A LINEAR ENVELOPE MODEL FOR MULTI-CHARGE STATE LINAC*

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Abstract

The traditional linear envelope tracking model is widely used in linac design and on-line tuning. However, for multi-charge state acceleration, where the transfer matrix acts differently on different charge-states, the linear envelope tracking model cannot be utilized. A direct way to handle multi-charge state acceleration is using multi-particle tracking, which is usually high in precision, but lacking in efficiency, therefore is not suitable for linac on-line beam tuning. In this paper, a new approach of adapting linear envelope tracking model to multi-charge state acceleration is proposed. The lattice of FRIB is used to test this technique in both linac segment and folding segment. The result is then benchmarked with a multi-particle tracking program IMPACT to ensure its precision with enhancement in efficiency.

INTRODUCTION

The traditional linear envelope tracking model is widely used. Instead of tracking every particle, the method is capable of keeping track of beam envelopes by knowing an initial theta matrix and the transfer matrix between the initial point and the objective point, which is shown below.

$$x_i^{(1)} = R_{ij} x_j^{(0)}$$

$$\sigma_{ij}^{(0)} = \frac{\sum_n \left(x_{ni}^{(0)} x_{nj}^{(0)}\right)}{N}$$

$$\sigma_{ij}^{(1)} = \frac{\sum_n \left(R_{ik} x_{nk}^{(0)} R_{jm} x_{nj}^{(0)}\right)}{N} = R_{ik} \sigma_{km}^{(0)} R_{jm}$$

Numbers of linac code are based on or contains this scheme, such as Trace 3D. The advantage of envelope tracking model is obvious: fast in calculation speed, simple and clear in physics, therefore, remains to be a preferable way in computational intensive applications like case-by-case based linac lattice global optimization, and time restricted applications like linac on-line beam tuning.

For traditional accelerators like electron or proton machines, only one charge state is presented and the traditional linear envelope tracking model can be directly adopted. However, Facility for Rare Isotope Beams (FRIB) is accelerating multiple charge states in order to enhance the beam current. For this kind of accelerators, the traditional linear envelope tracking model can no longer be adopted directly, extensions onto multi-charge states acceleration problems are needed.

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HOW TO DEAL WITH MULTI-CHARGE STATE ACCELERATION

There are three different schemes to handle multi-charge state acceleration problem. The most simple and straight forward one is multi-particle tracking. The advantage is we can expect minimum change of existing tracking code, attain high precision and detailed information. Particle tracking code like IMPACT, is utilizing this scheme. However, the shortcoming is high computational intensity and slow speed. And computer takes over all the calculation would result physics to be unclear.

The second way is to treat charge state deviation as momentum deviation for magnetic field. This method can give exact result on transverse coordinates and can reuse most part of existing code. But the problem the method can only handle all-magnetic field lattice. The imaginary extra momentum deviation can cause imaginary change in time-of-flight for non-relativistic cases. This method is sometimes quite useful when calculating multi-charge beam behaviour in a bending magnet.

THREE-STEP SCHEME

The third way is to treat different charge states separately using envelope tracking scheme. This method can be divided into three steps:

Step 1: Use an ideal particle with centre charge state to initiate the machine parameters, such as RF phases, bending magnet strength. Keep record of the particle and make it the reference of the whole beam.

Step 2: Choose the particle located at beam centre of each charge state as a reference particle and do single particle tracking according to the initialized lattice parameter. Keep record of each particle and make it the reference for each charge state beam.

Step 3: Do envelope tracking for each charge state beam. The transfer matrix should be adjusted according to its own charge state and reference orbit.

This method of handling multi-charge state is a combination of precision of particle tracking method and efficiency of envelope tracking. The calculation efficiency is quite high. Details can vary while the three step scheme concept is universal. For next part, the FRIB lattice would be analysed as an example using this method.

APPLICATION TO FRIB LATTICE

The FRIB project, built at the Michigan State University in corporation with the US Department of

Energy (DOE), will deliver all stable ion beams with energy more than 200MeV/u and target beam power more than 400kW. In order to meet this goal, multi-charge state acceleration is needed, especially for heavy ions, to increase the beam current and beam power. For the Uranium case, the 33+, 34+ charge states are accelerated in linac segment 1, and after a charge stripped and charge selector, the 76+, 77+, 78+, 79+, 80+ charge states are selected and accelerated all the way to the target. Fig. 1 shows detailed layout of FRIB lattice.

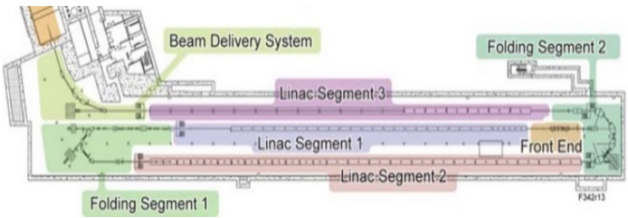

Figure 1: Layout of FRIB lattice.

Linac Segment

The linac segment is where heavy ions gain most part of its kinetic energy. For linac segment 1, superconducting quarter-wave resonators (QWRs), are used for accelerating ion beams while for linac segment 2, superconducting half-wave resonators (HWRs) are used. For both linac segment 1 and linac segment 2, superconducting solenoids are the main component used for focusing ion beams.

Linear Matrix Models: To track the energy and phase advance for synchronous particle in an RF cavity, the Drift-Kick-Drift thin lens model is utilized:

$$\begin{cases} W_f = W_i + qV_0 T(k)\cos\varphi_i - qV_0 S(k)\sin\varphi_i \\ \varphi_f = \varphi_i + \dfrac{qV_0}{2W_i} k[T'(k)\sin\varphi_i + S'(k)\cos\varphi_i] \end{cases}$$

T, T', S, S' are Transit Time factors. Two gap accelerating model is utilized to decrease the error caused by constant velocity assumption [1].

The longitudinal phase space transfer matrix comes from differentiation of the longitudinal Drift-Kick-Drift thin lens model and keeping the linear term of energy and phase deviation. The transverse phase space transfer matrix also comes from a Drift-Kick-Drift thin lens model. For each acceleration gap, the total transfer matrix can be decomposed into drift spaces between two focusing/ defocusing gaps and an acceleration.

A soft edge solenoid model is used in our model:

$$M_{soft\,sol} = M_{edge} M_{sol} M_{edge}$$

$$M_{edge} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\Phi & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\Phi & 1 \end{bmatrix}$$

M_{sol} is the traditional hard edge solenoid transfer matrix M_{edge} describes the edge effect. $\Phi = -\frac{g^2 a}{2}, = \frac{Bqc}{2\beta E_{tot}}$, and a is the solenoid radius [2].

These models have been verified by envelope tracking of single charge state after benchmark with a multi-particle tracking code IMPACT [3]. Then we can use the three step scheme to extend the model to handle multi-charge state acceleration.

Linac Segment 1: First step is to initialize the lattice parameters using a centre charge state. For linac segment 1 case where the two charge states are 33+ and 34+, either one can be used as reference charge state. Here we choose 33+ as the reference charge state and the lattice is initialized using an ideal 33+ particle. After initializing the lattice parameters, the centre particle for 33+ and 34+ charge states are tracked and results are recorded as reference orbit for each charge state. Then we can use envelope tracking scheme to track the envelope of 33+ and 34+ beam bunch separately. Transfer matrix can be influenced not only by charge states but also by reference orbit. Beam centre and RMS envelope evolution results can be seen in Fig. 2.

We can see from the figure that the 33+ charge state particle tends to have larger phase, because the 33+ charge state would always be dragged off and fall behind due to lack of acceleration efficiency compared to 34+. For transverse direction, the centre of beam for both charge states is coincident with the beam axis. Due to the different focusing strength for the different charge states, the focusing lattice cannot match both charge states at the same time, so we can see oscillation comes from mismatch for both charge states.

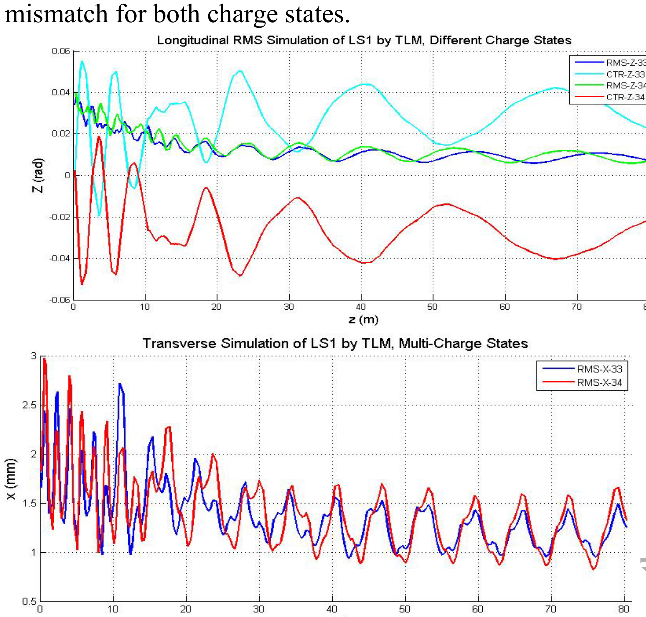

Figure 2: Upper: Linac segment 1 longitudinal phase centre deviation for the 33+ and 34+ two charge states and the phase spread RMS envelope in the centre of mass frame Lower: transverse beam RMS envelope for the 33+ and 34+ two charge states.

RMS Envelope Re-combine: For real experiment, we usually measure the whole bunch beam size. So, we need to calculate the total RMS beam size. In the centre of mass frame, $\bar{x}$ is centre of beam for each charge state, σ is the RMS size, N is particle number, foot note i represents different charge state, than the whole bunch RMS beam size equals:

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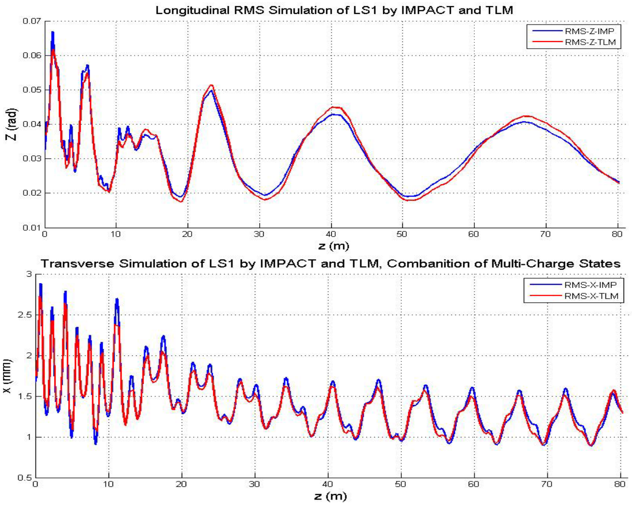

Figure 3: Re-combine of RMS envelope and benchmark with IMPACT. Upper figure, Re-combine of RMS envelope for the longitudinal direction, Lower figure, Re-combine of RMS envelope for the transverse direction.

$$\sigma = \sqrt{\frac{\sum_i N_i(\sigma_i^2 + \bar{x}_i^2)}{N}}$$

Using this scheme to recombine the beam size and benchmark with IMPACT, we can get the result shown as Fig. 3. We can see that the simple envelope model for handling multi-charge state benchmarks well with IMPACT simulation.

Folding Segment

There are two 180 degree benders linking three linac segments. For charge state far away from centre whose equivalent dispersion is quite high, higher order term may appear. In order to suppress the potential influence from higher order term, a new scheme has been proposed.

First, use a centre charge state to initialize lattice parameter, for uranium beam in bending segment 1 case where the charge states are 76+, 77+, 78+, 79+ 80+, the charge state 78+ is chosen to initialize the lattice. Then, for the centre particles of the remaining charge states, their own reference orbits are set up. Note that in order to suppress the possible higher order term coming from bending magnet, the exact transfer map is used to track the reference particles [4].

After setting up reference orbits, we can use envelope model to do envelope tracking. For this time, the traditional bending magnet transfer matrix for non-relativistic ion beams can be used.

By using the scheme, we can also obtain the beam centre and beam RMS size of each charge state in folding segment 1, the result is shown as Fig. 4. After recombination and benchmark with IMPACT, we arrive in the final result benchmark with IMPACT shown as Fig. 5. The thin lens model agrees well IMPACT result.

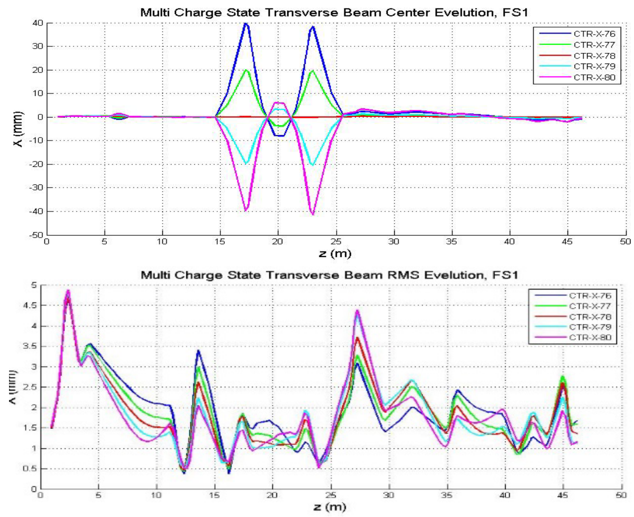

Figure 4: Transverse beam centre (upper) and beam RMS envelope (lower) evolution of five different charge states in folding segment 1.

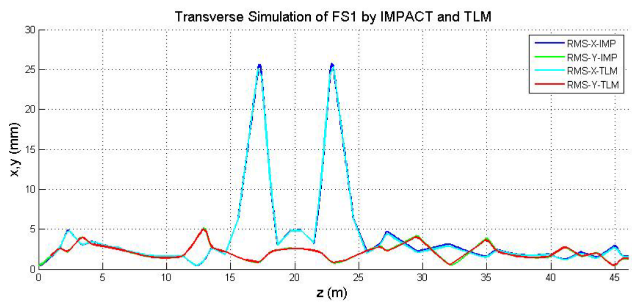

Figure 5: Re-combine of transverse RMS envelope of five different charge states in folding segment 1 and benchmark with IMPACT.

CONCLUSION

The multi-charge state envelope tracking model has proved to be precise and efficient. The result agrees well with multi-particle tracking. Very minor modification of currently available model is needed. The computational efficiency also turns out to be high. A beginning to end of FRIB lattice parallel computation of multi-particle tracking by IMPACT would take several minutes, while multi-charge state envelope tracking method would take only several seconds. The increment in efficiency makes the model promising in further development of on-line applications like fast beam tuning and lattice optimization for multi-charge state acceleration machine as FRIB.

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