SINGLE PASS ELECTRON COOLING SIMULATIONS FOR MEIC*

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Abstract

Cooling of medium energy protons is critical for the proposed Jefferson Lab Medium Energy Ion Collider (MEIC). We present simulations of electron cooling of protons up to 60 GeV. In the beam frame in which the proton and electrons are co-propagating, their motion is non-relativistic. We use a binary collision model which treats the cooling process as the sum of a large number of two-body collisions which are calculated exactly. This model can treat even very close collisions between an electron and ion with high accuracy. We also calculate dynamical friction using a δf PIC model. The parallel code VSim (formerly Vorpal) is used to perform the simulations. We compare the friction rates with that obtained by a 3D integral over electron velocities which is used by BETACOOL.

ELECTRON COOLING

In the simplest form of electron cooling [1], an electron beam is created with the same γ as a relativistic ion beam which we wish to cool. The two beams are then made to co-propagate over a section of a storage ring, the **cooling section**. In the **beam frame** traveling with the average velocity of the particles, their velocities are non-relativistic, and the particles interact by means of Coulomb collisions.

This paper explores the amount of dynamical friction which can be obtained in an (unmagnetized) cooling section of the Jefferson Lab Medium Energy Ion Collider. The particles to be cooled are protons with $\gamma = 65.4$. The cooling section is $s = 60$ m long. These simulations are done in 3D, where x and y are transverse coordinates and z is the direction of beam propagation.

The bunched electron beam is non-uniform, but suppose we consider a small region only a few Debye lengths wide in the center of the electron bunch. In this region, the electron density is nearly constant. We consider a distribution of electrons which is uniform in space and has a Maxwellian distribution in velocity

$$f_0(\vec{v}) = \frac{n_e}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z} \exp\left[-\frac{v_x^2}{2\sigma_x^2} - \frac{v_y^2}{2\sigma_y^2} - \frac{v_z^2}{2\sigma_z^2}\right],$$
(1)

where n_e is the density in the center of the electron bunch. In the beam frame the RMS velocities $\vec{\sigma}$ are non-relativistic, specific values are given in Table 1. Note that

Table 1: Electron Beam Parameters Used in the Simulations, all Parameters are in the Beam Frame

parameter	symbol	value	unit
electrons per bunch		2.5	10^9 elec.
peak density	n_e	1.32	$10^{15}e/m^3$
plasma freq.	ω_p	3.26	10^8/sec
plasma per.		3.07	nanosec
interaction time	τ	3.06	nanosec
trans. RMS bunch size	r_x	303	μm
long. RMS bunch size	r_z	1.3	m
trans. RMS velocity	σ_x	2.35	10^5 m/sec
long. RMS velocity	σ_z	0.36	10^5 m/sec
trans. Debye length	λ_x	114	μm
long. Debye length	λ_z	17.5	μm

z is the direction of beam propagation, and because this is an accelerated beam it is not isotropic, and $\sigma_z < \sigma_x = \sigma_y$.

Consider now a single ion moving at velocity $\vec{v}_{ion}$, and let $\vec{v}_{\rm rel} = \vec{v}_{ion} - \vec{v}$ be the velocity of each electron relative to the ion. Assuming each collision is symmetric in time, the dynamical friction force on the ion can be calculated by the 3D integral over the electron velocity space [2]

$$\vec{F}_\parallel = -\frac{n_e Z^2 e^4}{4\pi\epsilon_0^2 m_e} \int_{-\infty}^{\infty} \Lambda \frac{\vec{v}_{\rm rel}}{|\vec{v}_{\rm rel}|^3} f_0(\vec{v})\, d^3\vec{v}.$$
(2)

Here Z is the ion charge number (for protons, $Z = 1$), $e < 0$ is the electron charge, and m_e is the electron mass. In the simplest case, the Coulomb logarithm

$$\Lambda = \log(\rho_{max}/\rho_\perp),$$
(3)

where ρ_{max} and $\rho_\perp$ are often called "the maximum and minimum impact parameters". This terminology is unfortunate, because equation (2) actually includes contributions from collisions with arbitrarily small impact parameters [2]. $\rho_\perp$ is the impact parameter for 90 degree scattering

$$\rho_\perp(\vec{v}_{\rm rel}) = \frac{Ze^2}{4\pi\epsilon_0 m_e |\vec{v}_{\rm rel}|^2}.$$
(4)

In the actual beam, collisions with arbitrarily small impact parameters can occur. In simulations, the problem is that very small impact collisions are rare, and there may not be enough of them to accumulate good statistics. Therefore, in simulations it is advantageous to introduce a cutoff

ρ_c, and to ignore collisions with impact parameter less than ρ_c. A modified Coulomb logarithm for equation (2), which can take into account ρ_c as well as the finite interaction time is given by [2]

$$\Lambda = \frac{1}{2} \log \left[\left(\frac{\rho_{max}^2 + \rho_\perp^2}{\rho_\perp^2 + \rho_c^2} \right) \left(\frac{\rho_c^2 + d^2}{\rho_{max}^2 + d^2} \right) \right], \quad (5)$$

where $d = |\vec{v}_{rel}|\tau/2$ and $\tau = s/(\gamma\beta c)$ is the interaction time in the beam frame. Note that the Coulomb logarithm (5) reduces to (3) in the case where $\rho_c = 0$ and $d \gg \rho_{max} \gg \rho_\perp$.

In general, the Coulomb logarithm Λ is different for each electron because of the dependence of $\rho_\perp$ and d on $\vec{v}_{rel}$. We cannot hope to calculate the integral (2) analytically, but it is easily calculated numerically because the velocity distribution $f_0(\vec{v})$ decays exponentially.

If $\omega_p = (n_e e^2/(m_e \epsilon_0))^{1/2}$ is the plasma frequency, then the Debye lengths in x, y and z, respectively, are $\lambda_x = \sigma_x/\omega_p$, $\lambda_y = \sigma_y/\omega_p$ and $\lambda_z = \sigma_z/\omega_p$. For this accelerated beam the longitudinal Debye length is considerably less than the transverse Debye length.

Over one passage through the cooling section, suppose we need to accumulate more than N_c collisions of impact parameter less than ρ_c in order to get good statistics on these collisions. In [2] it is shown that for the isotropic case, N_c and ρ_c are related by the formula

$$\rho_c = \sqrt{\frac{N_c}{\langle|\vec{v}_{rel}|\rangle \tau \pi n_e}}, \quad (6)$$

where $\langle|\vec{v}_{rel}|\rangle$ is the average magnitude of the relative velocity over all electrons. This equation can be used in the numerical models to set the cutoff impact parameter ρ_c.

To estimate the dynamical friction in the actual beam, we numerically evaluate the friction (2) using the Coulomb logarithm (5) with $\rho_c = 0$ and $\rho_{max} = 3r_x$, where r_x is the transverse RMS bunch size. Figure 1 shows the friction force in eV/m obtained for protons moving at various velocities. If the electron velocity distribution were isotropic, Figure 1 would be radially symmetric in $|\vec{v}_{ion}|$. Instead it shows enhanced friction longitudinally, and the maximum friction force of 0.659 eV/m occurs for a proton moving purely longitudinally with $v_z = 0.90 \times 10^5$ m/s.

SIMULATION MODELS

Electron cooling is an extremely weak effect, and difficult to capture numerically. The Debye shielding cloud around one proton has a magnitude on the order of the proton charge, $|e|$. Therefore, the perturbation in the electron bunch due to the presence of a single proton is nine orders of magnitude smaller than the charge of the electron bunch, $(2.5 \times 10^9)e$.

The first model we consider is the **binary collision** model [2]. The Coulomb collisions are modeled as an n-body problem, although we ignore electron-electron interactions. The interaction between each ion and electron is
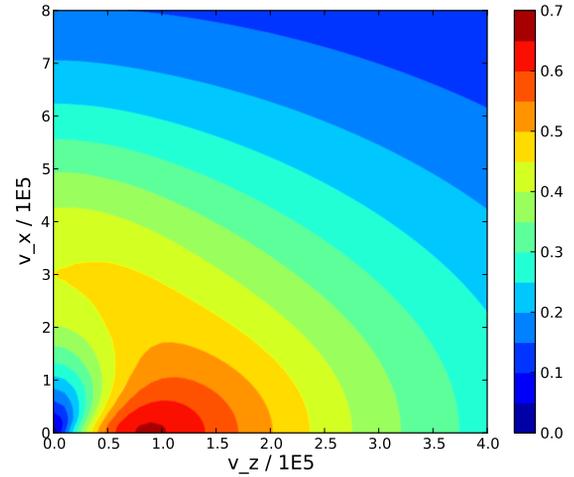
Figure 1: A contour plot of the friction force in eV/m as a function of the longitudinal and transverse ion velocity. This figure is only shown in one quadrant because it is symmetric in the other three.

calculated exactly as a 2-body problem, with the contributions summed over all particles. This produces an n-body solver which can accurately model any 2-body collision with a constant time step [2]. To accurately model 3-body and higher collisions the time step must be decreased during these interactions, however these collisions are rare and do not contribute appreciably to dynamical friction.

In order to reduce the noise in these simulations, we add a positron over each electron with identical location and velocity. When an electron and positron interact with a proton, the diffusive velocity kicks cancel while the dynamical friction accumulates. Without this trick the dynamical friction would be dominated by particle noise.

Figures 2 and 3 show friction estimates from the binary collision model, the two figures are for purely longitudinal or transverse motion, the two axes of Figure 1. However, the theoretical curves are not the same as in Figure 1, because we have adjusted the theory to take into account the size of the numerical domain (L), and use a ρ_c corresponding to $N_c = 5$. The simulation domain is a cube $L = 800$ μm on a side, and in [2] it is shown that we should use $\rho_{max} = \min\{3r_x, 0.43L\}$ and $d = \min\{|\vec{v}_{rel}|\tau/2, 0.43L\}$. These modifications of the Coulomb logarithm (5) reduce the friction by about 30% compared with the values in Figure 1. The theoretical friction values compare well with those predicted by the binary collision model. The error bars are derived by comparing friction rates for multiple ions. In the transverse case (figure 3) the errors can be large, probably because the electrons are hotter transversely ($\sigma_x > \sigma_z$).

The second model we consider is a **linear δf PIC model** [3–5]. We consider the shielding to be a perturbation from the equilibrium Vlasov-Poisson equation. The perturbation is modeled using weighted particles, where the particle weight represents the perturbation and all weights start at zero. The δf particles move in response to background

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A11 - Beam Cooling
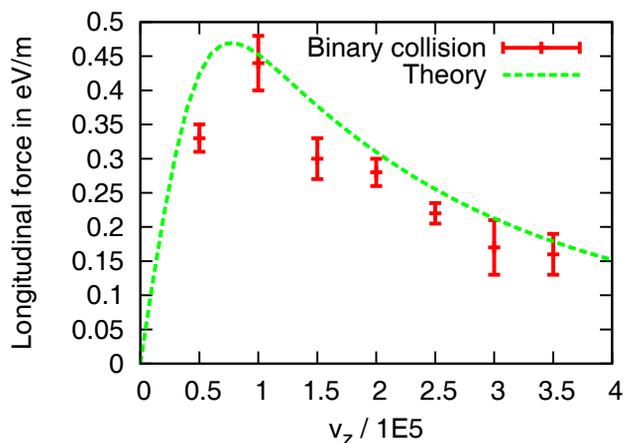
Figure 2: Theoretical longitudinal friction, eq. (2) versus that obtained from the binary collision model.
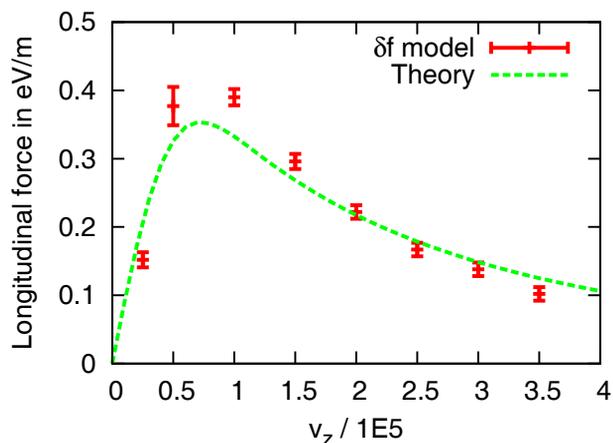
Figure 4: Theoretical longitudinal friction, eq. (2) versus that obtained from the δf model.
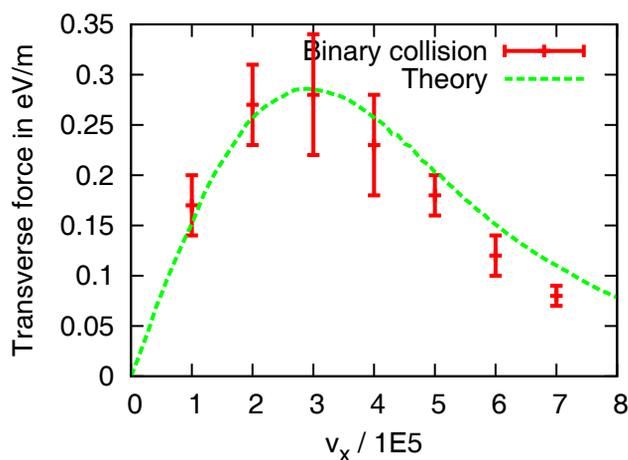
Figure 3: Theoretical transverse friction versus that obtained from the binary collision model.
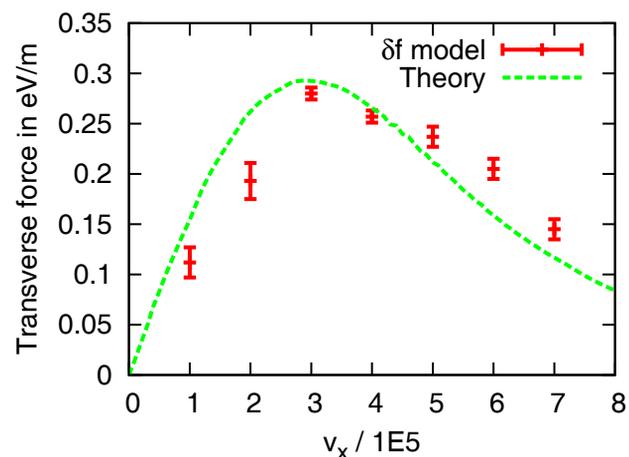
Figure 5: Theoretical transverse friction versus that obtained from the δf model.

fields, and their weights evolve to capture the shielding. The ions move in the electric field of the δf particles using the Boris push [6]. In all cases, the average force on each ion is calculated from the velocity change of the ion over the run. We note that a force of 0.5 eV/m results in a proton velocity change of 0.15 m/s over the cooling section, or about 1 part in 10^6 of the proton beam frame velocity.

Figures 4 and 5 show dynamical friction results for the δf model. The δf model uses a different domain size which is not cubical, therefore in the theoretical model we have inserted a different L. The error in the δf method is often very small, because different protons behave similarly. The δf method eliminates much of the particle noise present in the binary collision model.

SUMMARY

Theoretical estimates of dynamical friction (2) have been confirmed by two numerical simulations. The theoretical model includes effects from the finite interaction time τ, and also effects present in simulations such as finite domain size L and a lower cutoff in impact parameter, ρ_c. The nu-

merical simulations agree well with the theoretical model when we insert appropriate values of these parameters.

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