THE PROBLEM OF Q-DROP IN SUPERCONDUCTING RESONATORS REVISED BY THE ANALYSIS OF FUNDAMENTAL CONCEPTS FROM RF-SUPERCONDUCTIVITY THEORY

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Abstract
The Q-factor versus accelerating field experimental plot for superconducting cavities affected by the Q-slope can be generally divided in three regions: low field, medium field and high field. The low field increase of the Q-factor can be mathematically described by the presence of an overlayer made of a poor superconductor. The medium field Q slope is instead described by the fact that due to the motion of the superfluid the energy gap is decreased by the product of Fermi momentum by the supervelocity. In this paper it is shown that this parasitic term is negligible for values of the mean free path larger than the coherence length, while it becomes a serious limitation, for shorter values of the mean free path. Since the BCS surface resistance shows a minimum for values of the Residual Resistivity Ratio around 10, this means that experimentalists will never benefit simultaneously of extremely high Q values and high fields. It is then proved that Niobium sputtered cavities will never be usable at high accelerating gradients, unless Residual Resistivity Ratio values of at least 100 will be achieved in the Niobium film growth. The high field Q-slope instead is due to the instability of superconducting state due for high values of superfluid velocity.

INTRODUCTION

Last two decades have seen a strong push of R&D in the field of superconducting resonant cavities for particle accelerators. Results unimaginable before have been achieved, but still a strong limitation affects the expected performances: no matter the number of cells, the resonator Q-factor decays versus the accelerating field, so to inhibit the achievement of high fields at reasonable levels of power. The phenomenon is encountered unfortunately both for Nb bulk cavities and for Nb thin film sputtered Cu cavities, being the limitation even more severe in this latter case.

More precisely speaking, typical curves of Q versus field, as the one displayed in fig. 1, show three distinguished behaviours depending on the intensity of the field. At low field the Q-factor is found to increase with field; at medium field it decays exponentially; then at high fields the decay follows a even higher slope. Referring to basic concepts of fundamental theory, in this paper we provide a simple explanation for all these three regions.

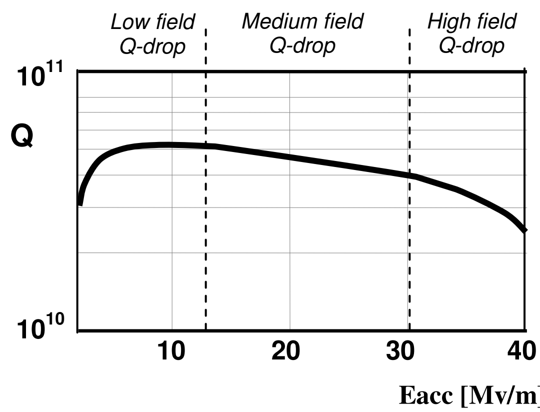

Fig. 1 Typical dependence for a re-entrant shape 1,3 GHz Niobium cavity measured at 1,8 K

THE LOW FIELD Q-SLOPE

Why the Q-factor should increase versus field is not immediately intuitive, and that's why this phenomenon has been either neglected for years or has been considered the result of instrumental errors. However the Q-increase versus field has been found since the early years and in so many laboratories all over the world that the probability of an instrumental error is absolutely low.

In this section we will show that the low field increase of Q factor can be easily explained by simply calculating the surface impedance of the bilayer system of fig. 2, made of the base superconductor (called SC2 in the following) coated by a thin layer of a poorer superconductor (called SC1) of a given thickness **a**. The penetration depths of the two superconductors are respectively λ_2 and λ_1. The author intuition was triggered by the experimental evidence that some of the most striking cases of Q-increase at low fields are found in cavities whose internal surface was either specially [1] or unintentionally [2] contaminated by the presence of over-layers.

For a semi-infinite conductor that fills the **+x** half-space and has a plane surface at **x = 0**, and a plane wave such as $H(x) = H(0)\, e^{(ikx-\omega t)}$, the surface resistance is defined as

$$Z \equiv \frac{E_y(0)}{\int_0^\infty J_y(x)dx} = \frac{4\pi}{c}\frac{E_y(0)}{H_z(0)}$$

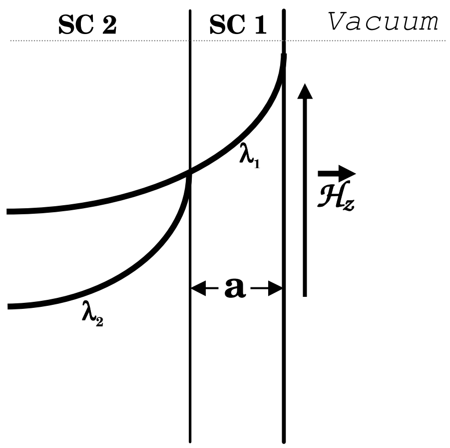

Fig.2 Bilayer system made of the bulk niobium (SC 2) and of an over-layer of a superconductor (SC 1) with poorer superconducting properties.

In the case of the two superconductors of fig. 2, it holds the following

$$\frac{1}{z} = \frac{\int_0^a J(x)dx + \int_a^\infty J(x)dx}{E_y(0)} = -\frac{c}{4\pi}\left\{\frac{H'_z|_0^a + H''_z|_a^\infty}{E_y(0)}\right\}$$

where the field **H(x)** is calculated within $\mathbf{SC_1}$, at the interface, and within $\mathbf{SC_2}$

.

$$X \le a; \qquad H'_z(x) = H'_z(0)\, e^{-\frac{x}{\lambda 1}}$$

$$X = a; \qquad H'_z(a) = H'_z(0)\, e^{-\frac{a}{\lambda 1}}$$

$$X \ge a; \qquad H''_z(x) = H''_z(a)\, e^{-\frac{(x-a)}{\lambda 2}} =$$

$$= H''_z(0)\, e^{-\frac{a}{\lambda 1}} e^{-\frac{(x-a)}{\lambda 2}}$$

Therefore by simple algebra, one arrives to the following assumption

$$\frac{1}{Z} = \frac{1}{Z_1}.\,(1 - e^{-a/\lambda_1}) + \frac{1}{Z_2} e^{-a/\lambda_1}$$

where $\mathbf{Z_1}$ is the surface impedance for the $\mathbf{SC_1}$, while $\mathbf{Z_2}$ is that for $\mathbf{SC_2}$. Analogously if $\mathbf{Q_1}$ is the Q-factor exhibited by $\mathbf{SC_1}$, and $\mathbf{Q_2}$ that for $\mathbf{SC_2}$,

being $\mathbf{\Delta Q = Q_2 - Q_1}$,

$$Q = Q_1(1 - e^{-a/\lambda 1}) + Q_2 e^{-a/\lambda 1} = Q_1 + \Delta Q \cdot e^{-a/\lambda 1}$$

that results into the effect displayed in fig. 3

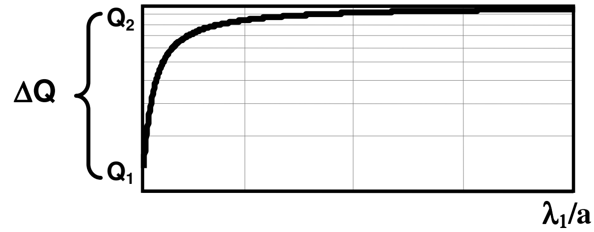

Fig. 3 The behaviour of the resulting Q-factor versus the penetration depth of the over-layer normalized on its thickness.

This graph is very important for our task, because of the following hypothesis: being the over-layer a contaminated film, for low field intensities, its penetration depth $\boldsymbol{\lambda_1}$ can depend on the reduced magnetic field $\mathbf{b = B/B_C}$ as in the following relation:

$$\lambda_1(B) = \lambda_1(0) + \left.\frac{\partial\lambda}{\partial b}\right|_0 b + \ldots \cong \lambda_1(0) + \alpha \cdot b$$

By only this statement, we can simply explain how, at low fields, the Q-factor can increase versus magnetic field. Indeed we can just say that, the more $\boldsymbol{\lambda_1}$ increases with magnetic field, the more the “clean and high performance” SC2 is involved.

Then we need to anticipate herewith the mechanism recognized by the author as responsible for the medium range Q-drop, i.e. the gap dependence on magnetic field in the superconductor SC2, $\boldsymbol{\Delta_2(b) = \Delta_2(0) - k_2\, b}$. This will result straightforwardly into an exponential decay of the SC2 related Q-factor, i.e. $\mathbf{Q_2 = Q_2\,(0)\; e^{-k2\, b}}$ and with little algebra we achieve a final relation for the bilayer quality factor $\mathbf{Q_{Tot}}$ **:**

$$Q_{Tot} = Q_1(0) \cdot (1 - e^{-\frac{a}{(\lambda 1 + \alpha b)}}) +$$

$$+ Q_2(0) \cdot e^{-k_2 b} e^{-\frac{a}{(\lambda 1 + \alpha b)}}$$

By this equation, and as plotted in fig. 4, it can be seen that the role of SC1 is strongly dissipative and that the SC1 related Q-factor is increasing versus field. As far as the second term is concerned instead, by increasing the field, the penetration depth in SC1 becomes higher and the losses are more and more shifted into the SC2, that is a pure superconductor and has lower losses. This mechanism however shows a maximum. Strong fields saturate the SC2, giving rise to normal dissipative fields. The composition of the two terms presents also a maximum at even lower field value.

The dependence of the maximum of $\mathbf{Q_{Tot}}$ versus the thickness **a** of SC1 can also be observed. At a first sight, the Q rise versus field can appear as a benefit, but it is easy to observe that for lower values of **a**, the value of the $\mathbf{Q_{Tot}}$ at the maximum also increases, proving that actually the presence of the overlayer is a source of losses and it is not beneficial. However the presence of a Q-increase versus field can give useful information about the superficial contamination: the higher is the field at which the maximum occurs, the thicker is the overlayer.

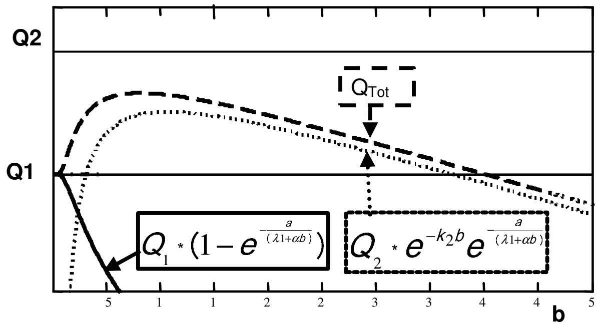

Fig. 4 Plot of the bilayer Q-factor versus the reduced magnetic field **b**. The former term and the latter term of the expression found for $\mathbf{Q_{Tot}}$ are plotted too.

The schematization of a contaminated surface as a bilayer is of course an approximation. In real cases one can have several different overlayers or even a layer with a continuous distribution of contamination. In such a case, the Q equation for two superconductors can be easily extrapolated to the case of **n** layers each one distinguished by its penetration depth $\boldsymbol{\lambda_j}$,

$$Q_{Tot} = Q_1 + \sum_{i=1}^{n-1} (Q_{i+1} - Q_i) e^{-a \sum_{J=1}^{i} \frac{1}{\lambda_j}}$$

and applying the limit for **n** that goes to infinity, the relation becomes an integral equation, that can be solved by solving the Volterra differential Equation

$$\lim_{n \to \infty; a \to 0} Q_{TOT} = \int_0^\infty \frac{dQ(x)}{dx} e^{-\int_0^x \frac{1}{\lambda(\bar{x})} d\bar{x}} dx$$

THE MEDIUM FIELD Q-SLOPE

Apriori the surface resistance in the superconducting state should be independent of magnetic field. In reality this happens very seldom and the increase of surface resistance versus field could be explained if the energy gap depended on applied magnetic field. The field dependence of the energy gap commonly reported by literature is

$$\Delta = \Delta_0 (1 - H^2/H_C^2).$$

This formula has been often considered in analyzing the problem of Q-slope, but since it does not fit experimental results, it has been concluded that the magnetic field has no effect on the gap depression. Now, this formula already contested in many further publications [4], does not apply in our case and, in the author opinion, it has caused a real damage to the superconducting cavity scientific community. As already proposed in a early paper [5] indeed, a more rigorous derivation of the energy gap depression due to magnetic field is the key variable in the Q-slope problem.

Being $\mathbf{p_F}$ the Fermi momentum and $\mathbf{v_S}$ the velocity of the superfluid, the dependence of the energy gap from magnetic field that should be taken into account is the following:

$$\Delta = \Delta_0 - p_F v_S$$

Stated already in 1962 by John Bardeen [6], this relation suggests that the displacement of the pairs causes an increase in free energy of the system which may be expressed simply in terms of supercurrents.

In the low temperature limit, there are no excitations formed and thus no change in Δ until the velocity $\mathbf{v_S}$ reaches the value for which it is favorable to form pairs of excitations, corresponding to transfer of an electron from one side of the Fermi sea to the other. This criterion is the depairing condition [7,8]:

$$\frac{1}{2} m ({p_F}/{m} + v_s)^2 - \frac{1}{2} m ({p_F}/{m} - v_s)^2 > 2\Delta$$

i.e. the superconducting state becomes unstable against pair creation when $\mathbf{v_S\, p_F > \Delta}$.

In other words as in superfluid helium, the superconductivity is destroyed by the critical velocity of superfluid. Actually the author considers very strange the fact that in the field of superconducting cavities, the critical current has been never considered. Indeed if for superconducting magnets the fundamental and independent parameters are: $\mathbf{T_C}$, $\mathbf{H_C}$ and $\mathbf{J_C}$, why for superconducting cavities, $\mathbf{J_C}$ disappears? Is 40 MV/m (1600 G) an rf field not strong enough to induce strong currents?

In the case of the gap depression mechanism,

$$R_{BCS} \approx e^{-\frac{\Delta}{kT}} = e^{-\frac{\Delta_0}{kT} + \frac{p_f v_s}{kT}}.$$

Being $\boldsymbol{\lambda}$ the London penetration depth, **e** the electron charge, ℓ the mean free path and $\boldsymbol{\xi}$ the coherence length, given that $J_S = -\frac{A}{\mu_0 \lambda^2}$; $\lambda^2 = \lambda_0^2 \cdot (\frac{\xi_0}{\xi})$ and,

$$\lambda_0^2 \mu_0 = \frac{m}{n \cdot e^2}$$

after some algebra, it easy to arrive to the following result:

$$\frac{p_f v_s}{kT} = \frac{B \cdot \lambda_0 \cdot e \cdot v_F}{kT} \cdot \left(\frac{n}{n_S}\right)^{1/2}, \text{ being}$$

$$\frac{n}{n_S} = \cot gh\left(\frac{\ell}{\xi_0}\right)$$

Looking at fig. 5 displaying the superfluid density $\mathbf{n_S/n}$ versus mean free path normalized over coherence length, it immediately appears that the energy gap depression is mainly determined by a key parameter that is the ratio ℓ/ξ_0.

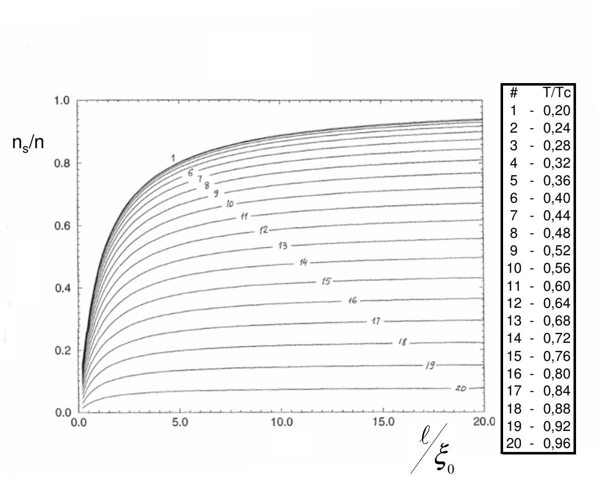

Fig. 5 Density of superelectrons as a function of the ratio ℓ/ξ_0

For Pure Niobium $\xi_0 \approx 380\text{Å}$ and $\ell \approx 24\text{Å} \cdot (\beta - 1)$, where β is the Residual Resistivity Ratio (RRR).

Graph gives access to the comprehension of the middle Q-slope mechanism: for $\ell > \xi_0$, that corresponds to the case of bulk Niobium, the parasitic term $\mathbf{P_F V_S}$ is negligible; but as soon as RRR value β becomes less than 100, the gap depression becomes important. This would explain why Niobium films have a terribly high Q-slope. Similarly, bulk Niobium cavities that are in situ baked at around 100 C before the rf test, have a higher slope too because the baking could promote the diffusion of impurities dissolved from the bulk to the surface layer, deteriorating the β value of the surface.

The BCS surface resistance in the superconducting state has a minimum around β =10, that is just the average value obtained for sputtered Niobium films. The minimum of Surface resistance corresponds to the maximum of Q, but according to the present paper, it corresponds to the maximum slope. In summary there is no hope to get rid from the Q slope for Nb sputtered Cu cavities unless a method is found for making films of RRR at least 100.

The sad conclusion of this analysis is that minimum of exist. The good aspect is instead that once found any experimental technique for increasing the RRR values, Nb coated cavities could become again an opportunity.

THE HIGH FIELD Q-SLOPE

The problem of the high field Q-slope in the author opinion is due to the instability of the superconducting state in case of superfluid velocity $\mathbf{v_S}$ close to the critical velocity $\mathbf{v_C}$.

Indeed in the local electrodynamics, the total current is $\mathbf{J = J_1 + J_2}$, and for small supervelocity $\mathbf{J_S = n_S\ e\ v_S}$. At larger $\mathbf{v_S}$ there is a depairing effect due to the current and the Meissner current becomes

$$j_1 = n_s e v_s \left(1 - \frac{v_s^2}{v_c^2}\right)$$

At $\mathbf{v_S = v_C}$, the energy gap closes, however, as sketched in fig. 6 the supercurrent reaches a maximal value $\mathbf{v_m}$. Region of $\mathbf{v_S}$ from $\mathbf{v_S}$ to $\mathbf{v_C}$ is unstable. A superconductor can be forced by an AC current however to values of $\mathbf{v_S}$ higher than $\mathbf{v_m}$, but the state is highly dissipative.

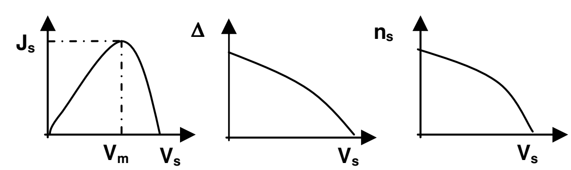

Fig. 6 Plot of the Supercurrent, energy gap and number of superelectrons versus the velocity of superfluid. Over $\mathbf{v_m}$ the superconducting state become unstable.

CONCLUSIONS

The experimental plot of the Q-factor of superconducting cavities affected by the Q-slope problem can be generally divided in three regions: low field, medium field and high field.

The low field Q-slope detected as an increase of the Q-factor versus field can be mathematically described by the presence of an overlayer made of a poor superconductor SC1 over the reference superconductor SC2.

In this case we have shown that the Q factor of the bi-layer is expressed by the formula

$$Q = Q_1(1 - e^{-a/\lambda 1}) + Q_2 e^{-a/\lambda 1}$$

Medium field Q slope is instead described by the fact that due to the motion of the superfluid the gap decreases linearly versus superfluid velocity

$$\Delta = \Delta_0 - p_F v_S$$

The parasitic term $p_F v_S$

$$p_f v_s = B \cdot \lambda_0 \cdot e \cdot v_F \cdot \left(\cot gh \frac{\ell}{\xi_0} \right)^{1/2}$$

is negligible for values of the mean free path larger than the coherence length, it becomes a serious limitation, for shorter values of the mean free path. Since the BCS surface resistance presents a minimum for values of the RRR around 10, this means that experimentalists will never benefit simultaneously of extremely high Q values at high fields. It is then easily understandable that Niobium sputtered cavities will never be usable at high accelerating gradients, unless Residual Resistivity Ratio values of at least 100 will be achieved in the Niobium film growth.

As regards the high field Q-slope, using the Ginzburg Landau result for the Meissner current,

$$j_1 = n_s e v_s (1 - \frac{v_s^2}{v_c^2})$$

the state become dissipative as it approaches the critical velocity.

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