# HELICAL SELF FOCUSING AND COOLING ACCELERATING STRUCTURE

S.V. Kuzikov#, S.V. Mishakin, A.A. Vikharev, IAP RAS, Nizhny Novgorod, Russia

*Abstract*

A new helical RF accelerating structure, which can provide emittance control of the electron beam, is proposed. This structure combines properties of a linear accelerator and a damping ring simultaneously. It makes acceleration of straight on-axis beam as well as beam cooling due to the synchrotron radiation of particles. These properties are provided by specific slow eigen mode which consists of two partial waves, $TM_{01}$ and $TM_{11}$ (near to cut off). The longitudinal field of the first wave is synchronous with electrons, the transverse fields of the second wave are far from synchronous condition and they cause electron wiggling like it occurs in RF undulator. As a result the emittance control might be employed at linear trajectory of the high-energy beam without decrease of the average gradient. Calculations show that surface electric field at level ~0,31 (relative to accelerating field) and shunt impedance ~20 MOhm/m at 30 GHz are achievable. Cooling rate at gradient 100 MV/m corresponds to the equivalent magnetic field ~0,75 T.

## INTRODUCTION

High luminosity of colliding particles in meeting point is achievable for compact, well focused bunches. Such bunches are achievable using systems of particle's cooling. At energies up to several GeV dampling rings provides effective cooling [1]. In these rings particles move in spatially-periodic fields of DC magnets. There is cooling effect due to synchrotron radiation of particles [2]. In order to reach TeV level of energy, behind a cooling ring particles are directed in main linac. Unfortunately emittances of bunches propagating in high-gradient accelerator have tendency to increase, because long trains of particles inevitably excite wakefields which spoil beam quality.

At high energies (more than 10 GeV) rings cannot be used, because too high level of energy dissipation caused by radiation, and too big radius of a ring would be necessary. That is why, cooling undulator can be installed at the linear trajectory only. However, DC magnets around the accelerating structure would conflict with feeding, focusing, and diagnostic systems, and inevitably large period does not allow reaching small emittances, because the smallest achievable emittance is proportional to squared wiggler period [3]. Suggested in the paper [3] accelerator with alternating accelerating sections and wigglers would reduce effective gradient.

An accelerating RF structure, which also plays a role of RF undulator, does not reduce gradient. Such structure might be based on helical corrugated waveguide, where the operating normal mode consists of the $0^{th}$ spatial harmonic (with positive phase velocity), which is actually the accelerating mode, and the $-1^{st}$ harmonic (with negative phase velocity) which is responsible for particle wiggling. The transverse non-synchronous field components can provide emittance control and beam focusing due to ponderomotive force which depends on transverse electric field gradient of the $-1^{st}$ harmonic wave.

The enumerated properties allow to distinguish our new helical self focusing and cooling structure (HSFC) [4] among other helical structures investigated earlier [5, 6]. Nevertheless, there are important properties to be common for all helical structures. For example, like other helical structure a new structure has smooth shape of the constant circular cross-section (no expansions or narrowings) and big aperture (no small irises). A new technology of the mass production, based on a "corkscrew" in a copper mandrel, seems also possible which allows avoiding junctions inside sections.

## TM01-TM11 HELICAL ACCELERATING STRUCTURE

Let us consider the accelerating structure based on helical waveguide described in cylindrical system of coordinates (r, z, $\varphi$) by equation:

$$r(z,\varphi) = R + a \cdot \sin\left(\frac{2\pi \cdot z}{L} + m\varphi\right). \qquad (1)$$

where R – is a waveguide radius, a and L – are amplitude and period of the corrugation. Period of the corrugation is close to $2\pi/h_{TM01}$, where $h_{TM01}$ – is a propagation constant of the partial $TM_{01}$ mode in smooth circular waveguide of the radius R. If index m equals 1, such helical structure couples partial waves $TM_{01}$ and rotating on azimuth $TM_{11}$. The $TM_{01}$ does not perturb transverse movement of on-axis electrons, the $TM_{11}$ does not have longitudinal $E_z$ component on axis.

Table 1: Parameters of $TM_{01}$-$TM_{11}$ HSFC Structure

| | |
|---|---|
| Radius, R | 6.09 mm |
| Period, L | 8 mm |
| Corrugation amplitude, a | 1.25 mm |
| Azimuth index, m | 1 |
| Frequency, f | 28.2 GHz |
| Phase advance, $h_0L$ | 4.727 |
| Q-factor (copper) | 10800 |
| Norm. accelerating field, $E_a / E_s^{max}$ | 0.307 |
| Shunt impedance, $R_{sh}/L$ | 18.9 MOhm/m |

#kuzikov@apppl.sci-nnov.ru


ISBN 978-3-95450-138-0

Parameters of helical structure $TM_{01}$-$TM_{11}$ was optimized to reach maximum of the accelerating field relative to maximum of the arisen surface field $E_a / E_s^{max}$, maximum of shunt impedance $R_{sh}$ was also necessary. Results of optimization for geometry at frequency 28GHz are shown in the Table 1. At acceleration gradient 100 MV/m the equivalent magnetic field is as high as 0.75 T. Dispersion relation for the optimized structure in a form of dependence of the eigen frequency f on phase advance $h_0L$ ($h_0$ – is a Floquet propagation constant) is plotted in Figure 1. The line 1 in Fig. 1 is actually light cone which corresponds to the condition $h_0$=k. The operating mode (curve 2) has low group velocity (several per cents of light velocity). Of course, the operating point is the cross point of the curves 1 and 2.

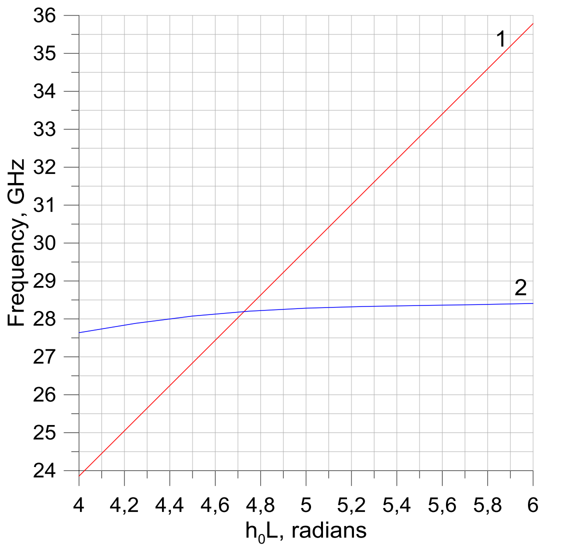


Figure 1: Dispersion characteristics: 1 – light cone line, 2 – dispersion curve of $TM_{01}$-$TM_{11}$ wave in HSFC structure.

Field structure of the operating mode is shown in Fig. 2 (at middle plane) and in Fig. 3 (at two cross-sections). Accelerating field magnitude is uniform at beam line (Fig. 2).

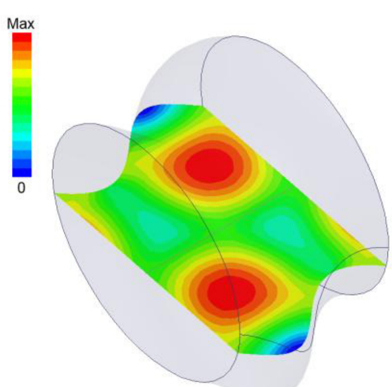


Figure 2: Module of electric field in middle plane.

Transverse field components are also uniform (Fig. 3), but phase velocities of longitudinal component and both transverse components have opposite signs (Fig. 4). Phase velocities of the transverse magnetic components are directed like phase velocities of transverse electric field components.

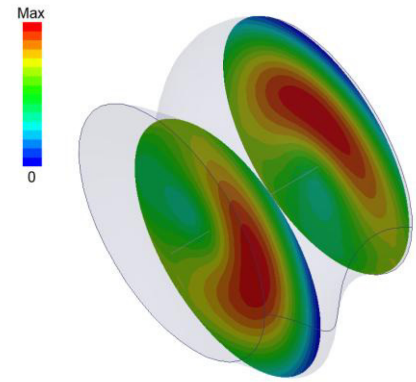


Figure 3: Module of electric field in two sequent cross-sections.

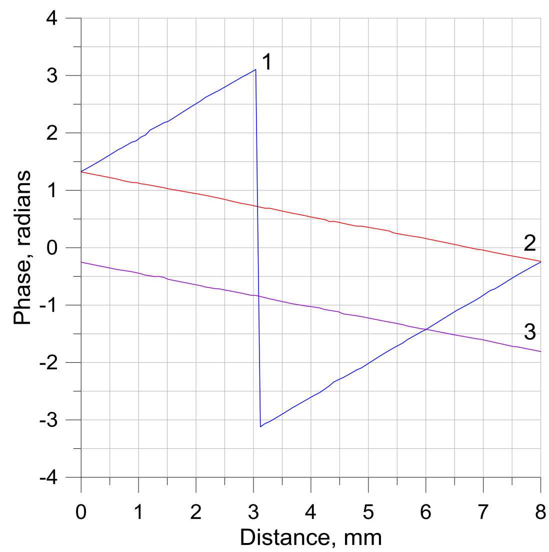


Figure 4: Phases of $E_z$ (longitudinal) field component (curve 1) and phases of $E_x$ (curve 2) and $E_y$ (curve 3) field components.

The helpful feature of $TM_{01}$-$TM_{11}$ HSFC structure is a flexible focusing of electrons. Physically the reason is that electric field of the -1$^{st}$ spatial harmonic has non-uniform distribution with minimum at axis of the structure. In such system moving electrons "sees" focusing ponderomotive (Miller) force:

$$F_{\perp} = -\frac{e^2}{4m(\omega + |h_{-1}|v_e)^2 \gamma^2} \nabla_{\perp} |E|^2 . \qquad (2)$$

where e and m – are charge and mass of an electron respectively, $v_e \approx c$ - is a velocity of electrons, $\gamma$ - Lorentz's factor, $h_{-1} = h_0 - 2\pi / L$ - is a propagation constant of the -1$^{st}$ spatial harmonic.

Near cut off condition the $TM_{11}$ mode has field maxima determined by $E_z$ components which are far from axis. That is why, there is the focusing force. In the rotating $TM_{11}$ mode $E_z$ component is distributed uniformly on azimuth, but the superposition of $TM_{11}$ and $TM_{01}$ produces field distribution with maxima rotating on azimuth and moving along z coordinate (Fig. 3). The average value of this force does not depend neither on azimuth nor distance.

In order to show focusing effect in $TM_{01}$-$TM_{11}$ HSFC structure, we calculated by CST Microwave Studio 100 MV/m accelerating section consisted of ten periods. Initial energy of electrons at entrance of the structure was 10 GeV, bunch length was 1 ps, bunch diameter was taken as 3 mm, full charge was as high as 100 nc. In Fig. 5a one can see input transverse bunch population, in Fig. 5b output bunch population is shown. One can see that particles near the centre of the bunch are not perturbed by RF fields, but bunches at bunch periphery are shifted in direction to the origin. This shift is caused by the mentioned action of the ponderomotive force.

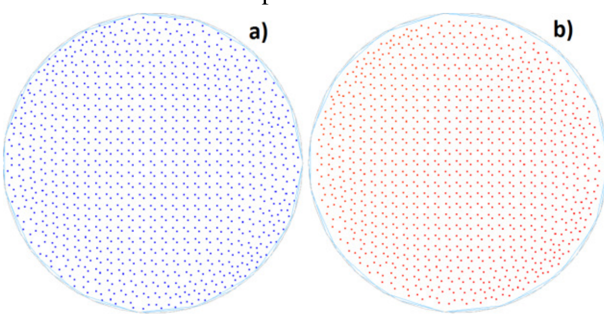


Figure 5: Bunch population at entrance of structure (a) and bunch population at output of structure (b).

Calculations of long HSFC structure, aimed to provide gradient as high as 100 MV/m, show that at distance 3 km energy spread is reduced by factor 40%, transverse emittance becomes less by factor 50%, full energy is less in comparison with classical structure without cooling by factor 17%.

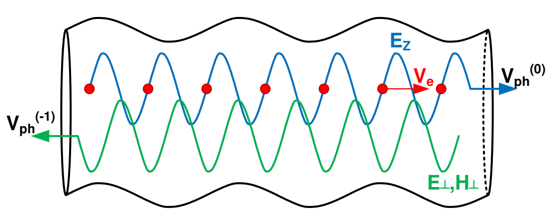


Figure 6: RF undulator excited by bunches itself.

Note that HSFC structure can be used as RF undulator only. In this case there are several appealing features. In particular, the same bunches (if its quality is high enough) could be used, in order to excite the operating mode, and in order to produce undulator radiation. The second reason to use HSFC structure as RF undulator could be important for sources of THz radiation where low-energy (MeV) electrons are used. These bunches experience an influence of Coulomb force, so longitudinal bunch size grows very fast on length. The longitudinal size could be controlled by $E_z$ wave component in the helical structure, if one synchronizes bunches with RF field, like it is shown in the Fig. 6.

## OTHER SCHEMES

Schemes, where only properties of RF undulator or only properties of acceleration occur, are also possible.

In particular, helical accelerating structures $TM_{01}$-$TM_{21}$ or $TM_{01}$-$TE_{21}$ of elliptic cross-sections do not have on-axis transverse fields. Nevertheless, properties of beam focusing in these structures still exist. The first mentioned structure has $E_a / E_s^{max} \approx 0.3$, but the shunt impedance is less than in the considered $TM_{01}$-$TM_{11}$ structure.

In the structure $TE_{11} - TM_{01}$, where the $0^{th}$ harmonic is $TE_{11}$, and the $-1^{st}$ harmonic is $TM_{01}$, the shunt impedance is high enough, but the parameter $E_a / E_s^{max}$ is less than in the $TM_{01}$-$TM_{11}$ structure.

## CONCLUSION

Helical waveguides might be used as accelerating structures. These waveguides allows to accelerate electrons (or positrons), to control beam emittances and energy spread due to cooling, to provide self focusing due to transverse ponderomotive force or to combine all the mentioned effects simultaneously.

## ACKNOWLEDGMENT

The authors would like to thank Prof. J.Hirshfield for his stimulating interest to this work and for helpful discussions with him.

## REFERENCES

[1] G.K. O'Neill. Storage Ring Synchrotron: Device for High Energy Physics Research. Physical Review, No. 5, v.102, pp.1418–1419, 1956.

[2] J.M.J. Madey. Stimulated Emission of Radiation in Periodically Deflected Electron Beams. Journal of Applied Physics, Vol. 42, No. 1, pp. 1906-1913, 1971.

[3] H.H. Braun, M. Korostelev, F. Zimmermann. "Potential of Non-standard Emittance Damping Schemes for Linear Colliders". Proceedings of APAC 2004, Gyeongju, Korea, pp. 306-308, 2004.

[4] S.V. Kuzikov, A.A. Vikharev, J.L. Hirshfield. "Helical Accelerating Structure with Controllable Beam Emittance", 1st European Advanced Accelerator Concepts Workshop, Isola d'Elba, 2013. https://agenda.infn.it/conferenceOtherViews.py?confId=5564&view=standard

[5] Wilson, J. L., Fathy, A. E., Kang, Y. W., & Wang, C. "Applications of twisted hollow waveguides as accelerating structures". IEEE Transactions on Nuclear Science. Vol. 56, No. 3, pp. 1479-1486, 2009.

[6] Awida M. H. "Investigation of Beam-RF Interactions in Twisted Waveguide Accelerating Structures Using Beam Tracking Codes". Proceedings of PAC09, Vancouver, BC, Canada, 2009, WE2PBC03, pp. 1855-1857.




ISBN 978-3-95450-138-0