IMPROVEMENT OF DIGITAL FILTER FOR THE FNAL BOOSTER TRANSVERSE DAMPERS

T. Zolkin*, The University of Chicago, Department of Physics, Chicago, IL 60637–1434, USA
N. Eddy†, V. Lebedev‡, FNAL, Batavia, IL 60510–5011, USA

Abstract

Fermilab Booster has two transverse dampers which independently suppress beam instabilities in the horizontal and vertical planes. A suppression of the common mode signal is achieved by digital notch filter which is based on subtracting beam positions for two consecutive turns. Such system operates well if the orbit position changes sufficiently slow. Unfortunately it is not the case for FNAL Booster where the entire accelerating cycle consists of about 20000 turns, and successful transition crossing requires the orbit drifts up to about 10 μm/turn, resulting in excessive power, power amplifier saturation and loss of stability. To suppress this effect we suggest an improvement of the digital filter which can take into account fast orbit changes by using bunch positions of a few previous turns. To take into account the orbit change up to N-th order polynomial in time the system requires $(N+3)$ turns of "prehistory". In the case of sufficiently small gain the damping rate and the optimal digital filter coefficients are obtained analytically. Numerical simulations verify analytical theory for the small gain and predict a system performance with gain increase.

INTRODUCTION

Consider a paraxial transverse motion of the particle with equilibrium energy in a circular accelerator. For further calculations we will be neglecting a coupling between transverse planes and consider only one of them for simplicity. Neglecting nonlinear effects, in an approximation of linear optics, the dynamics of a particle can be described by the "time" evolution of the betatron state-vector, $\tilde{\zeta}(s) = [q(s), p(s)]^{\mathrm{T}}$, where the variable s is the path length along the closed orbit and plays the role of time, q is the particle deviation in considered transverse direction with respect to the closed orbit, and $p = d\,q(s)/ds$ is canonically conjugated momentum.

The canonical transformation to the so-called normalized coordinates $(p, q) \to (\mathcal{P}, \eta)$:

$$\begin{aligned} \eta(s) &= q(s)/\sqrt{\beta(s)}, \\ \mathcal{P}(s) &= p(s)\sqrt{\beta(s)} + q(s)\,\frac{\alpha(s)}{\sqrt{\beta(s)}}, \end{aligned}$$

where $\beta(s)$ is a beta-function and $\alpha(s) = -\frac{1}{2}\,d\beta(s)/ds$, is performed in order to reduce the motion (from an initial position s_1 to a final one s_2) to the pure rotation of new normalized vector $\zeta(s) = [\eta(s), \mathcal{P}(s)]^{\mathrm{T}}$:

$$\begin{bmatrix} \eta(s_2) \\ \mathcal{P}(s_2) \end{bmatrix} = \begin{bmatrix} \cos\mu_{12} & \sin\mu_{12} \\ -\sin\mu_{12} & \cos\mu_{12} \end{bmatrix} \begin{bmatrix} \eta(s_1) \\ \mathcal{P}(s_1) \end{bmatrix},$$

where $\mu_{12} = \psi(s_2) - \psi(s_1)$ and $\psi(s)$ is a betatron phase advance

$$\psi(s) = \int_0^s \frac{d\overline{s}}{\beta(\overline{s})}.$$

Damper System

A general transverse damper is an active feedback system consisting of pickup station which measures the beam position, and a kicker which is located downstream and damps the betatron oscillations of the beam as a whole by applying an appropriate kick (see Fig. 1).

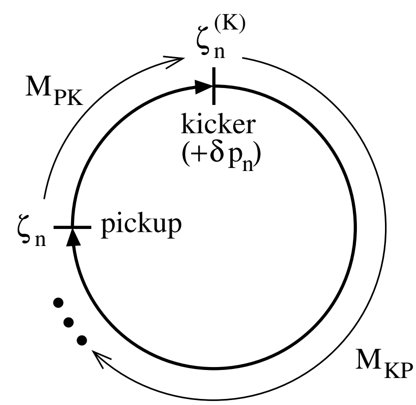

Figure 1: Schematic plot of the one turn transformation of the betatron state-vector in a lattice with damper.

For further considerations we will denote as ζ_n and $\zeta_n^{(\mathrm{K})}$ the normalized betatron-state vector on the n-th turn at the locations of the pickup and the kicker respectively, as $M_{\mathrm{PK}}(\mu_1)$ and $M_{\mathrm{KP}}(\mu_2)$ the transport matrices between pickup and kicker and between kicker and pickup, where $\mu_{1,2}$ are phase advances corresponding to those matrices, and as δp_n the change of particle momentum applied at the n-th turn by kicker. The above determines the betatron tune $\mu_0 = \mu_1 + \mu_2$.

Thereby, a one-turn map that recursively defines a new state-vector at the pickup location can be written as, [1]:

$$\zeta_{n+1} = M_{\mathrm{K}P}\,\zeta_n^{(\mathrm{K})} = M_{\mathrm{K}P}\left(M_{\mathrm{P}K}\,\zeta_n + \begin{bmatrix} 0 \\ \delta p_n \end{bmatrix}\right). \quad (1)$$

* zolkin@uchicago.edu
† eddy@fnal.gov
‡ val@fnal.gov

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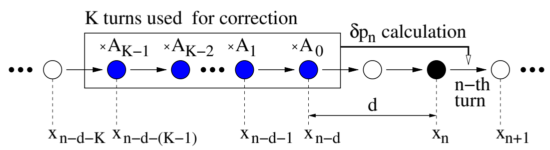

Figure 2: Schematic diagram representing the work of digital filter. Correction algorithm uses $\mathcal{K}$ turns of prehistory (blue circles) in order to calculate the change in momentum δp_n, which will be applied d turns later (for this diagram $d = 2$) after the current turn (n-th turn shown with a black filled circle).

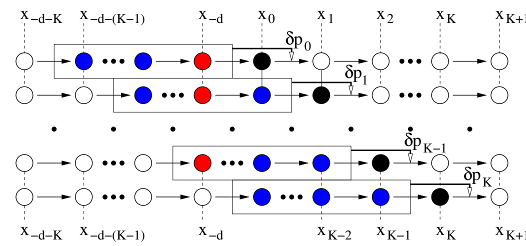

Figure 3: Schematic diagram representing the work of digital filter for $\mathcal{K}$ turns with single erroneous measurement ($(-d)$-th turn shown with a red circle).

DAMPING DYNAMICS FOR SMALL GAIN

In most general form the the angle correction algorithm which is based on $\mathcal{K}$ previous measurements is given by:

$$\delta p_n = \frac{g}{\sqrt{\beta_{\rm P}\beta_{\rm K}}} \sum_{k=0}^{\mathcal{K}-1} A_k \left(q_{n-d-k} + \delta q_{n-d-k}\right),$$

where g is the dimensionless gain, δq_i is an error of measurement introduced by pickup at i-th turn, A_i are the coefficients which define the properties of digital filer, d is an integer which determine the delay in turns after which the correction to be applied (typically $d = 0, 1, 2$ depending on the system), and, $\beta_{{\rm P},K}$ denote a beta functions at the pickup and the kicker locations respectively. The schematic diagram of the algorithm is shown in Fig. 2.

For further simplification one can introduce a complex normalized variable $z = \eta - i\,\mathcal{P}$, which allow to rewrite matrix Eq. (1) as

$$\begin{aligned} z_{n+1} &= e^{i\mu_2} \\ &\times \left[e^{i\mu_1} z_n - ig \sum_{k=0}^{\mathcal{K}-1} A_k \left(\Re\, z_{n-d-k} + \delta\eta_{n-d-k}\right)\right], \end{aligned}$$

where $\delta\eta_i = \delta q_i/\sqrt{\beta_{\rm P}}$ is the renormalized error, and $\Re\, z$ is a real part of z. Below we will consider the beam dynamics in a presence of damper for the case of small gain.

Damping Rate

Temporarily omitting the heating term, $\delta\eta_i$, and taking into account that the contribution of the z^* term is averaged out for the case of small gain, the last equation can be solved by the use of the first-order perturbation theory. Looking for the solution in the form $z_n = z_0\, e^{i\mu n}$, where $\mu = \mu_0 + ig_d$ and g_d is a damping rate presumed to be small, finally gives

$$g_d = i\,\frac{g}{2}\, e^{-i(\mu_1+\mu_0 d)} \sum_{k=0}^{\mathcal{K}-1} A_k e^{-i\mu_0 k}. \qquad (2)$$

As one can see, the imaginary part of the damping rate should vanish in order to have critically damped system, i.e. $\Im\, g_d = 0$. This imposes an additional restriction on coefficients A_i.

Emittance Growth

To estimate the emittance growth excited by noise of the pickup measurements, we will keep only the heating term, while damping term will be omitted. If only one measurement is erroneous, let say $\delta\eta_0$, then summing the effect of $\mathcal{K}$ turns one obtains

$$z_{\mathcal{K}} = \left(z_0 - 2\, g_d\, \delta\eta_0\, e^{i\mu_0 d}\right) e^{i\mu_0 \mathcal{K}},$$

where Eq. (2) was used to find the sum. One can see that every single error $\delta\eta_i$ will contribute $\mathcal{K}$ turns multiplying the effect (see Fig. 3). That yields an increase of the emittance due to a single kick

$$\delta\epsilon = \frac{\overline{\langle \delta z \delta z^* \rangle}}{2} = 2\,\overline{\delta\eta^2}\left[(\Re\, g_d)^2 + (\Im\, g_d)^2\right] = 2\,|g_d|^2 \frac{\overline{\delta q^2}}{\beta_{\rm P}},$$

where in addition to the averaging over initial phases and amplitudes (operator $\langle\ldots\rangle$), we performed averaging over the kick amplitudes (operator $\overline{(\ldots)}$), and

$$\left(\overline{\delta q^2}\right)^{1/2} = \sqrt{\beta_{\rm P}}\, \left(\overline{\delta\eta^2}\right)^{1/2}$$

is the rms error of a single measurement. As one can see the emittance growth does not depends on the choice of coefficients A_i.

DESIGN OF THE DIGITAL FILTER

In general the digital filter should not be sensitive to the equilibrium orbit offset relative to the geometrical center of pickup. Otherwise, the system attempts to correct this offset which leads to increased voltage on the plates of the kicker and inefficiency of the algorithm. It is usually called a notch filter condition and it gives a constraint on coefficients A_i:

$$\sum_{k=0}^{\mathcal{K}-1} A_k = 0.$$

However, it is insufficient if the beam orbit is changing fast. For example, in FNAL Booster, successful transition crossing with high intensity beam requires fast change of equilibrium orbit in time. The present digital notch filter is not able to effectively suppress beam oscillations around transition.

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & (N+1)^0 & (N+2)^0 \\ 0 & 1 & 2 & \cdots & (N+1)^1 & (N+2)^1 \\ 0 & 1 & 4 & \cdots & (N+1)^2 & (N+2)^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 1 & 2^N & \cdots & (N+1)^N & (N+2)^N \\ \cos(\delta\mu) & \cos(\mu_0+\delta\mu) & \cos(2\mu_0+\delta\mu) & \cdots & \cos\left[(N+1)\mu_0+\delta\mu\right] & \cos\left[(N+2)\mu_0+\delta\mu\right] \end{bmatrix}. \quad (3)$$

The filter which would not be sensitive to the orbit change up to N-th order polynomial ($q_n^{\mathrm{eq}} \propto n^N$) requires at least $\mathcal{K} = (N+2)$ coefficients. It is easy to show, that they are related to the binomial coefficients and up to a common factor

$$A_k = (-1)^k C_k^{N+1}, \qquad k = 0,\ldots,N+1.$$

An additional constraint which optimizes the damping, $\Im\, g_d = 0$, gives

$$\sum_{k=0}^{\mathcal{K}-1} A_k \cos(\mu_0 k + \delta\mu) = 0,$$

where $\delta\mu = \mu_1 + \mu_0 d$. That increases the required number of turns to $\mathcal{K} = (N+3)$. If minimum possible number of coefficients is used, one can write matrix equation

$$\mathbf{M}\cdot[A_0, A_1, \ldots, A_{N+2}]^{\mathrm{T}} = 0$$

with the solution

$$\begin{aligned} A_k \; = \; (-1)^k &\left[C_k^{N+1} \sin\left(\frac{N+3}{2}\,\mu_0 + \delta\mu + N\frac{\pi}{2}\right)\right. \\ &\left. + \, C_{k-1}^{N+1} \sin\left(\frac{N+1}{2}\,\mu_0 + \delta\mu + N\frac{\pi}{2}\right)\right], \end{aligned}$$

where $k = 0,\ldots,N+2$, and matrix $\mathbf{M}$ is determined by Eq. 3. A common factor, ± 1, in front of all A_i must be added in order to have $\Re\, g_d > 0$.

Numerical Example for the FNAL Booster

The parameters of the Booster damper system are listed in Table 1. A numerical example of design of the digital filter which suppresses effect of orbit offset, and, its linear and quadratic changes with time is presented in Fig. 4. The real and imaginary part of the gain, g_d, as a function of particle frequency, ν, for horizontal and vertical degrees of freedom is shown in top figure. Optimal values of the coefficients for the digital filter, A_i, as a function of Booster betatron frequency, ν_0, for horizontal (middle figure) and vertical (bottom figure) degrees of freedom are presented as well. All calculations were performed for parameters listed in Table 1.

REFERENCES

[1] R. De Maria, D. Valuch, G. Arduini, W. Höfle, G. Kotzian and V. A. Lebedev, "Suppression of Emittance Growth by Excited Magnet Noise with the Transverse Damper in LHC in Simulations and Experiment," Conf. Proc. C **110904**, 508 (2011).

Table 1: Parameters of the FNAL Booster Damper System

	Horizontal	**Vertical**
Number of missing turns		
d	0	0
Betatron frequency		
ν_0	6.75	6.83
Fractional part of tune		
$\{\mu_0\} = \mu_0 - 6\times 360°$	270°	298.8°
Pickup-kicker phase advance		
$\{\mu_1\} = \mu_1 - 6\times 360°$	67.5°	288.5°

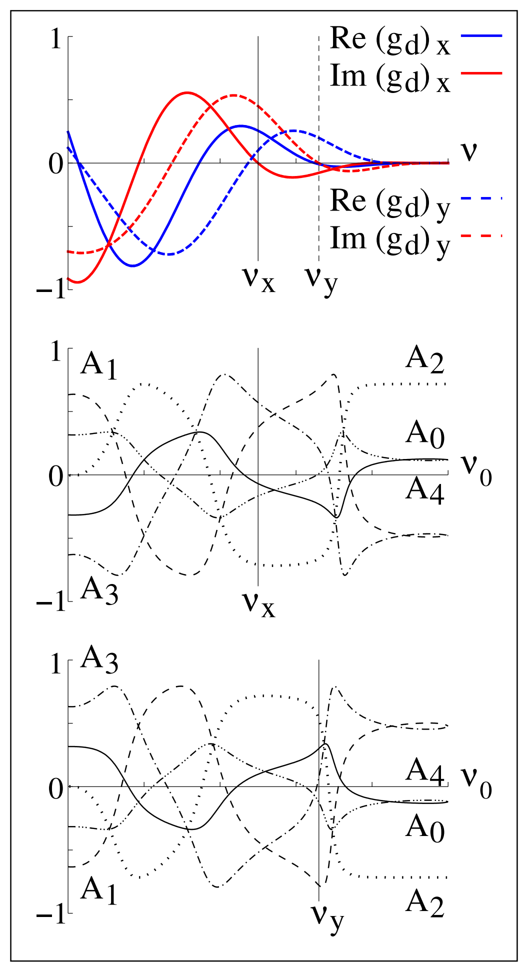

Figure 4: Numerical example of the digital filter design for FNAL Booster, $N = 2$. Both scales for ν and ν_0 are from 6.5 to 7.0.

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