# APF DTL DESIGN BASED ON IMPAPF

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#### Abstract

Alternative phase focusing (APF) DTL has advantages in price and space. However, the designing of APF is difficult because of the jumping phases. In order to design and simulate a proper APF, a code iMpAPF(i Multiparticle APF) has been developed. RK4 is introduced in this code for calculation, and the soul of the code is smoothness. Theoretically, the particle tracing figures out that the theoretic phase advance ratio between longitudinal direction and transverse direction is close to 2. Based on this code, a C<sup>5+</sup>, 200 MHz, medium energy APF as a linear injector of a synchrotron used for cancer therapy has been designed. This paper focuses on the development of iMpAPF, and also several results obtained from the code are shown.

### **INTRODUCTION**

Alternative phase focusing (APF) DTL refers to a special DTL used for accelerating heavy ions with low energy. There are no conventional focusing elements, and the longitudinal and transverse focusing are realized by alternating electromagnetic filed instead. The synchronous phases are negative and positive alternatively to acquire the longitudinal and transverse focusing in APF DTL.

APF DTL was first proposed in 1953 to accelerate heavy ions with low energy[1]. This method was researched globally; however, it did not find practical applications because of small longitudinal acceptance due to rapidly jumping phases, until it was used as an injector of a synchrotron for cancer therapy in HIMAC[2][3][4][5][6].

APF is more suitable for low energy machine because of electric focusing. For there is no conventional elements inserted in the cavity, the capacitance and inductance are much smaller than those in traditional DTL cavity, which leads to a higher shunt impedance and a small size.

# MATHEMATICAL THEORY OF APF DESIGN

# Equivalent Quadrupole

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Electromagnetic field in a RF gap satisfies Laplace's equation under quasistatic approximation. Using Maxwell's equation, the relationship between  $E_z$  and  $E_r$  is given[7]

$$\frac{\partial E_r}{\partial r} = -\frac{1}{2} \frac{\partial E_z}{\partial z}.$$
 (1)

Without acceleration, a RF gap can be considered as a specific quadrupole, in which the focusing strengths are K and -2K transversely and longitudinally, respectively, ignoring induced magnetic field because of its small contribution. In order to obtain focusing in both directions, alternative focusing force is necessary.

#### Dynamics of FD Structure

There is an assumption that the RF gaps are continuous rather than discrete, and the focusing force is independent of time along the whole DTL cavity. Taking an alternative periodic structure as an object, the simplest case is that the former half is focusing and the latter half is defocusing in transverse direction: The lattice is FD.

The scalar potential of the structure forms parabolic saddle surfaces. The saddle surfaces change directions periodically along the cavity, and there is an odd point between halves, and one particle would be lost there.

The presentation in phase space is that the trajectory of one particle is an unsmoothed curve at the odd point. Mathematically, the Twiss parameter  $\alpha$  is continuous but not smooth.

# Dynamics of Smooth Structure without Acceleration

Exponential function is of the excellent quality because of infinite differentiability or smoothness. Its imaginary part represents periodicity which is the right thing the accelerator needs. Thereby, a conception occurs that a periodic cosine K(s) instead of square wave K(s) along the structure is used to control beam.

Replacing trigonometric function term by exponential function, the Bessel functions are the analytical solutions.

$$X'' + K_0 \cos\left(\frac{2\pi s}{L}\right) X = 0 \tag{2}$$

$$X'' + K_0 e^{\frac{2\pi s}{L}i} X = 0 (3)$$

In the equations, L=21, is the periodic structure length, s is longitudinal coordinate, K0 is the maximum focusing strength.

Solving the equation with RK4, one particle's motion is quasiperiodic oscillation, and its track forms particular cloud which has a strict boundary.

There are 5 eigen frequencies, corresponding to 5 oscillation modes with amplitudes decrease orderly: K0 L/55,  $1/L \pm K_0 L/55$  and  $2/L \pm K_0 L/55$ .

Figure 1 gives an oscillation of x and x' along a cavity in phase space when  $K_0 = 8$ , L=0.6, and  $x_0=2$  mm,  $x_0'=0$ . The

particular cloud is shown in blue and the particle's oscillation is shown in red.

If  $K_0$  and L are given, the design of the perfect structure is simplified to be a match between beam parameters and structure parameters. Figure 2 shows transverse Twiss parameters along longitudinal direction, and  $\beta$ ,  $\alpha$  and  $\gamma$  are plotted in red solid line, blue dotted line and pink broken line, respectively.



Figure 1: Oscillation cloud and trajectory of x and x' in phase space when  $K_0 = 8$ , L=0.6, and  $x_0=2$  mm,  $x_0'=0$ .



Figure 2: Transverse Twiss parameters along longitudinal direction, and  $\beta$ ,  $\alpha$ , and  $\gamma$  are plotted in red solid line, blue dotted line and pink broken line, respectively.

### Dynamics of Smooth Structure with Acceleration

The beam dynamics equations of smooth structure with acceleration in transverse and longitudinal directions are as follows, and the corresponding trajectory in phase space is shown in Figure 3[8].

$$R'' + \ln(\beta\gamma)' R' + K_R R = 0 \tag{4}$$

$$Z'' + \ln(\beta \gamma^{3})' Z' + K_{z} Z = 0$$
 (5)

Where R represent x and y. For arbitrary time,  $Kz \approx -2K_R$ . The logarithm components in the equations express the effect of increasing effective mass with velocity increasing[9].



Figure 3: Trajectory in x-x' phase space with acceleration.

#### Structure Discretization

Structure discretization is a iteration process to make sure the cell length calculated by neighboring phases which is expressed by Eq.(6) matches the real drift that the synchronous particle experience assuming a RF gap as a thin gap[10].

$$L_{cell} = \frac{\beta_{i+\frac{1}{2}\lambda}}{2\pi} (\phi_{i+1} - \phi_i + \pi)$$
(6)

where  $\beta_{i+1/2}$  means the reduced velocity that the particle experiences the middle point of the cell in the continuous structure. The flow chart of structure discretization is shown in Figure 4.

### **RESULTS FROM IMPAPF**

Based on this code, a  $C^{5+}$ , 200 MHz, medium energy APF as a linear injector of a synchrotron used for cancer therapy has been designed. The injected energy is 0.59 MeV/u, and the extracted energy is 5.5MeV/u.

Initial and final particle distributions of the continuous structure in phase space are shown in Figure 5. Figure 6 shows the cell lengths of discrete structure.

The equivalent gap voltage can be obtained by integrating the voltage along one cell, and normalized to the middle of the cell.

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Figure 4: The flow chart of structure discretization.

# CONCLUSION

The code iMpAPF has been developed to design APF. The soul of the code is smoothness which is the footstone of stability. The fatal drawback followed the smoothness is resonance which can be ignored in APF DTL because of the unconspicuous super-periods and the weak intensity.

The results from iMpAPF directly would not meet the requirements, and it is just an initial design. It can be inserted into PSO as an initial seed to search a much better result which would be the local optimization around the seed[11].



Figure 5: Initial and final particle distributions in phase space of the continuous structure.



Figure 6: Cell lengths of discrete structure.

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