SIMULATION OF WAKEFIELDS FROM AN ELECTRON BUNCH IN A METAMATERIAL WAVEGUIDE*

J. S. Hummelt[1], B. J. Munroe[1], S. M. Lewis[1], M. A. Shapiro[1#], and R. J. Temkin[1]
[1]Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Abstract

A metamaterial waveguide is proposed for use in a linear accelerator. The waveguide walls are made of a metamaterial structure composed of complementary split-ring resonators (CSRRs). In such a metamaterial waveguide, a TM-like mode exists that can be excited by an electron bunch. A complementary split-ring resonator is formed by slots machined in a metallic plate. The fact that the metamaterial waveguide is both planar and composed of elements that are direct machined makes it both easy to build and easily scalable to higher frequency linear accelerators. The metamaterial waveguide is coupled to single mode rectangular waveguides. These waveguides can then be connected to a microwave network to couple power into or out of the waveguide. The metamaterial waveguide that is presented was designed using electromagnetic simulations in HFSS, and wakefield as well as PIC simulations of the structure were carried out using the CST Particle Studio code.

INTRODUCTION

Metamaterial (MTM) structures [1] are periodic structures that are both sub-wavelength and operate in a frequency range in which the structures are resonant. In contrast to the disk-loaded waveguides used as accelerator structures, spatial harmonics do not play a role in the accelerating properties of MTM structures. Basic studies of MTMs for use in or as accelerator structures, beam diagnostics, and interaction structures of higher power microwave sources are of interest due to the potential benefits of using frequency selective materials in microwave generation and the advances in MTM fabrication that may allow for high frequency design.

A complementary split-ring resonator (CSRR) with negative permittivity has been introduced [1]. A medium formed of an infinite array of parallel plates with CSRRs machined into them was proposed as an accelerator structure [2,3]. This system can be considered as an infinite array of electric dipoles, and the effective permittivity and permeability of this MTM were determined and simulations showed that this MTM has a negative refractive index.

The CSRR MTM has been modified to be used as an accelerator structure. Instead of using an infinite medium, a MTM waveguide is formed by placing the CSRRs in a waveguide that is below cutoff. In this paper we study the dispersion properties of the MTM waveguide and the electron beam interaction with its field.

METAMATERIAL WAVEGUIDE

The MTM waveguide proposed (Fig. 1) is an assembly of four rectangular waveguides that have each had CSRRs machined into one of their faces as shown in Fig. 2. The waveguides are assembled such that the waveguide walls into which the CSRRs have been machined divide a 'cross' shaped pipe into four outer waveguide regions and one central waveguide region through which the electron beam passes. One period of the structure along the axial direction of the waveguide includes two split ring slots with openings facing each other, with total period p = 7 mm (Fig. 2). The MTM waveguide is designed in the S-band operating at 2.6 GHz. As the period is 7 mm, it is sub-wavelength at this frequency along the axial direction. The transverse dimension of the split rings is 41 mm. The slot width is g=2 mm and the CSRR thickness is 1.6 mm. In addition to the cross shaped structure shown in Fig. 1 we have investigated square, hexagonal, and triangular geometries as well.

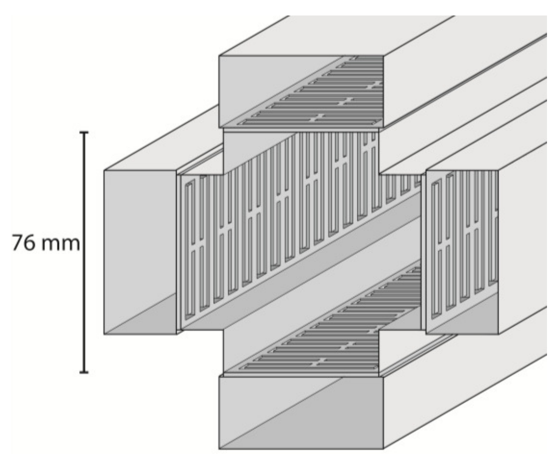

Figure 1: Metamaterial waveguide assembled from four WR187 waveguides (47.55 x 22.15 mm) with CSRRs machined into inner face of assembled structure.

*Work supported by AFOSR MURI Grant FA9550-12-1-0489 Administered by Univ. New Mexico and by DOE, High Energy Physics (Grant No. DE-SC0010075)
#shapiro@psfc.mit.edu

ISBN 978-3-95450-138-0

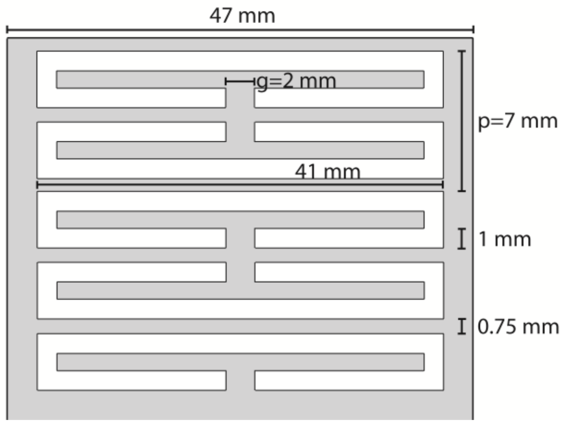

Figure 2: Metamaterial waveguide made from standard WR187 waveguide with CSRRs machined into one face of the guide.

Because of the electric response of the CSRRs, the MTM waveguide formed by the assembled waveguide structure has negative permittivity at 2.6 GHz. The structure's negative permeability is caused by the operation of the waveguide below its cutoff frequency for TM (HE) modes. The cutoff frequency of a waveguide with the same dimensions as the central region is near 5 GHz. The combination of negative permittivity and negative permeability provides a negative refractive index.

The HFSS simulation results showing the dispersion of the MTM waveguide are presented in Fig. 3. The frequency is shown as a function of phase advance per one period in degrees.

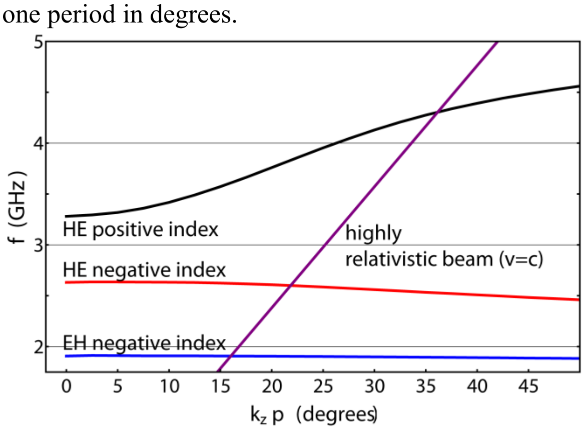

Figure 3: Dispersion curves of TM modes in the metamaterial waveguide. Two modes exist that couple to the electron beam, a negative refractive index mode at 2.5-2.6 GHz (red) and a positive index mode at 3.5-4.5 GHz (black). The relativistic electron beam dispersion line is also shown ($2\pi f=k_z c$, purple).

The negative group velocity, negative refractive TM-Hybrid mode is found at the frequency range of 2.5-2.6 GHz. In addition to this mode, the presence of the CSRRs also supports a lower group velocity mode at 1.9 GHz. The differences between these two modes have to do with the symmetry of fields between adjacent split rings. There is antisymmetry between split rings in the lowest order mode, which results in field cancellation and a very small axial field in the structure. Therefore, this mode is not expected to couple to the electron beam as well as the HE modes. At higher frequencies (3.5-4.5 GHz) a positive index mode is found. Shown in Fig. 3 is the dispersion line of a highly relativistic ($v \approx$ c) electron beam. The beam excites the MTM modes at the frequencies where this beam line intersects the modal dispersion curves.

WAKEFIELD SIMULATION

The CST Particle Studio wakefield solver was employed to simulate the wakefield created from a single impulse bunch travelling through the structure. While the wakefield solver lacks the ability to characterize the full beam-wave interaction as a PIC code would, it is an efficient way to estimate the relative coupling of an electron beam to any particular mode in a structure when we examine the wake impedance as a function of frequency. The wake impedance as a function of frequency $Z_z(\omega)$ is found by taking the Fourier Transform of the wake potential $W_z(s)$, which is defined by the relation

$$Z_z(\omega) = -\frac{\int_{-\infty}^{\infty} W_z(s)e^{-i\omega s}ds}{\int_{-\infty}^{\infty} \lambda_z(s)e^{-i\omega s}ds}$$

where λ_z is the longitudinal charge distribution and s is the distance behind the impulse bunch. The wake potential created by the test bunch, shown in Fig. 4, is displayed in Fig. 5 as a function of s.

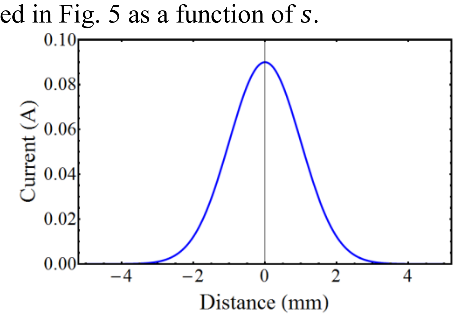

Figure 4: Test bunch current profile used in the wakefield simulations.

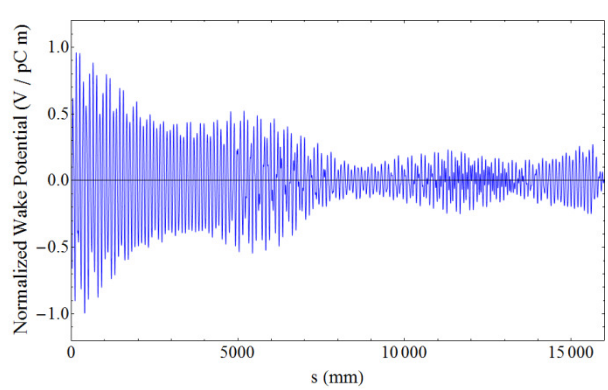

Figure 5: Longitudinal wake potential as a function of the distance behind the impulse bunch (s).

ISBN 978-3-95450-138-0

For this simulation the bunch length used is much shorter than the wavelength of interest. Fig. 6 shows the longitudinal wake impedance as a function of frequency. The on-axis bunch excites the TM modes in the MTM waveguide and the longitudinal wake impedance indicates narrow band excitation at the frequencies of 2.6 GHz and 4.5 GHz. In addition to these two modes, there is slight excitation of the mode at 1.9 GHz because it does have a small, but nonzero axial field. The frequencies of each mode are in good agreement with the HFSS electromagnetic simulations.

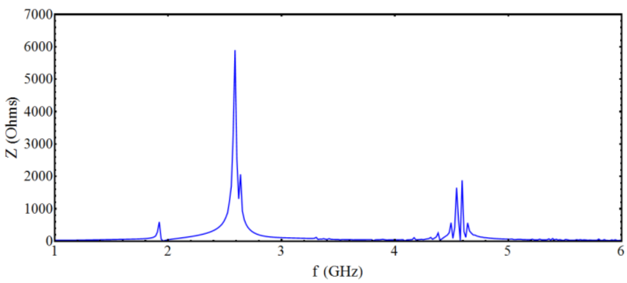

Figure 6: Longitudinal wake impedance shown as a function of frequency.

RADIATION OF A TRAIN OF BUNCHES

PIC simulations of a train of bunches in the 2.6 GHz MTM waveguide were performed using the CST Particle Studio code. This research is of interest for potential application at the beam line of the Haimson Research Corporation (HRC)/MIT 17 GHz linear accelerator, where the MTM waveguide simulated at 2.6 GHz is easily scalable to 17 GHz due to the frequency selectivity and planar geometry of the MTM. The HRC/MIT accelerator produces a train of sub-picosecond bunches with the repetition rate of 17.14 GHz. Radiation from this 17 GHz train of bunches has been used for bunch diagnostics, particularly bunch length measurement [4]. Radiation of the train of bunches in a photonic band gap structure was characterized in Ref [5].

These simulations were done for the S-band MTM waveguide and a train of bunches with a repetition rate of 2.6 GHz, an injected energy of 500 keV, and a time average current of 1 Amp. Fig. 7 shows the 1.1 m length of MTM waveguide with the electron bunches injected from the left.

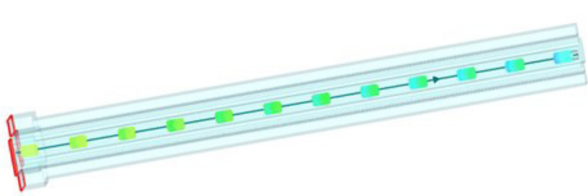

Figure 7: CST simulation of metamaterial waveguide structure with a train of bunches.

The radiation output is at the injection end of the structure because the radiation propagates backward, as predicted from the negative dispersion of the HFSS simulation. The energy distribution along the length of MTM waveguide in the train of bunches is shown in Fig. 8. Bunches give up their energy to radiation and the energy spread increases as the energy decreases.

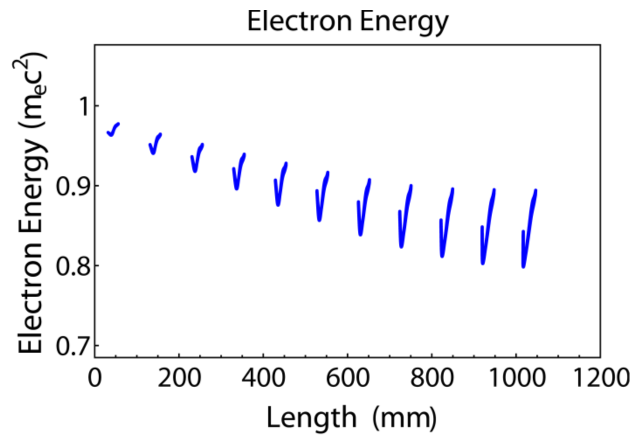

Figure 8: Electron energy distribution of the train of bunches as they travel through the structure. The bunches are injected at an energy of 500 keV (0.98 m_ec^2).

CONCLUSIONS

A metamaterial waveguide is proposed for use in an accelerator as a diagnostic or accelerating structure. The MTM waveguide is designed at the S-band but can be scaled to higher frequencies due to the simple and easily machined planar elements that give the waveguide its frequency response. A negative refractive index mode was identified for this MTM waveguide using the code HFSS, and this mode was also confirmed to both exist and couple strongly to the electron beam using the wakefield solver of CST Particle Studio. The excitation of the MTM waveguide by the electron beam was modelled and studied using the PIC solver of CST. A possible experiment using the HRC/MIT linac is presented.

REFERENCES

[1] R. Marques, F. Martin, and M. Sorolla. Metamaterials with Negative Parameters. Theory, Design, and Microwave Applications. John Wiley & Sons, 2008.
[2] M. A. Shapiro, J. R. Sirigiri, R. J. Temkin, and G. Shvets, Proc. of PAC09, Vancouver, BC, Canada, p. 2992.
[3] M. A. Shapiro, S. Trendafilov, Y. Urzhumov, A. Alu, R. J. Temkin, and G. Shvets, Phys. Rev. B 86, 085132 (2012).
[4] S. E. Korbly, A. S. Kesar, J. R. Sirigiri, and R. J. Temkin, Phys. Rev. Lett. 94, 054803 (2005).
[5] R. A. Marsh, M. A. Shapiro, R. J. Temkin, E. I. Smirnova, and J. F. DeFord, Nucl. Instr. & Meth. Phys. Res. A 618, pp. 16-21 (2010).

ISBN 978-3-95450-138-0