COMPARISON OF SHAPES OF MULTICELL CAVITY CELLS*

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Abstract

Comparison of cell shapes for a multicell cavity can be done in terms of (1) the aperture radius for a given wave length, (2) the peak electric field normalized to acceleration field and (3) the wall slope angle. All other important figures of merit, when this choice is done, become a matter of optimization. Several geometries of cells of superconducting cavities are compared from this standpoint.

The elliptic shape used for optimizations not always reflects the actual shape of cells. Influence of the weld seams on the main cavity figures of merit is also discussed.

INTRODUCTION

The main figures of merit for an elliptical-cell design of a superconducting cavity are E_{pk}/E_{acc}, GR/Q, and H_{pk}/E_{acc}. They determine field emission limit, wall loss, and breakdown field, respectively. These values can be treated as functions of geometrical parameters of the cell [1].

The aperture radius, R_a of the cavity is responsible for all three above-listed values; but first of all, for higher order modes (HOMs) propagation. Wakefields depend on this value reversely proportional from 2nd to 4th power [2, 3] and this geometric parameter can be treated directly as one more an important figure of merit to be used for different cases.

The geometry of elliptical cavities generally consists of two ellipses connected by a tangential line and is thus defined by 7 parameters. These parameters can be chosen to find optimal merit values for cavities. When the cavity frequency and the phase velocity are fixed, 5 degrees of freedom remain to optimize the merit values. In this paper we derive several properties of this optimization: (A) One cannot use the 5 degrees of freedom to simultaneously optimize every one of the figures of merit. When the following 3 additional conditions constrain the geometry parameters: the iris radius, the wall-slope angle, and the first merit value E_{pk}/E_{acc}, two degrees of freedom remain to optimize the other two merit values. We observe that: (B) The two degrees of freedom obtained by optimizing the second merit value GR/Q are always very close to those optimizing the third, H_{pk}/E_{acc}. (C) Using the two degrees of freedom to either optimize GR/Q or H_{pk}/E_{acc}, leads to an optimized merit value that depends monotonically on each of the three additional constrains: the iris radius, the wall-slope angle, and on E_{pk}/E_{acc}.

A comparison of shapes of a multicell accelerating cavity was done in terms of (1) the aperture radius R_a for a given wave length, (2) the wall slope angle α, and (3) E_{pk}/E_{acc} the peak electric field normalized to the acceleration field. The choice of these three primary parameters makes easier the trade-off when any particular project is discussed.

It becomes also clear that one can't compare shapes of two cavities with different 2 or all 3 primary parameters and tell, for example, that "the reentrant cavity (small α) has lower GR/Q having the same aperture (R_a)" not mentioning that for comparison was taken the RE cavity with bigger E_{pk}/E_{acc}.

It is also a wrong statement that "...reentrant cavities have potentially higher gradients but smaller apertures and hence larger wakefields" sounded at one of the ILC workshops (May 2006). Let us compare the cavities with **same** apertures, and you will have with the RE cavities the **same** wakefields but **lower** losses, **lower** H_{pk}/E_{acc} and **higher** cell-to-cell coupling as it will be shown in the present paper.

To compare values of aperture we have to use the dimensionless ratio R_a/λ, where λ is the working wave length, or to refer to the same frequency. We will use f = 1300 MHz, used for the TESLA cavities [4] and chosen for the Cornell Energy Recovery Linac. All the other figures of merit to be discussed here do not depend on the size but on the shape of cells only.

We will discuss the shape of the inner cells of a multicell cavity; however, the main statements are also valid for the end cells too.

ADVANTAGES OF THE REENTRANT SHAPE

For illustration of the advantages of the reentrant shape, Fig. 1, we reproduce, to create a holistic picture, some previous results [1], Fig. 2 and 3, and add the recent calculations of the cell-to-cell coupling for the cells optimized for minimal H_{pk}/E_{acc}, Fig. 4. Normalization in Figures 2 and 3 are done so that for the TESLA cavity [4] they are close to 1: $h = H_{pk}/42E_{acc}$ (actually in our calculations $h = 0.99$ for the TESLA cavities with this normalization), $gr/q = (GR/Q)/30800$, where the

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geometric factor times specific shunt impedance $GR/Q = 30800\ \text{Ohm}^2$ for the TESLA cavity.

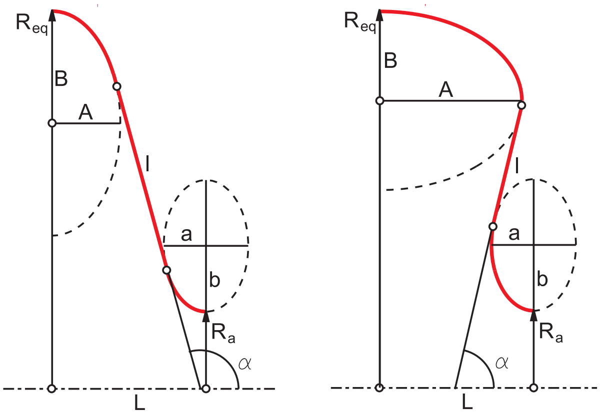


Figure 1: Geometry of the inner cell: non-reentrant (left) and reentrant shapes.

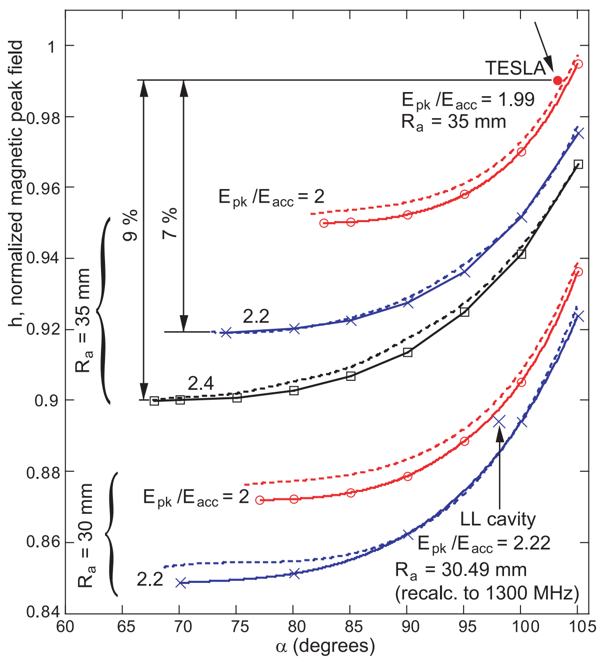


Figure 2: Normalized magnetic peak field for different wall slope angle of wall slope. Solid lines present optimization for min h, dash lines are for max. GR/Q.

Moving along the curves of Figures 2 to 4 to smaller angles, one can see how much the most important properties of the cavity can be improved keeping the same R_a (and, hopefully, HOM properties), and E_{pk}/E_{acc} (the x-raying threshold). Even higher advantages in cryogenic losses and decreasing the peak magnetic field can be achieved if we can to afford higher overvoltage on the iris. And this gain is higher at lower angles.

One can see that the reentrant cavities, i.e. the cavities with $\alpha < 90°$, have lowest losses and minimal peak magnetic field for any given values of aperture radius R_a or E_{pk}/E_{acc}. Moreover, cell-to-cell coupling for the inner cells optimized for minimal H_{pk}/E_{acc} increases when passing from the non-reentrant to the reentrant geometry. Even if this benefit is small, about 0.1 %, it denies the anxiety that the coupling can be lower for the reentrant case.

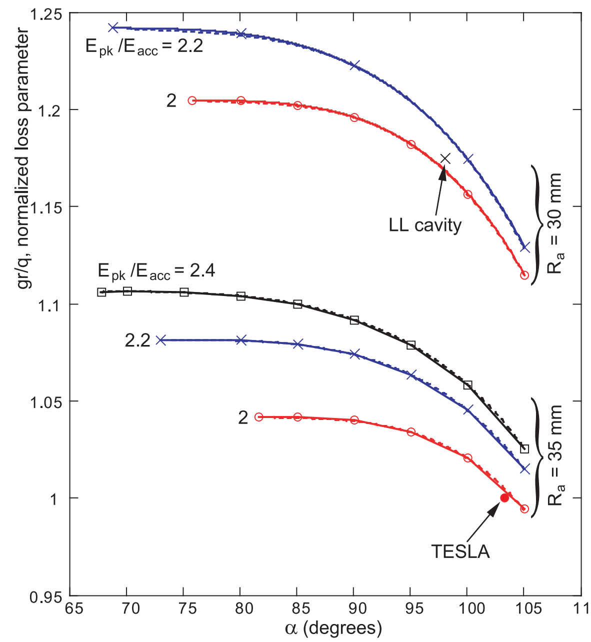


Figure 3: Normalized loss parameter for different angles of slope. Solid lines are for max. GR/Q, dash lines are for minimal h (graphically both lines nearly overlap).

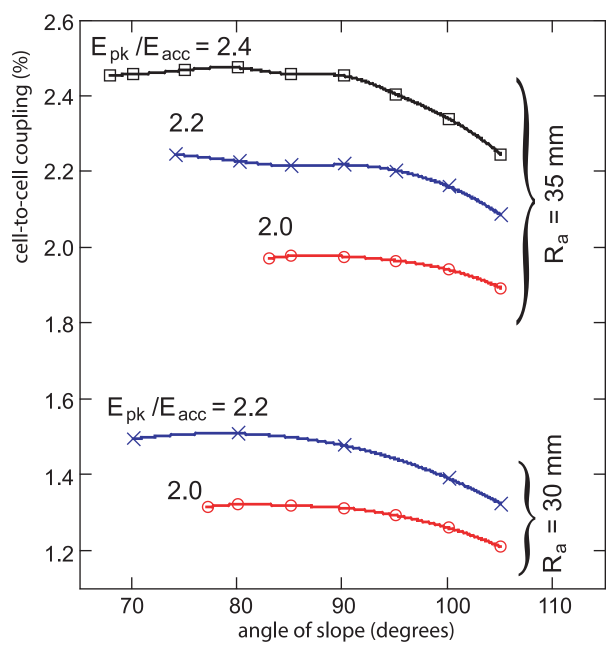


Figure 4:. Cell-to-cell coupling vs angle of slope for inner cell optimized for minimal h.

COMPARISON OF SOME CELL SHAPES

There are also shown in the graphs of Fig. 2 and Fig. 3, the cell of the TESLA cavity [4] and the low-loss (LL) cavity of JLab [5]. Position of the LL cavity cell on the graph is defined by the slope angle (98.0°),

$E_{pk}/E_{acc} = 2.22$, and $R_a = 30.49$ mm (recalculated to 1300 MHz). Linear inter- or extrapolation between the curves with $E_{pk}/E_{acc} = 2.2$ and 2.4 and $R_a = 30$ and 35 mm presented on the graphs gives for this point the place that it actually has. The same is true for the TESLA cavity inner cell, confirming the fact that both cavities are well optimized.

In a recent paper presented at LINAC08, a description of a cavity was done, which the authors called a Low Surface Field (LSF) cavity [6]. It is stated that this design "has both the E_{pk}/E_{acc} and H_{pk}/E_{acc} minimized to alienate potential side effects of high surface fields". From my point of view, this statement is incorrect because you can only find a minimum of one value keeping another one given. So it appears in reality: see Fig. 5 and 6.

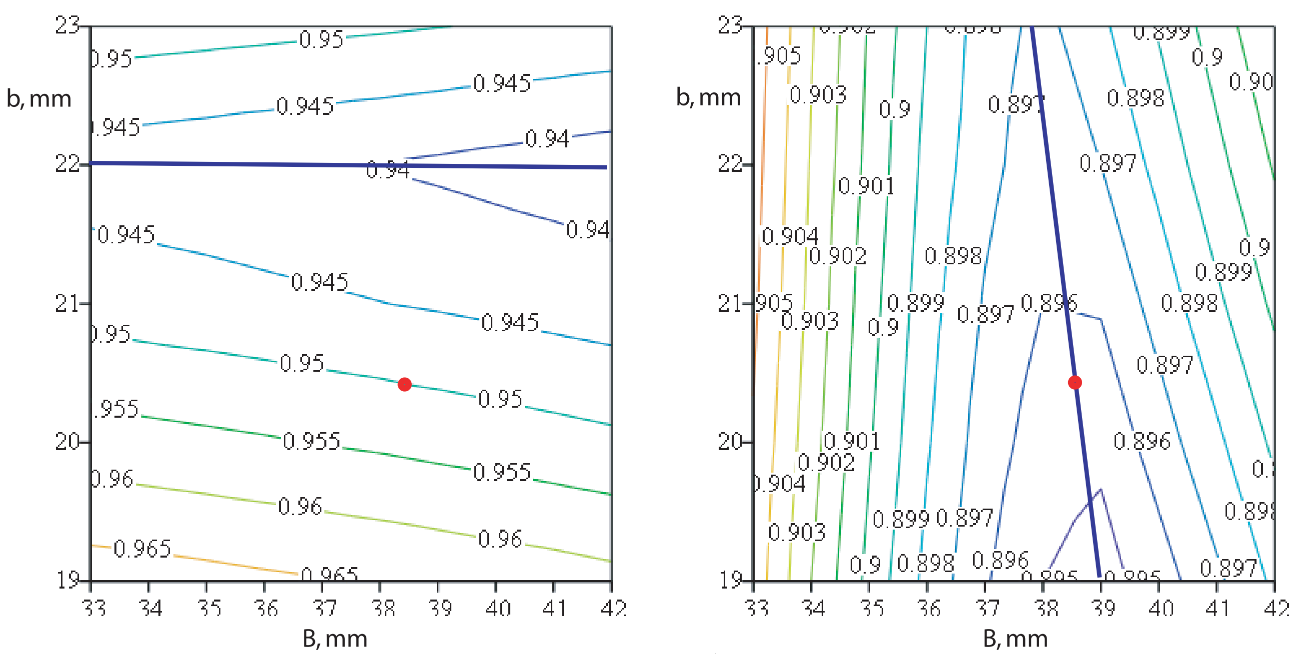
In Fig. 5, the choice of half-axes *B* and *b* is shown (designations are in Fig. 1). The value $e = E_{pk}/2E_{acc}$ is the normalized to TESLA value of the peak electric fields. As can be seen on the right picture, the point defining *B* and *b* is chosen in the "valley" with lowest *h*. However, one can take any point on the solid line shown in the picture. Choosing *e* 5 % lower than in TESLA with *B* = 38.5 and *b* = 20.5 mm, see the left picture, one makes a choice of E_{pk}/E_{acc} and nothing can be said about both minimized values. The crossing of two valleys shown with solid lines can be not the best value too because we can sacrifice one value in favor of another one. (A small deviation of data compared to [6] is due mainly to different value of the half-cell length. Here it is taken $c/4f = 57.652$ and *A* = 45.85 mm instead of 57.692 mm and 45.9 mm as in [6].)

Analogously can be analyzed the next "best point", to the crossing of two "valleys" but the choice is done and no absolute "both minimized" *e* and *h* values exist.

At the same time, the geometrical data for the universal curves presented above (Fig. 2 to 4) which can be found in our internal report [7] give for the point with $R_a = 30$ mm, $E_{pk}/E_{acc} = 2$, and $\alpha = 90°$ close values to the values of the LSF cavity: *A* = 47.49, *B* = 34.52, *a* = 10.16, and *b* = 16.99 mm. The value of H_{pk}/E_{acc} for this geometry is 37.10 Oe/(MV/m) that is also close to the values for the LSF shape: 37.11 (with *a* = 10.5 mm) and 37.70.

It should be noted that all 4 half-axes which are found in [7] are the result of a 4-D optimization, whereas the values of *A* and *a* in [6] are the estimations made on the basis of comparison with other shapes and, possibly, of intuition.

The next new shape which appeared recently [8] is the New Low Surface Field Cavity (NLSF). In this paper, the iris thickness (*a*) was assigned to that of LSF design and the equator radius (R_a) to that of LL. (I believe that this assignment was arbitrary, at least no explanation followed). All other parameters were not varied, other than the cavity parameters *b* and *B*. "Thus a two parameter optimization was performed...". One can see that actually only one parameter (*B* or *b*) was used for the optimization when the other was used for tuning to the work frequency. Further optimization consisted in changing the aperture and actually followed 1-parameter optimization. It was shown that cell-to-cell coupling (or bandwidth) increases with aperture – the well known fact. Some data for comparison are, of course, useful but no new shape was actually found.



e h

Figure 5: Normalized peak fields for the data from [6]. Choice of half-axes *B* and *b* for given values of *A*=45.85 and *a*=11.8 mm.

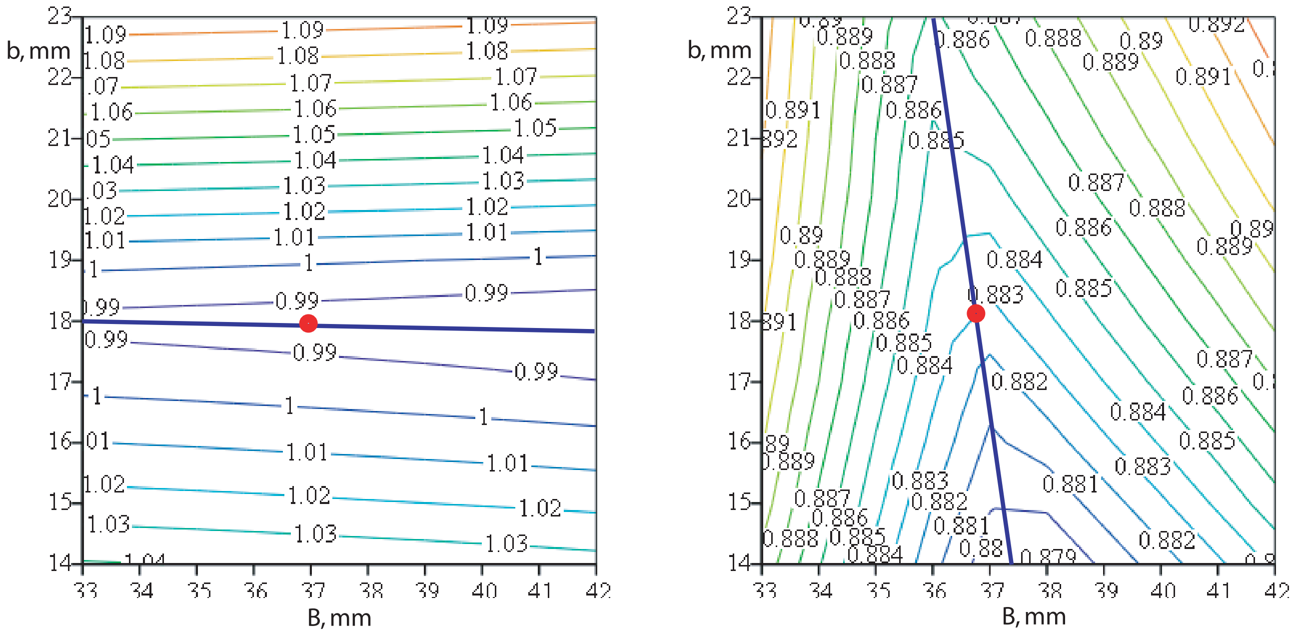


Figure 6: The next best solution of the LSF shape: with A = 47.15 and a = 10.5 mm.

The case of upright walls is easier for analysis because the procedure of optimization becomes 3-dimensional, instead of 4-D for the general case. This is possibly the reason why it is exploited again and again. Possibly, the easier design work with this shape is also a reason. So, it is understandable in the case of Ichiro cavity. However, the chemical treatment becomes nearly the same problem with the upright walls, as it is claimed to be in the case of RE shape. Decrease of losses and the peak magnetic field becomes smaller in the transition from upright to RE case, and this simplification of the design can be taken into account.

LIMITATIONS OF THE ELLIPTIC SHAPE: FLAT WELDING SEAMS

All the cavity shapes usually discussed have elliptic arcs forming the iris and also elliptic equator area. The shape of the iris is mainly responsible for the peak electric field, the correct shape near the equator helps to decrease the peak magnetic field.

However, the real cavity consists usually of half-cells welded along the equator and at the iris. It is difficult to keep the elliptic shape in the area of the seam; there is a flat surface where the fields can be different from the calculated ones. The width of the flat area can be about 3 mm, and even up to 6 mm (see Acknowledgement).

We will analyze changes of some figures of merit on the example of the optimized inner cell of Cornell's ERL.

The influence of the flat seam near the equator is negligible because of a big value of curvature radius. As it can be seen in Fig. 7, the change of the equatorial radius is only 20 mcm when the flat part of the surface is 1.5 mm wide (half of the seam width). E_{pk}/E_{acc} increases less than 2 %, and changes of frequency and H_{pk}/E_{acc} are negligible.

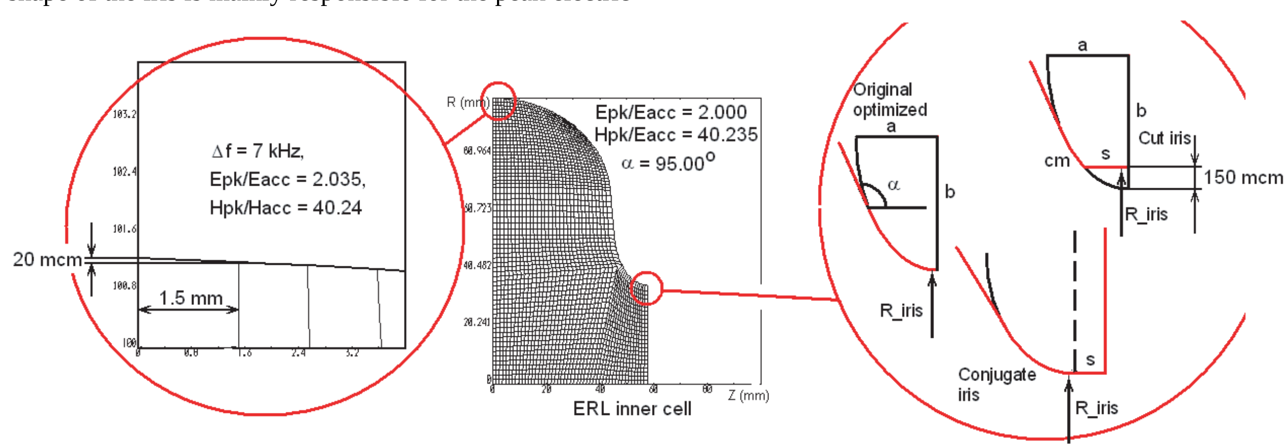


Figure 7: Flat welding seams at the equator and different shapes of the iris area.

Table 1: Parameters of cells with and without a flat seam area

Geometry / Parameter	Original optimized	Cut iris	Conjugate iris (tuned)
A, mm	43.99	43.99	41.4
B, mm	35.08	35.08	32.5
a, mm	12.53	12.53	12.1
b, mm	20.93	20.93	22.91
s, mm	0	1.5	1.5
R_iris	35	35.151	35
α, degrees	95.00	95.00	95.00
Req, mm	101.205	101.205	100.583
Δf, kHz	0	51	0
Epk/Eacc	2.00	2.13	2.00
k, cell-to-cell coupl., %	1.965	1.969	1.768
Hpk/Eacc,Oe/(MV/m)	40.23	40.25	40.71
G*R/Q, Ohm^2	31839	31820	31799 (-0.1 %)

Influence of the iris flattening is much larger. A simple cut (flat after melting) of the tip increases its radius by 150 mcm and has a higher effect on the peak electric field, it increases by 6.5 %. Possibly, such an increase had a place in the cavities with best performances and elimination of this shortcoming will help to further decrease H_{pk}/E_{acc} using higher nominal values of E_{pk}/E_{acc}, and increase the accelerating field. Optimization of the cell with flat segments on the contour line conjugate to the elliptic arc leads to thicker iris and lower cell-to-cell coupling if one keeps the original values of aperture, wall slope angle, and normalized peak electric field as can be seen in the Table.

If the width of the seam will be 6 mm (s = 3 mm, see Fig. 7) the effect of the increase of E_{pk}/E_{acc} can be quadratic relative to s, and this value can increase up to 2.5. A thorough work should be done to control the shape of the welding seam.

CONCLUSION

Three primary parameters, R_a, E_{pk}/E_{acc}, and the wall slope angle α are a good basis for comparison of cavities' figures of merit because most of them depend monotonously on these parameters. All the main properties of the RE shape appear to be the best if compared with other shapes having same R_a and E_{pk}/E_{acc}. Different proposed shapes of the cavities either fit the proposed universal curves or are worth in terms of GR/Q, H_{pk}/E_{acc}, or cell-to-cell coupling.

The real cells can be different from the elliptic shape having flat welding seam areas and this can influence much more significantly on the figures of merit than some optimizations.

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