

NEW TECHNIQUE TO MEASURE THE EMITTANCE OF BEAMS WITH SPACE CHARGE

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Abstract

Characterization of the transverse phase space of electron beams with space charge is an important requirement for high brightness particle accelerators. We present a novel technique to measure the transverse rms emittance. The method uses the beam divergence, measured, e.g. with optical transition radiation interferometry, and the beam radius, obtained by imaging the beam, to determine the cross correlation term and thus all the terms in the equation for the rms emittance. The experimental data is obtained with a lens-drift-screen setup similar to what is used in a quadrupole scan. However, the analysis of the scan data is unique. It involves taking the cross-correlation term as a control variable in a procedure which matches the beam envelope radius and its derivative calculated from the envelope equation, with the measured radius and divergences at the screen. A linear space charge model is used in the envelope equations, hence the errors in the measurement relate to the nonuniformity of the beam transverse distribution. The technique is tested using simulated data.

INTRODUCTION

It is important to measure the transverse beam emittance in accelerators to quantify the beam quality and match the optics in an accelerator beam line. Most beams of interest are space-charge-dominated near the source and low energy transport section, where the beam dynamics are mostly determined by interparticle forces rather than the beam pressure represented by emittance. Space charge usually modifies and degrades the performance of emittance measurement methods such as quadrupole scan techniques [1]. Reference [2] presents a comprehensive analysis of space charge force effects on such measurements. Several amendments have been proposed in literature to enhance the accuracy of quadrupole scan method for high intensity beams [3].

From the symmetry of the expression for the rms emittance

$$\bar{\epsilon}^2 = \langle x^2 \rangle \langle x'^2 \rangle - \langle x \cdot x' \rangle^2$$

it is obviously possible to determine emittance with a quadrupole scan technique using measurements of either the beam radius or divergence. The usual implementation of the method is to perform a quadrupole scan of beam size only. However, it is possible to obtain simultaneous, high-quality measurements of beam size and divergence [4]. In this paper, we propose a method for measuring emittance, assuming a linear space charge model, from a small number of measurements of divergence and radius.

In order to use these observables to determine the rms emittance at arbitrary focus points we need a methodology which relates the measured observables, i.e. divergence and beam size to the cross-correlation ($\langle x \cdot x' \rangle$) term. We will show how this can be done by taking the cross-correlation term as a control variable for matching beam envelopes to their actual envelopes with the constraint that the beam radii and divergences at the screen are same as the measured values.

MEASURING EMITTANCE FOR BEAM WITH SPACE CHARGE

Figure 1 shows a typical quadrupole scan setup. The focal length of the quadrupole (or solenoid) is scanned to achieve a minimum spot size on the screen located downstream at a distance L from the quadrupole.

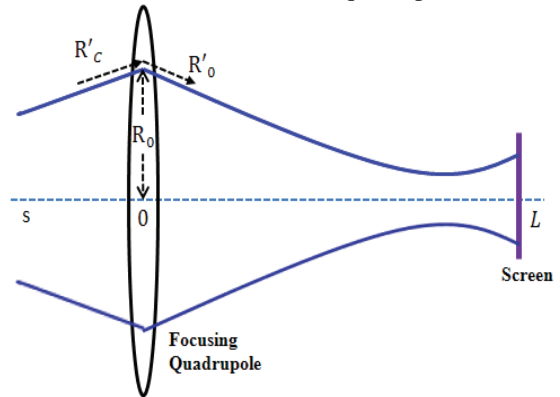


Figure 1: Diagram of a quadrupole scan setup to measure emittance. Beam enters the quadrupole with envelope R_0 and slope R'_c and leaves it with slope R'_o .

For a uniform round beam distribution where space charge is linear, the beam envelope evolution in a drift region is described by the following pair of coupled nonlinear ordinary differential equations (ODE):

$$R_j''(s) - \frac{K}{R_j(s)} - \frac{\epsilon_j^2}{R_j(s)^3} = 0 \tag{1}$$

where j ranges over transverse coordinates x and y , and dimensionless quantity

$$K = \frac{qI_b}{2\pi\epsilon_0 m(c\beta\gamma)^3} \tag{2}$$

is defined as the generalized perveance representing space charge defocusing forces. In Eq. (2), q is charge of the beam particles and I_b is the beam peak current.

We present a method for deriving emittance from two samples of beam radius and divergence for a round beam. As R_x and R_y are equal in this case, we drop the index j and treat all parameters for x .

Multiplying Eq. (1) by R' and then integrating with respect to s leads to the following first order ODE:

$$R'(s) = \pm \left[R_0'^2 + \epsilon^2 \cdot \left(\frac{1}{R_0^2} - \frac{1}{R(s)^2} \right) + 2K \cdot \ln \left(\frac{R(s)}{R_0} \right) \right]^{1/2}, \quad (3)$$

where R_0 and R_0' , denote the radius and slope of the envelope at the lens. We need to have the beam envelope relation in terms of at-screen radius and slope quantities. This can be done simply by switching R_0 and R_0' with beam quantities at the screen, $R(L)$ and $-R(L)'$, and applying a minus to the LHS of Eq. (3).

Such minuses are necessary as we are treating the envelope evolution along $-s$. Thus, envelope ODE in terms of envelope parameters at screen can be expressed as:

$$R'(s) = \mp \left[R(L)^2 + \epsilon^2 \cdot \left(\frac{1}{R(L)^2} - \frac{1}{R(s)^2} \right) + 2K \cdot \ln \left(\frac{R(s)}{R(L)} \right) \right]^{1/2}. \quad (4)$$

This form of envelope ODE can easily be solved by numerical integration in packages like MATLAB.

As there is no closed form solution to this equation for large K , we cannot provide a closed form answer for emittance. The following procedure is devised to determine the emittance.

We start with a guess for cross-correlation term and try to infer focal setting applied to the lens by solving Eq. (4). Beam radius and divergence measurements at the screen are translated to $R(L)$ and $R(L)'$ and therefore inferred focal length is going to be a function of the guessed cross-correlation. We use the error between calculated focal length with actual focal length to correct our guess for cross-correlation and reiterate the procedure. Finally, after several steps, as the inferred focal length converge toward the actual one we come within a close vicinity of the actual cross-correlation.

Numerical Procedure

The detailed steps of the procedure are described here. As before we need two sets of beam radius and divergence measurements at two distinct focal lengths f_1 and f_2 :

$$\left\{ \begin{matrix} f_1 \\ \langle x_1^2 \rangle \\ \langle x_1'^2 \rangle \end{matrix} \right\}, \quad \left\{ \begin{matrix} f_2 \\ \langle x_2^2 \rangle \\ \langle x_2'^2 \rangle \end{matrix} \right\}$$

Henceforth, we use subscript n , denoting the step number, alone or besides the measurement numbers 1 or 2 on all parameters that change over subsequent steps.

1. $XC1_n$, initial guess for the cross-correlation term at f_1 , is chosen according to the discussion in next section.

2. $XC2_n$, the cross-correlation term at f_2 , is calculated as:

$$XC2_n = \pm \sqrt{XC1_n^2 + \langle x_2^2 \rangle \langle x_1'^2 \rangle - \langle x_1^2 \rangle \langle x_2'^2 \rangle} \quad (5)$$

3. Next, ϵ_n (not to be confused with normalized emittance) and $R'_i(L)$ for two measurement sets ($i = 1, 2$) are calculated as:

$$\begin{cases} \epsilon_n = 4[\langle x_1^2 \rangle \langle x_1'^2 \rangle - XC1_n^2]^{1/2} \\ R'_1(L)_n = 2 \frac{XC1_n}{\sqrt{\langle x_1^2 \rangle}} \\ R'_2(L)_n = 2 \frac{XC2_n}{\sqrt{\langle x_2^2 \rangle}} \end{cases} \quad (6)$$

Note that all three relations are in term of $XC1_n$.

4. ODE Eq. (4) can now be solved for finding at-lens envelope radius and slope conditions of measurements 1 and 2:

$$\begin{cases} R(0)_{1n} & , & R'(0)_{1n} \\ R(0)_{2n} & , & R'(0)_{2n} \end{cases}$$

5. For f_{1n} and f_{2n} estimates of focal lengths we have:

$$\begin{cases} f_{1n} = \frac{R(0)_{1n}}{R'_c - R'(0)_{1n}} \\ f_{2n} = \frac{R(0)_{2n}}{R'_c - R'(0)_{2n}} \end{cases} \quad (7)$$

Canceling the unknown R'_c between these two equations and replacing f_{2n} with its final value f_2 leads to:

$$f_{1n} = \frac{f_2 \cdot R(0)_{1n}}{R(0)_{2n} + f_2 \cdot (R'(0)_{2n} - R'(0)_{1n})} \quad (8)$$

This equation is used to update f_{1n} at each step n . As before, the necessity for two pair of measurements arises from R'_c .

6. It can be easily checked that f_{1n} is a function of $XC1_n$. i.e. $f_{1n} = g(XC1_n)$. $XC1_n$ should be modified so that reiteration of the procedure from entry 2 makes f_{1n} closer to the target value f_1 . In other words, $XC1_n$ is zero of this equation:

$$g(XC1_n) - f_1 = 0 \quad (9)$$

Since derivative of the function g is not known, a modified form of Newton method [5] was used to find $XC1_n$ as zero of this equation.

After updating $XC1_n$ the procedure is repeated from entry 2 until f_{1n} converges with desired precision toward f_1 . The process can also be stopped when the variation on emittance calculated at two consecutive steps is less than some $\delta\%$ of the calculated emittance, where δ is usually chosen between 1 and 10. Finally, beam effective emittance is the last ϵ_n calculated in entry 3.

SIMULATION RESULTS

In this section, we present results of tests of our proposed approach with simulated beams. We used the software WARP [6] to simulate the lens-drift experiment. Based on presumed beam parameters ϵ and K and an initial envelope radius at the lens, R_0 , WARP generates a set of beam radius and divergence rms values $\langle x^2 \rangle$ and $\langle x'^2 \rangle$ at end of the drift section with length L where beam divergence and size are supposedly measured, e.g., using OTRI. The focal length of the lens, f , is calculated according to a hard edge model of the lens. The performance of the method for the several grades of the space charge was tested.

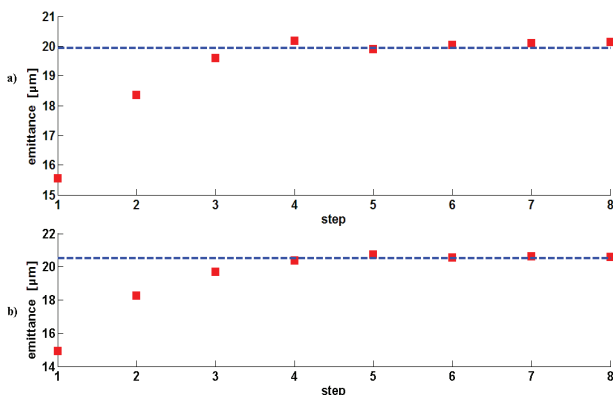


Figure 2: Plot showing convergence of emittance for (a) a medium space charge beam ($K = 2.83 \times 10^{-4}$), and (b) for a high space charge beam ($K = 7.06 \times 10^{-4}$).

Figures 2.a and 2.b show emittance convergence curves for space charge dominated beams with $K = 2.83 \times 10^{-4}$ and $K = 7.06 \times 10^{-4}$ respectively. The convergence is fast and errors in calculation of emittance are satisfactory.

To see effectiveness of the proposed procedure we tried to compare mentioned results with the emittance measured by conventional Courant Snyder (CS) parameter fitting and the minimum beam size method.

In figure 3 we compare the emittance obtained with different methods as a function of the parameter K . As expected the error in determining the emittance obtained using the CS quadratic fitting technique and the minimum beam size method, which are both accurate for emittance dominated beams, becomes increasingly large as space charge increases. In contrast, our two points fit method gives acceptable values for the emittance for all values of the K parameter shown.

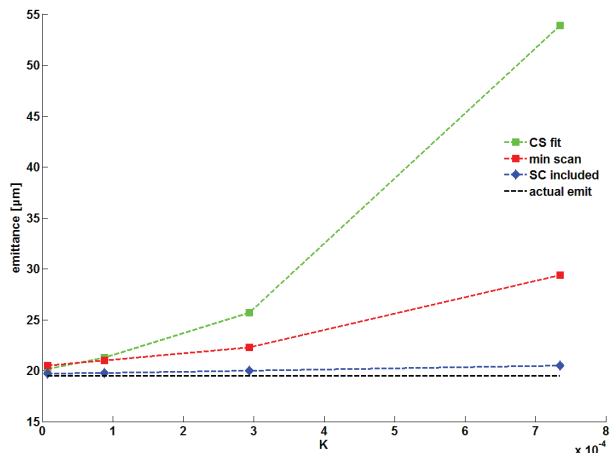


Figure 3: Comparison of the emittance measured using different methods as a function of beam perveance (K): dashed green curve: CS parameter fitting method, red curve: cross-correlation determination at minimum of quad beam size scan, blue curve: two value method including space charge, showing small deviation from the actual emittance.

ACKNOWLEDGMENT

We are grateful for inspiring discussions with J. G. Power, C. F. Papadopoulos, and I. Haber. We also thank J. L. Vay and D. P. Grote for their outstanding support of the WARP code. The work is supported by the U.S. Department of Energy, Office of High Energy Physics contract no. DEFG0207ER41489, the US Department of Defense, Office of Naval Research contract no. N000140711043 and the Joint Technology Office.

REFERENCES

- [1] B. E. Carlsten., J. C. Goldstein, P. G. O'Shea, and E. J. Pitcher, "Measuring emittance of non-thermalized electron beams from photoinjectors," *Nuclear Instruments and Methods in Physics Research, Section A* 331 (1993) 791.
- [2] S. G. Anderson, J. B. Rosenzweig, G. P. LeSage, and J. K. Crane, "Space-charge effects in high brightness electron beam emittance measurements," *Phys. Rev. ST Accel. Beams* 5 (2002) 014201.
- [3] C. G. Limborg, S. M. Gierman, and J. G. Power, "A Modified Quad Scan Technique for Emittance Measurement of Space Charge Dominated Beams," in *Proceedings of the IEEE PAC 2003*, p. 2667 (2003).
- [4] R. B. Fiorito and D. W. Rule, "Optical transition radiation beam emittance diagnostics," *AIP Conf. Proc.* 319, p. 21 (1994).
- [5] G. J. Borse, *Numerical Methods with MATLAB* (PWS Publishing Company, 1997).
- [6] A. Friedman, "Simulation of Intense Beams for Heavy Ion Fusion," *Nucl. Instrum. Meth. Phys. Res. A* 544 (2005) 160.