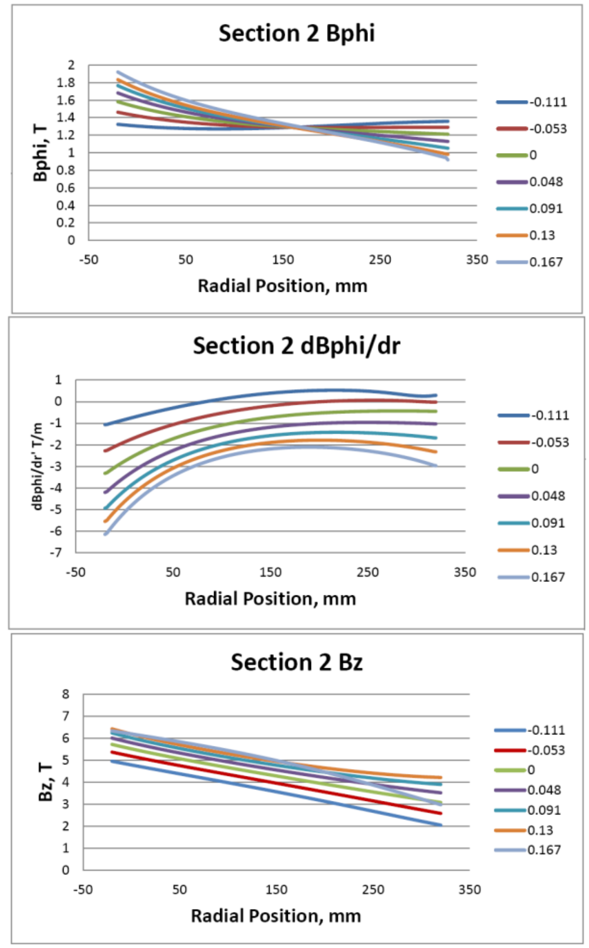

Figure 2: B_ϕ dB_ϕ/dr, and B_z vs. r for section 2. The different colors represent different ellipse asymmetries.

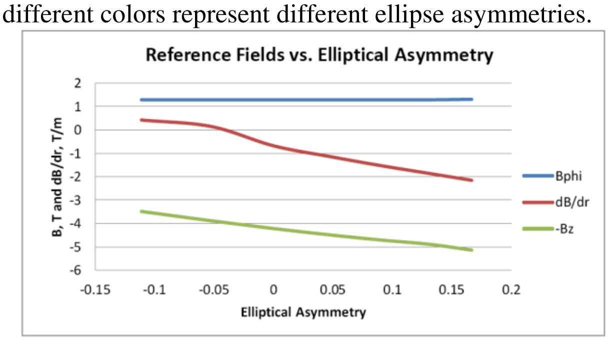

Figure 3: B_ϕ, and $-B_z$ in Tesla and dB_ϕ/dr in T/m on the reference orbit for different ellipse asymmetries.

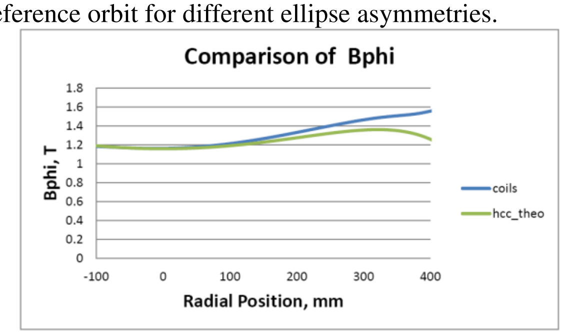

Figure 4: Comparison of the coil description of B_ϕ with the theoretical Bessel function description for section 2.

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Figure 4 compares the B_ϕ field from the HS description with the theoretical Bessel function description. The coil description does not yet correct for the sextupole which affects the field at large radius. For section 2, an oblate ellipse description with A=-0.14 produces a reasonable representation of the Bessel function field.

Section 6

Section 6 requires a large magnetic field and the coils must be fairly thick in order to produce that field. Figure 5 shows B_ϕ, dB_ϕ/dr and B_z for different elliptical asymmetries for section 6. Figure 6 shows B_ϕ, dB_ϕ/dr and B_z on the reference orbit as a function of elliptical asymmetry. Figure 7 compares the B_ϕ field calculated from the HS coils to the Bessel function description of the field for the case A=0.106 which approximates the theoretical description over most of the useful beam aperture. The external solenoid field is 14.2 T which can be made with Nb_3Sn conductor. The maximum field at the HS coils is 20 T which suggests that those coils could be wound with Bi-2212 conductor.

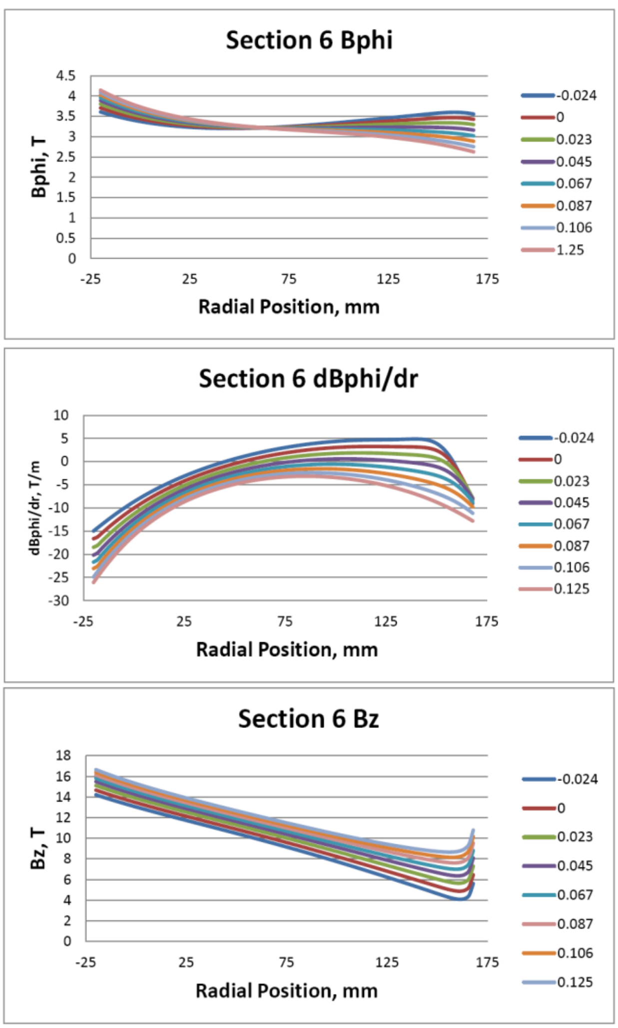

Figure 5: B_ϕ, dB_ϕ/dr, and B_z vs. r for section 2. The different colors represent different ellipse asymmetries.

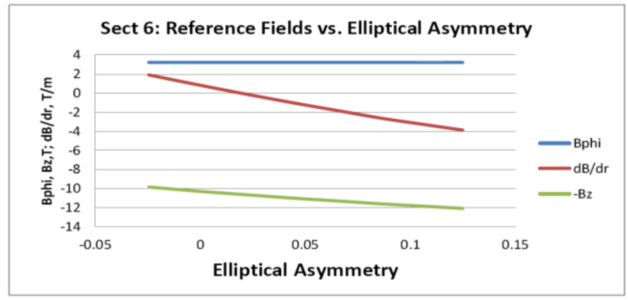

Figure 6: B_ϕ, dB_ϕ/dr, and $-B_z$ on the reference orbit as a function of elliptical asymmetry.

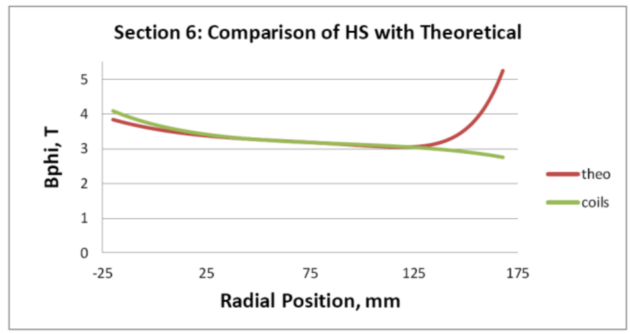

Figure 7: Comparison of B_ϕ calculated from the Bessel function description with that from HS coils.

CONCLUSIONS

Using elliptically shaped instead of circularly shaped coils provides an additional degree of freedom that can be used for matching the HS coils the Bessel function description of the field used for the HCC. Field maps made from the Biot-Savart integration of elliptical coils in the HCC can be used to describe the field for the muon cooling simulation program. This is a work in progress.

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