SINGLE SPOKE RESONATOR INNER ELECTRODE OPTIMIZATION DRIVEN BY REDUCTION OF MULTIPOLES*

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Abstract

Accelerating cavities based on coaxial resonators, half wave and spoke resonators for example, do not have azimuthal symmetry. This lack of symmetry introduces a transverse field perturbation which affects negatively the beam dynamic, since the particles traveling through the structure are crossing two accelerating gaps separated by the inner electrode. The field asymmetry induces an asymmetric transverse momentum gain which, once expanded in multipoles, appears to be due to a quadrupole perturbation. Depending on the cavity geometry and particle velocity, the influence of electric and magnetic fields may vary quite significantly. One solution to obtain symmetric transverse fields in spoke resonators consists in modifying the inner electrode shape from a pole to amore elaborated structure resembling an X or a Y letter shape. The application of these changes symmetrizes both electric and magnetic fields and reduces the multipoles amplitudes to negligible values. This paper presents the study aimed at reducing the multi-pole amplitudes for SSR2 cavity for Project X; the presented procedure, in general, is valid for any spoke cavity.

INTRODUCTION

Coaxial resonators are widely used for low-beta applications At Fermilab one family of half wave resonators (HWR) and two families of single spoke resonators (SSR) will be used in Project X. The geometry of both HWR and SSR is obtained from a coaxial line approximately half of a wave length long. The accelerating gaps of these cavities are formed by the inner and outer conductors of the coaxial resonator. Particles are travel along an axis perpendicular to the inner electrode. Let us assume that the particles are traveling through the gaps along the z axis, the lack of azimuthal symmetry leads to a different transverse momentum gain on x and y axes, acting as a quadrupole on the beam [1]. This issue has already been examined at Fermilab [1], [2] and a solution has been implemented to overcome HWR field asymmetry [3]: the central part of the inner electrode was modified using a radially symmetric circular shape. Such modification allows to obtain regular transverse electric field reducing asymmetric effects on the beam envelope. When the same approach was used on SSR2, the second family of spoke resonators for Project X, results were quite unexpected: the field asymmetry was even higher than before. This was caused by the magnetic transverse fields, which are not negligible in SSR2 cavities.

TRANSVERSE FIELD ASYMMETRY

Half wave and spoke resonators accelerating mode is quasi-TEM mode, that has non-zero transverse components on paths slightly off the ideal electric axis of the cavity. Considering the z axis as the longitudinal direction, electric and magnetic fields will be present in x and y. In figure 1 transverse field components of SSR2 cavity are plotted along z, for an offset of ten millimetres on the x-y plane: along x axis E_x and H_y are present, while for an offset on y direction E_y and H_x.

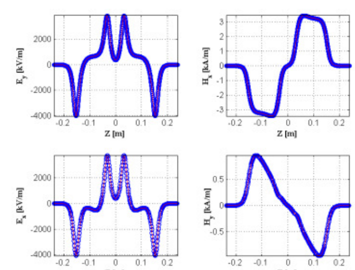

Figure 1: SSR2 electric and magnetic transverse fields.

A particle traveling with a radial offset r will experience a transverse kick due to the electric and magnetic fields:

$$\Delta p_x(r,\alpha)c = \int_{z_i}^{z_f}\left(\frac{E_{x(r,\alpha)}}{\beta} - i\boldsymbol{Z}_0 H_y(r,\alpha)\right)e^{i\frac{kz}{\beta}}dz \quad (1)$$

$$\Delta p_y(r,\alpha)c = \int_{z_i}^{z_f}\left(\frac{E_y(r,\alpha)}{\beta} + i\boldsymbol{Z}_0 H_x(r,\alpha)\right)e^{i\frac{kz}{\beta}}dz. \quad (2)$$

Where $\boldsymbol{Z}_0$is the impedance of free space, $\beta = v/c$ and α is an angle, taken with respect to the x axis: α=0 corresponds to x axis and $\alpha=\pi/2$ refers to y axis. The momentum gain can be calculated in the whole beta range for the accelerated particles, and its components can be evaluated separately to highlight the difference between electric and magnetic contribution. The electric field line are directed from the spoke to the outer walls, while the magnetic field lines spin around the inner post, hence the magnetic field component orthogonal to the electrode will be higher than the one parallel to it. Figure 2 shows SSR2 single spoke geometry, the green arrows on the left (a) represent a sketch of the electric field lines while the orange circles on the right (b) show the magnetic field spinning around the electrode.

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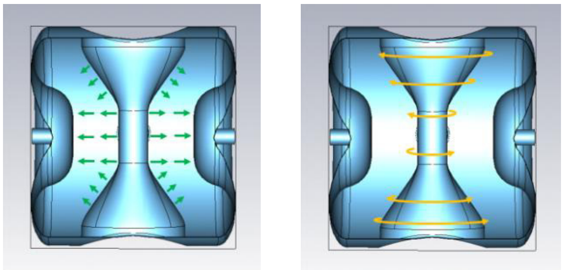
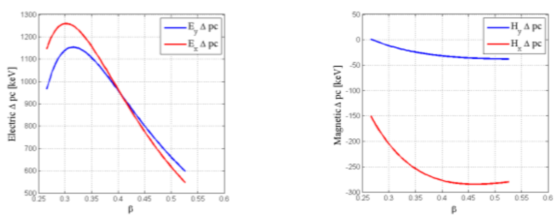


(a) (b)

Figure 2: field line sketch of a spoke resonator (a) electric and (b) magnetic [1].

Assuming that the inner post lies along y, the magnetic field component orthogonal to the spoke will be H_x and the parallel one will be H_y; see plots in fig. 1 for comparison. All the transverse momentum gain components have been plotted in figure 3, the entire energy range of SSR2 cavity in Project X has been analysed (approximately from 40 MeV to 160 MeV). The left plot shows the electric field contribution; on the right the magnetic momentum gain is shown. Looking at fig. 3, it is immediately clear that the magnetic asymmetry is not negligible, especially for the higher beta region.



(a) (b)

Figure 3: (a) electric and (b) magnetic transverse momentum gain [1].

As a figure of merit for field asymmetry the following parameter Q can be used:

$$Q = \frac{\Delta p_x(r,0)c - \Delta p_y(r,\pi/2)c}{(\Delta p_x(r,0)c + \Delta p_y(r,\pi/2)c)/2}. \quad (3)$$

This parameter, evaluated on the full particle energy domain of SSR2, has been plotted in figure 4; its value is independent from the gradient or peak field because it is normalized over the sum of the two components: $\Delta p_x(r,0)c$ and $\Delta p_y(r,\pi/2)c$.

Calculating a multipole expansion of the transverse kick, it is possible to show that the asymmetry parameter is a measure of the quadrupole amplitude over the monopole.

ELECTRODE DESIGNS SHOWING NEGLIGIBLE FIELD ASYMMETRY

In order to bring symmetry to the cavity geometry it is necessary to re-design the inner electrode of the coaxial resonator. This modification consists in replicating half of the electrode with periodicity in the x-y plane, so the result will be an electrode resembling the shape of a letter Y or X. These two designs show very weak transverse field asymmetry for both electric and magnetic components. In the following subsections each design and its fields are described.

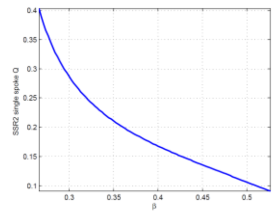
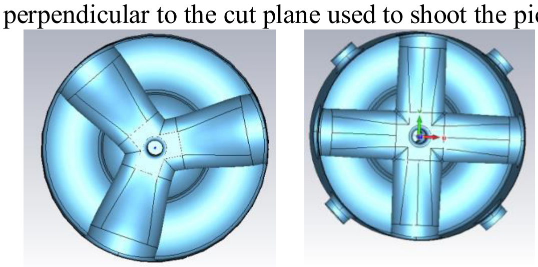


Figure 4: SSR2 single spoke asymmetry parameter vs. particle beta

Y-Shaped Spoke

The Y-shaped spoke, see fig. 5(a), consists of an inner electrode replicated and equally spaced at 120 degrees in the x-y plane. In figure 5(a) the x-y cross section of the Y-spoke design is shown, particles travel along z axis which is perpendicular to the cut plane used to shoot the picture.



(a) (b)

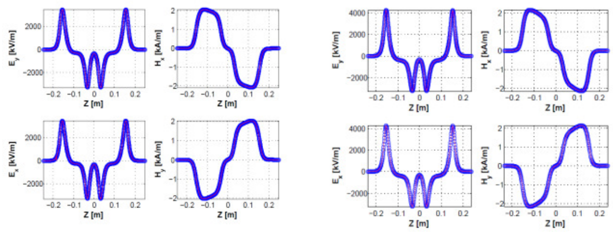
Figure 5: (a) Y-spoke design cross section, (b) X-spoke x-y section.

The transverse electric and magnetic fields benefit from the modified electrode. Transverse fields on a 10 mm offset are plotted in figure 6(a), electric on the left magnetic on the right. The x and y components of the two fields are in separate plots because an overlapping will not allow distinguishing one from the other, since they are almost identical. In figure 7 (a) the asymmetry parameter Q has been plotted versus the particle beta in the energy range of Project X, its maximum value is just ≈ 3% of the maximum Q for the single spoke. Figure 8 shows the magnetic (b) and the electric (a) transverse momentum gain on a 10 mm offset from the centre of the beam tube, the magnetic components (on the right) show some residual asymmetry.

X-Shaped Spoke

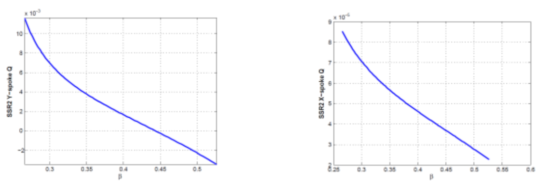
The X-spoke design has been made by intersecting two single spoke electrodes rotated by 90 degrees from each other; figure 5(b) shows the x-y cross section of the cavity geometry. Transverse field components and the transverse kicks are almost identical; this design does not have noticeable field asymmetry. Figure 9 shows the transverse momentum gain calculated for the full beta range, electric components are plotted on the left (a) and magnetic on the right (b). Since $\Delta p_x c$ and $\Delta p_y c$ electric and magnetic are identical in the plots, it is worth looking at the parameter

Q versus particle beta: figure 7 (b) shows how small the residual asymmetry is. The parameter *Q* for this cavity dropped to the 10^{-5} range.



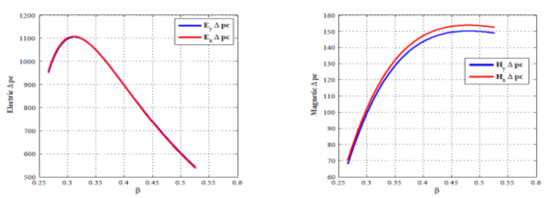
(a) (b)

Figure 6: SSR2 transverse field on 10 mm offset (a) Y-spoke, (b) X-spoke.



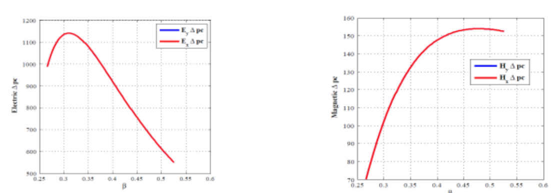
(a) (b)

Figure 7: Q parameter vs. beta (a) SSR2 Y-spoke, (b) SSR2 X-spoke.



(a) (b)

Figure 8: SSR2 Y-spoke transverse kicks vs. beta, (a) electric and (b) magnetic.

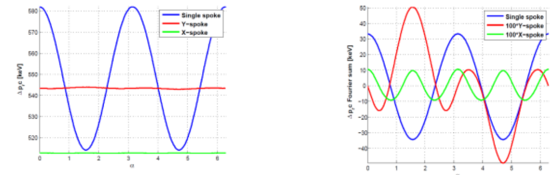


(a) (b)

Figure 9: X-spoke transverse kick components, (a) electric and (b) magnetic.

MULTIPOLE ANALYSYS

A more complete approach to the problem of transverse field asymmetry involves the expansion of $\Delta p_R c$ (the radial component of the transverse kick) in multipoles. One should to start by calculating the transverse kick at a given radial value, for all the angles on the x-y plane. Subsequently it is possible to decompose $\Delta p_R c(r, \alpha)$ in harmonics to evaluate the dipole, quadrupole, sextupole and octupole amplitudes. Figure 10(a) shows the angular dependence of $\Delta p_R c(r,\alpha)$ for all the three different designs of SSR2, calculated at r=10 mm for a synchronous particle (beta = 0.47) longitudinal energy gain of 4.5 MeV.



(a) (b)

Figure 10: (a) $\boldsymbol{\Delta p_R c}$ as a function of the angle α, (b) Fourier sum of the radial kick, constant component excluded.

Figure 10(b) shows the Fourier series of $\Delta p_R c$, the plot does not include the constant radial component, included in 10(a) instead. In 10(b) the amplitudes of X and Y-spoke have been multiplied by a factor 100. Table 1 reports the values of the multipoles amplitudes, up to octupole; it confirms how beneficial the modification to the inner electrode is, quadrupole amplitude is highlighted.

Table 1: Multipoles Amplitudes Calculated at r=10 mm and 4.5 MeV Synchronous Particle Energy Gain

Amplitudes	Single spoke	Y-spoke	X-spoke
1^{st}[keV]	5.812e-14	2.57E-01	6.31E-14
2^{nd}[keV/mm]	3.391	1.76E-03	0.000717
3^{rd}[keV/mm^2]	4.331e-16	2.43E-03	2.77E-16
4^{th}[keV/mm^3]	4.747e-4	9.52E-07	9.67E-05
5^{th}[keV/mm^4]	4.957e-18	3.58E-07	5.58E-18
6^{th}[keV/mm^5]	2.338e-08	1.53E-07	2.89E-08
7^{th}[keV/mm^6]	1.367e-19	4.31E-09	2.91E-20
8^{th}[keV/mm^7]	2.719e-10	3.49E-10	6.35E-10

CONCLUSIONS

Alternative designs for a spoke resonator have been presented; the work has been driven by reduction of the field asymmetry which leads to a quadrupole perturbation on beam dynamic. Quadrupole amplitude has been reduced significantly and the X and Y-spoke designs have symmetric transverse fields (electric and magnetic). The approach to cavity design presented in this paper can be used for any spoke resonator which shows transverse field asymmetry.

REFERENCES

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