

# PHYSICS OF POLARIZED PROTONS IN ACCELERATORS\*

M. Bai

Brookhaven National Laboratory, Upton, NY 11973, USA

## Abstract

Polarized proton beams have been one of the essential elements in fundamental research such as unveiling the deep secret of proton spin structure. Polarized proton beams can also be the tool for direct measurement of the proton's electric dipole moment (EDM). However, due to the interaction between spin motion and electric and magnetic fields, it is very challenging to overcome various depolarizing mechanisms through acceleration, and necessary spin manipulations at a store energy to meet the physics program requirements. Several decades of efforts have been devoted to develop techniques to preserve polarization and spin manipulation, as well as further our understanding of spin dynamics. These efforts directly led to the successful spin program at the Brookhaven Relativistic Heavy Ion Collider (RHIC), the world's first high energy polarized proton collider. This tutorial presentation introduces basic physics which governs the spin dynamics in accelerators. A brief history of polarized protons development, as well as what have been achieved at RHIC are also reported.

## INTRODUCTION OF SPIN DYNAMICS BASICS

Just like charge and mass, spin is also an intrinsic property of elementary particles. Spin vector  $\vec{S}$  for a particle is then defined as

$$\vec{S} = \langle \psi | \vec{\sigma} | \psi \rangle, \quad (1)$$

where  $\vec{\sigma}$  is the Pauli matrices, and  $\psi$  is the particle spin state. Proton, a spin of  $\frac{1}{2}$  particle, has two eigenstates, i.e.  $\psi_+ = |\frac{1}{2}, +\frac{1}{2}\rangle$  (up state) and  $\psi_- = |\frac{1}{2}, -\frac{1}{2}\rangle$  (down state). The intrinsic magnetic moment of a particle is then given by

$$\vec{\mu} = g \frac{e}{2m} \vec{S} \quad (2)$$

where  $e$  and  $m$  are the electric charge and rest mass of the particle.  $g$ -factor for a perfect point-like particle is  $\frac{1}{2}$ . For proton, the anomalous  $g$ -factor  $G = \frac{g-2}{2}$  is 1.7928474.

For a beam of spin- $\frac{1}{2}$  particles, polarization  $P$ , on the other hand, is a classical concept defined as

$$P = \frac{N_+ - N_-}{N_+ + N_-} \quad (3)$$

where  $N_{\pm}$  are the number of particles in the two spin states, respectively [1].

In a circular accelerator, the spin motion is governed by the Thomas-BMT equation [2]. In the frame of particles

orbital revolution, it is given by

$$\frac{d\vec{S}}{dt} = \frac{e}{\gamma m} \vec{S} \times [G\gamma \vec{B}_{\perp} + (1+G)\vec{B}_{\parallel} + (G\gamma + \frac{\gamma}{\gamma+1}) \frac{\vec{E} \times \vec{\beta}}{c}], \quad (4)$$

where  $\vec{S}$  is the spin vector in the particle's rest frame,  $\vec{B}_{\perp}$  and  $\vec{B}_{\parallel}$  are the transverse and longitudinal components of the magnetic fields in the laboratory frame with respect to the particle's velocity  $\vec{\beta}c$ ,  $\vec{E}$  is the electric field particle encounters and  $\gamma$  is the relativistic Lorentz factor. Here, Eq. 4 shows that in a perfect planar circular accelerator, the spin vector precesses  $G\gamma$  times in one orbital revolution.  $Q_s = G\gamma$  is then defined as spin tune.

In general, the effect on spin motion from electric field in an accelerator is negligible. Equation 4 can then also be expressed as

$$\frac{d\vec{S}}{ds} = \vec{n} \times \vec{S} = [G\gamma \hat{y} + G\gamma \frac{B_x}{B\rho} \hat{x} + (1+G) \frac{B_s}{B\rho} \hat{s}] \times \vec{S}, \quad (5)$$

where  $\hat{x}$ ,  $\hat{y}$ ,  $\hat{s}$  are the unit vector along the direction of radial, vertical and longitudinal, respectively.  $ds = \rho d\theta$ , where  $\theta$  is the bending angle. The equation of motion for the spin state in Eq. 1 also known as spinor for a  $\frac{1}{2}$  particle is

$$\frac{d\psi}{d\theta} = -\frac{i}{2} (\vec{\sigma} \cdot \vec{n}) \psi = -\frac{i}{2} H \psi. \quad (6)$$

For a constant  $H$ , Eq. 6 can be rigorously solved as

$$\psi(\theta_2) = e^{-\frac{i}{2} H (\theta_2 - \theta_1)} \psi(\theta_1) = M(\theta_2 - \theta_1) \psi(\theta_1) \quad (7)$$

where  $M(\theta_2 - \theta_1)$  is a  $2 \times 2$  spinor transfer matrix. For a main bending sector dipole, the spinor transfer matrix is just  $M(\theta_2 - \theta_1) = e^{-\frac{i}{2} G\gamma (\theta_2 - \theta_1) \sigma_3}$ . For a thin quadrupole, the spinor transfer matrix is  $M = e^{-\frac{i}{2} (1+G\gamma) kl y \sigma_1}$ , where  $kl$  is the normalized integrated strength of the quadrupole and  $y$  is the vertical displacement of the particle from the center of the quadrupole [1]. For any spin rotator which rotates spin vector by  $\phi$  around an axis  $\hat{n}_{rot}$  is  $M = e^{-\frac{i}{2} \phi \hat{n}_{rot} \cdot \vec{\sigma}}$ . In general, in a circular accelerator, the one turn spin transfer matrix for a particle on the closed orbit is

$$M = e^{-\frac{i}{2} Q_s 2\pi \vec{\sigma} \cdot \hat{n}_{co}}, \quad (8)$$

where  $Q_s$  is the spin tune, and  $\hat{n}_{co}$  is the stable spin direction. Equation 8 shows that for the particle's spin vector always returns to the same direction if it is initially aligned with the stable spin direction  $\hat{n}_{co}$ . Otherwise, the spin vector just simply precesses around the stable spin direction  $\hat{n}_{co}$   $Q_s$  times per orbital revolution. For a particle in a perfect circular accelerator where it only sees vertically aligned guiding magnetic fields, the stable spin direction is vertical and spin tune is  $Q_s = G\gamma$ , linearly proportional

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to the particle's energy. With the presence of any localized spin kick of  $\psi$  around an axis of  $n_e$  at location  $\theta$ , the one turn transfer spinor transfer matrix of a particle on closed orbit then becomes

$$M(\theta + 2\pi, \theta) = e^{-\frac{i}{2}G\gamma(2\pi-\theta)\sigma_3} e^{-\frac{i}{2}\phi\vec{\sigma}\cdot\hat{n}_e} e^{-\frac{i}{2}G\gamma\theta\sigma_3}, \quad (9)$$

and both spin tune and stable spin direction are changed [1].

In general, both spin tune and stable spin direction are function of particle's phase space coordinate, i.e.

$$\hat{n}_{co}(\vec{J}_z, \phi_z, \theta) = \hat{n}_{co}(\vec{J}_z, \phi_z + 2\pi, \theta) \quad (10)$$

where  $\vec{J}_z$  and  $\phi_z$  are particle's 6-D phase space coordinates. Spin tune is the number of spin precessions around  $\hat{n}_{co}$  in one orbital revolution. Unless betatron tune is integer, spin tune and stable spin direction can no longer be computed by the simple one turn spin transfer matrix approach. Algorithms like SODOM, SLIM were developed to compute the stable spin direction for particles off closed orbit [3, 4]. For a single isolated resonance, an analytical solution of  $\hat{n}_{co}$  was also found by Mane [5]. In the presence of overlapping resonances, the stable spin direction can be computed by numerical spin tracking using stroboscopic averaging developed by Heinemann and Hoffstaetter [6].

## CHALLENGES IN ACCELERATING POLARIZED PROTONS TO HIGH ENERGY

### Depolarizing Resonances

In a high energy planar accelerator, accelerating polarized protons operation is challenged by the presence of non-vertical magnetic fields due to magnetic field errors, quadrupole misalignments or vertical betatron oscillations, which kick the spin vector away from vertical direction. When the frequency of the perturbation on the spin motion coincides with the spin precession frequency, the kicks on the spin vector can be coherently added and result in polarization loss, i.e. a depolarizing spin resonance [1].

Depending on the source of the spin perturbing magnetic fields, there are two types of spin depolarizing resonances. The imperfection spin resonances at  $G\gamma = k$  are due to dipole errors and quadrupole misalignments. Here  $k$  is an integer. The strength of this resonance is proportional to the size of the vertical closed orbit distortion. The intrinsic spin resonances at  $G\gamma = kP \pm Q_y$ , on the other hand, are driven by vertical betatron oscillation in the absence of coupling as well as other high order effects. Here,  $P$  is the super-periodicity of the machine and  $Q_y$  is the vertical betatron tune. The stronger the betatron oscillation, the stronger the intrinsic spin resonance. In general, for a resonance at  $Q_s = K$ , the resonance strength  $\epsilon_K$  is defined as

$$\epsilon_K = \frac{1}{2\pi} \oint [(1 + G\gamma) \frac{B_x}{B\rho} + (1 + G) \frac{B_s}{B\rho}] e^{iK\theta} ds. \quad (11)$$

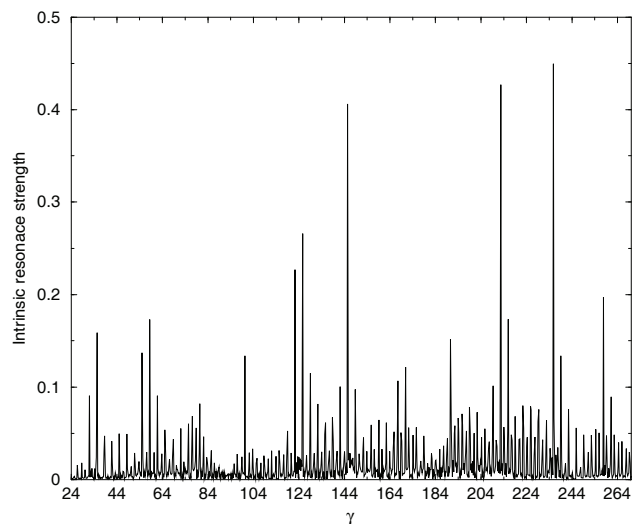


Figure 1: Strength of intrinsic spin resonances from injection energy to 250 GeV in RHIC. These are calculated from a RHIC lattice without any spin manipulation devices. The intrinsic spin resonance strength is calculated with a single particle at an emittance of  $10\pi$  mm-mrad.

Evidently, for an imperfection resonance, its resonance strength is independent of particle's betatron oscillation amplitude. Intrinsic resonance strength, on the other hand, is different for particles at different betatron amplitude since they experience different magnetic field in a quadrupole. Fig. 1 shows the strength of intrinsic resonances as a function of energy in RHIC, calculated with Depol [7]. A total of 423 imperfection resonances lies between the RHIC injection energy and its designed store energy at 250 GeV, and the higher the beam energy, the stronger the imperfection resonance. The strong intrinsic spin resonances in RHIC are located at  $81 \pm (Q_y - 12)$ ,  $81 + (Q_y - 6)$ ,  $2 * 81 + (Q_y - 12)$ ,  $3 * 81 - (Q_y - 12)$  and  $5 * 81 \pm (Q_y - 12)$ . Clearly, no beam polarization can survive through all these depolarizing resonances.

The polarization change after crossing a resonance depends on the resonance strength as well as how fast the resonance is crossed. For a single isolated resonance, the ratio of the polarization after crossing the resonance  $P_f$  to the initial polarization  $P_i$  is given by the Froissart-Stora formula [8]

$$P_f = P_i (2e^{-\frac{\pi|\epsilon_k|^2}{2\alpha}} - 1) \quad (12)$$

where  $\alpha = \frac{dQ_s}{d\theta}$  is the resonance crossing rate and  $\epsilon_k$  is the strength of the spin resonance. As indicated in Eq. 11, since intrinsic spin resonance strength is a function of the betatron oscillation amplitude, the average beam polarization of a Gaussian beam crossing an intrinsic resonance then becomes

$$P_f = P_i \frac{1 - \frac{\pi|\epsilon_k|^2}{\alpha}}{1 + \frac{\pi|\epsilon_k|^2}{\alpha}}, \quad (13)$$

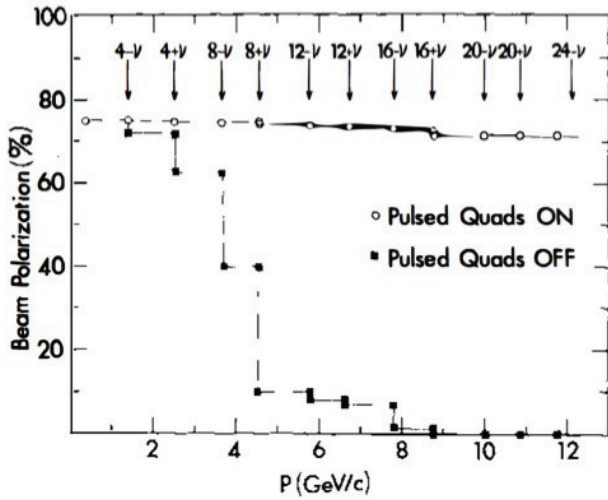


Figure 2: This plot shows the beam polarization as a function of the beam energy in the ZGS.

For a Gaussian beam, a polarization profile, i.e. polarization as function of betatron coordinates can be developed, of  $P(x, x', y, y') = P_0 e^{\frac{x^2 + x'^2}{2\sigma_{xp}^2}} e^{\frac{y^2 + y'^2}{2\sigma_{yp}^2}}$  can be developed, The average beam polarization is  $\langle P \rangle = P_0 \frac{1}{(1+R_x)(1+R_y)}$  [9]. Here  $P_0$  is the beam polarization of particle in the beam center,  $\sigma_{x,p}$  and  $\sigma_{y,p}$  are the rms size of horizontal and vertical polarization profile, respectively.  $R_{x,y} = \sigma_{x,y}^2 / \sigma_{x(y),p}^2$  where  $\sigma_{x,y}$  is the rms beam size.

### Overcome Imperfection Spin Resonance and Intrinsic Spin Resonance

Both imperfection resonances and intrinsic resonances can be overcome by correcting the closed orbit distortion at  $G\gamma = K$  and fast jumping betatron tune at  $G\gamma = kP \pm Q_y$  [1], respectively. Both techniques were first developed at the Argonne ZGS (Zero Gradient Synchrotron) [10, 11] in the early 70s. Fig. 2 shows the tune jump at each intrinsic resonance during acceleration. Both techniques were also applied to the AGS at BNL. With six fast tune jump quadrupoles and harmonic orbit corrections, polarized protons were accelerated to 22 GeV with 40% polarization in the AGS [12]. However, to achieve this, months of dedicated beam time were required [13].

For strong intrinsic spin resonances, one can use an RF dipole to excite a large amplitude coherent betatron oscillation to effectively enhance the intrinsic resonance strength and achieve full spin flip with regular resonance crossing rate. The advantage of this technique is to preserve the beam emittance by energizing the RF dipole adiabatically [14]. This technique was first successfully developed at the AGS [15] to obtain full spin flip at the four strong intrinsic resonances  $G\gamma = 0 + Q_y$ ,  $G\gamma = 12 + Q_y$  and  $G\gamma = 36 \pm Q_y$ . However, this technique is not applicable to weak intrinsic resonances due to the limit of physical aperture.

### Partial Snake

For medium energy accelerators, the tedious and lengthy setup time of orbital harmonic correction for each imperfection resonance can be a serious impediment. Hence, a novel technique of using a partial snake, a magnetic which rotates spin vector around an axis in the horizontal plane by an angle of  $\psi < 180^\circ$ , to keep the spin tune away from all integers was proposed [16]. Equation 14 shows that spin tune, as a function of beam energy, becomes discontinuous at each integer. Hence, all imperfection resonances are avoided.

$$\cos(\pi Q_s) = \cos(G\gamma\pi) \cos\left(\frac{\psi}{2}\right). \quad (14)$$

This technique was first developed at the BNL AGS [17]. A 5% solenoid snake, i.e. it rotates spin vector by 5% of  $180^\circ$ , was installed in the AGS in the mid 1990s to overcome all the imperfection resonances [18].

In order to avoid the depolarization due to the weak resonances in the AGS, a dual partial snake scheme was developed in 2006. A 5.9% room temperature helical snake plus a super-conducting helical snake which can provide an maximum strength of 20% at the AGS extraction energy were installed located  $\frac{1}{3}$  of the ring apart. With this configuration, spin tune then becomes [19]

$$\cos\pi Q_s = \cos G\gamma\pi \cos\frac{\psi_c}{2} \cos\frac{\psi_w}{2} - \cos G\gamma\frac{\pi}{3} \sin\frac{\psi_1}{2} \sin\frac{\psi_2}{2}. \quad (15)$$

Here,  $\psi_c$  and  $\psi_w$  are the amount of spin rotations of the strong super-conducting snake and the 5.9% snake, respectively. Equation 15 implies that not only the spin tune is forbidden in a gap around each integer but also that the width of the gap is modulated at an integer multiple of 3. The gap reaches its maximum width at each integer multiple of 3 where the two snakes are coherently added and otherwise reaches its minimum width when the two snakes are subtracted. Since the AGS has a super-periodicity of 12, all the strong intrinsic resonances are located at the integer multiple of 3 where the spin tune forbidden gap reaches its maximum. Hence, by placing the vertical betatron tune inside the spin tune forbidden gap, all imperfection and intrinsic spin resonances are avoided during the AGS acceleration [20].

### Full Siberian Snake

To reach even higher energy polarized protons, it is operationally impossible to individually overcome each depolarizing resonance due to the amount of spin depolarizing resonances during the acceleration. Much stronger resonances also make it very difficult to preserve polarization with just partial snakes. Thanks to the invention of the Siberian snake by Derbenev and Kondratenko in 1976 [21], the polarized proton acceleration to high energies became practical. The Siberian snake is a special device which rotates the spin vector by  $180^\circ$  around an axis in the horizontal plane. Equation 9 shows in a ring with a single full snake, the spin tune becomes half integer, independent of

beam energy and both imperfection resonances and intrinsic resonances are avoided [21]. This technique was first experimentally demonstrated in the cooler ring of Indiana University Cyclotron Facility in the 1980s[13].

For high energy accelerates like RHIC, intrinsic resonance can be so strong that more than one snakes are needed. In a ring with two full Siberian snakes located at  $180^\circ$  apart from each other are used, the one turn spin transfer matrix is

$$M(\theta+2\pi, \theta) = e^{-\frac{i}{2}G\gamma(\pi-\theta)\sigma_3} (i\hat{n}_1 \cdot \vec{\sigma}) e^{-\frac{i}{2}\pi\sigma_3} (i\hat{n}_2 \cdot \vec{\sigma}) e^{-\frac{i}{2}\theta\sigma_3} \quad (16)$$

where  $\hat{n}_1, 2$  are the axes of the two snakes, both are in the horizontal plane with an angle of  $\Delta\phi$ . It can easily show that the spin tune is  $Q_s = \frac{1}{\pi}\Delta\phi$ , where  $\Delta\phi$  is the angle between the axes of the two snakes. With the axes of the two snakes perpendicular to each other, the spin precession tune is  $\frac{1}{2}$ . It is also easy to show that the stable spin direction stays vertical all around the ring. RHIC employs two full Siberian snakes in each of its two accelerators to preserve polarization through acceleration as well as collision at a store energy, and each RHIC snake consists of four helical dipoles [22].

The spin tune and stable spin direction mentioned above for one snake as well as dual snake setup are for a particle at zero betatron oscillation amplitude in a perfect ring. In reality, both spin tune and stable spin direction can become strongly dependent on the particle's phase space coordinates in the presence of strong spin depolarizing resonances, and can no longer be calculated through the simple one turn matrix approach as illustrated in Eq. 16. For a single isolated spin resonance case, an analytic solution was found by Mane [5]. Based on Mane's model, stable spin direction exhibits singularity at location of  $mQ_y = Q_s + k$ , and leads to significant depolarization. Here,  $m, k$  are integers,  $Q_y$  is the vertical betatron tune and  $Q_s$  is the spin precession tune. This is the so-called snake resonance, and was first observed during numerical simulation by Lee and Tepikian [23]. They are also experimentally observed at IUCF [24] as well as RHIC [25].  $m$  is also called the order of the snake resonance. In an accelerator with only a single snake, an intrinsic spin resonance can drive both even order snake resonances ( $m$  is an even number) as well as odd order snake resonance ( $m$  is an odd number), and available betatron tune space for polarized beam operation can be rather limited.

With dual snake setup like RHIC, on the other hand, all the odd order snake resonances are eliminated and only the even order snake resonances exist in the absence of closed orbit distortion. This greatly opens up the available tune space for accelerating polarized beams. However, snake imperfections and imperfection resonances can bring back even order snake resonances, and result in significantly reduction of betatron tune space for preserving polarization. In addition, the imperfection of snake can deviate spin tune away from half integer, and causes each snake resonance to split [1]. Hence, optimizing snake configuration as well as

minimizing closed orbit distortion are critical in avoiding snake resonances.

Careful choice and precise control of betatron tune to stay away from any snake resonances are critical to preserve polarization. For  $Q_s = \frac{1}{2}$ , one can see that snake resonance is forbidden at locations of  $Q_y = \frac{n}{2m+1}$ . For RHIC,  $Q_y = 29.673$ , at the vicinity of  $3^{rd}$  order resonance, was chosen for accelerating polarized protons beyond 100 GeV, where three strong very strong intrinsic resonances are present.

In reality, the presence of snake imperfections and imperfections resonances brings back even order snake resonances. The imperfection of snake can deviate spin tune away from half integer, and causes each snake resonance to split [1]. Both can significantly shrink the available space for the betatron tune during the acceleration. Hence, optimizing snake configuration as well as minimizing closed orbit distortion are critical in minimizing polarization loss. In addition, it is also very critical to keep the spin tune as close to half integer as possible. In RHIC, spin tune shift can come from the error in the angle between the axes of the two snakes, and the horizontal orbital angle at the two snakes [26].

In the presence of very strong intrinsic resonance, i.e.  $\epsilon_k > 0.5$ , the snake resonances can be overwhelming and the tolerance on closed orbit and beam parameters can be at a level of impractical for operation. In this case, more snakes option should be explored. To accelerate He-3 in RHIC, because of the large anomalous g-factor, the intrinsic resonance for He-3 is about a factor of 2 stronger than the resonances for protons. Fig. 3 shows the snake resonance spectrum with current RHIC dual snake configuration. However, this can be greatly mitigated by adding two more pairs of snakes as shown in the bottom plot of Fig. 3. The 6-snake setup not only opens up available betatron tune spaces for acceleration, but also greatly relaxes the tolerance on other beam parameters [27]. As derivated from the first order spin resonances like intrinsic resonance, the strength of snake resonance is tightly associated with strength of its driving resonance. Generally, the stronger the intrinsic resonance, the more harmful the snake resonance is. However, it is also worthwhile to point out the polarization crossing a snake resonance does not obey the Froissart-Stora formula. Currently, this is only studied through numerical spin tracking.

### Polarization Lifetime

In high energy colliders like RHIC, at the store energy carefully chosen to be distant from major depolarizing resonances, both beams undergo beam-beam interaction, which leads to betatron tune shift as well as incoherent tune spread for each beam. For RHIC, the beam beam tune spread can be as large as 0.0075 per collision for polarized proton physics operation. This can populate particles in the beam sitting on a snake resonance, and polarization deterioration can happen during long hours of store as observed at RHIC [28].



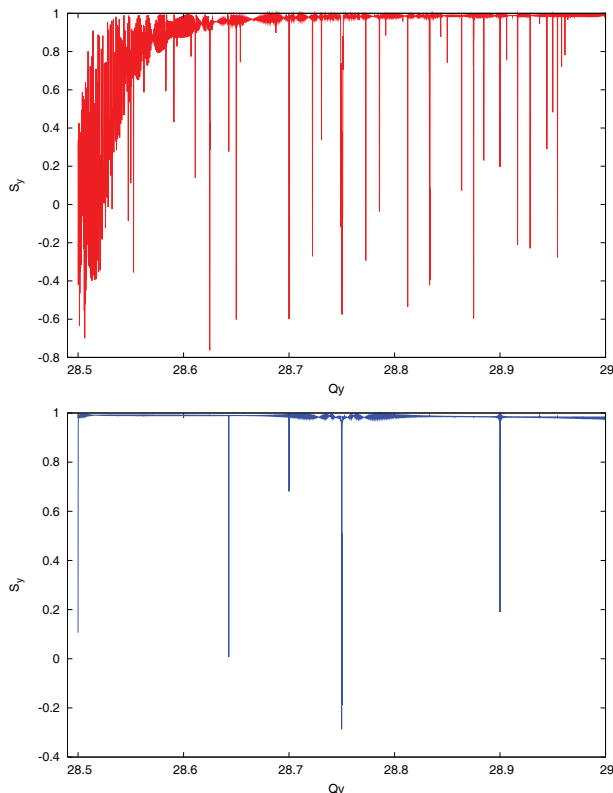


Figure 3: Top plot shows the snake resonance spectrum with RHIC dual snake setup, while bottom plot corresponds to 6-snake setup. Both plots show the vertical component of the spin vector as function of vertical betatron tune.

In addition to its impact on betatron motion, the beam-beam force also provides kick to spin motion. This effect was investigated by Batygin and others [29]. For a Gaussian distributed round relativistic beam, the electric field and magnetic field are  $\vec{E} = \frac{qn}{2\pi\epsilon_0 r} [1 - \exp(-\frac{r^2}{2\sigma_2})] \hat{r}$  and  $\vec{B} = \frac{1}{c} \vec{\beta} \times \vec{E} \hat{\phi}$ , where  $qn$  is the charge line density of the beam,  $\sigma$  is the transverse beam rms size,  $\hat{r}$  and  $\hat{\phi}$  are the unit vector along the radial and azimuthal direction of beam. The corresponding beam beam tune shift is  $\xi = \frac{Nr_0\beta^*}{4\pi\gamma\sigma^2}$ , where  $r_0$  is the classic radius of the particle. Apply this to the Thomas-BMT equation, one can then estimate the maximum spin kick  $2\pi\epsilon_{bb}$  from the beam-beam interaction as

$$\epsilon_{bb} = 2G\gamma\xi\sigma \frac{r}{\sigma} \frac{(1 - \exp(-\frac{r^2}{2\sigma_2}))}{\frac{r^2}{2\sigma_2}}. \quad (17)$$

Fig. 4 shows the calculated maximum beam-beam spin kick as a function of distance from the center of beam for a beam of  $15\pi$ mm-mrad emittance (95%, normalized) at RHIC store energy of  $G\gamma = 487$ . 0.7 m  $\beta$ -function at collision point and beam-beam tune shift  $\xi = 0.01$  are assumed. In general, this is comparable to very weak lattice driven spin resonances. Thus, the beam-beam driven snake resonances are very weak for accelerators like RHIC that are equipped

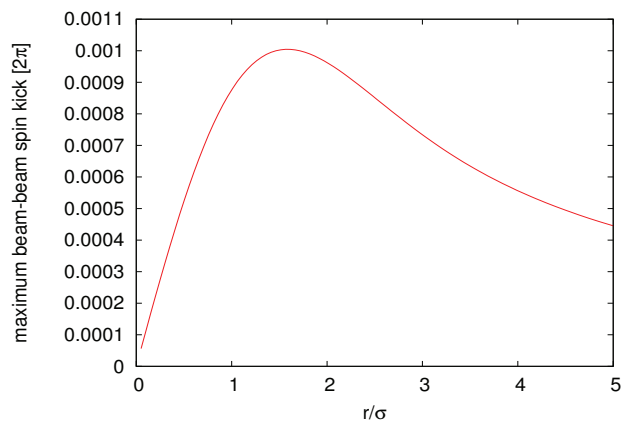


Figure 4: Maximum spin kick from beam-beam for a Gaussian round beam at RHIC store energy of  $G\gamma = 487$ . The  $\beta^*$  is 0.7m, and normalized 95% beam emittance is  $15\pi$ mm-mrad for the calculation. The beam-beam parameter is chosen as 0.01 for this calculation.

with full snakes, and should be fully benign to beam polarization at store as long as the betatron tune stays away from the snake resonances. So far, detailed studies and analysis of the RHIC beam-beam impact on the polarization lifetime indicate that beam polarization deterioration at store is still due to the snake resonance at  $5Q_y = Q_s + 3$  [28]. However, for accelerators with no snakes, beam-beam kick can drive additional spin resonances and result in polarization losses as observed at electro-positron collider LEP [30] and HERA, a polarized electron and proton collider [31]. Hence, it is critical for the choice of the working point, i.e. the tunes without collision to ensure long polarization lifetime.

## CONCLUSION

Accelerating polarized protons to high energy is challenged by various depolarizing mechanisms driven by magnetic fields from manufacturing errors, misalignments, betatron oscillation and etc. Motivated by the need of having high energy polarized protons to study proton spin structure as well as other spin dependent physics, continuous efforts over the past couple of decades have been made in seeking ways to overcome spin depolarizing resonances and preserve polarization to high energy. Table 1 lists the achieved polarized proton performance at various facilities. The operating period for each facility is also listed. In Table 1, RHIC polarization is measured with an absolute polarimeter using H-jet [32]. It reflects the polarization averaged over a store, typically, 8 hours.

Physics of polarized protons in accelerators is a very rich topic, and this tutorial presentation can only cover the very basics of spin dynamics in a circular accelerator. The non-linear aspects of spin dynamics, the effect of synchrotron radiation on polarization as well as challenges in robust numerical spin tracking are not covered in this presentation due to the complexity of each topic and limited space. In

Table 1: Performance of Polarized Protons Facilities

facility	operating period	beam energy [GeV]	max polari- -ration	beam intensity $10^{11}$
ZGS	1969-1979	12	71%	0.9
AGS	1979-present	23	70%	2
IUCF	1982-1995	0.2	N/A	N/A
COSY	1985-present	3	N/A	N/A
RHIC	2000-present	255	57%	1.8

addition, this presentation also couldn't cover the topics of spin manipulation driven either by the physics program or by the needs of diagnostics like non-destructive spin tune measurement. The author can only hope this presentation serves as a brief introduction to attract those talented colleagues in the community to join the journey of this rich and interesting physics of polarized beams in accelerators.

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