THE CHALLENGE OF MONTE CARLO METHOD IN TWO SIDED MULTIPACTOR

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Abstract

Superconducting RF cavities have been used in modern accelerators. SRF cavities have the limitation on maximum RF voltage and maximum delivered RF power. The major limitation comes from Multipacting in the cavity and waveguide. To design a Multipacting-free RF structure, numerical tracing calculations are required. Here, Monte Carlo method within a plane parallel model is employed using a wide range of parameters. For more accurate predictions, a long history of electron trajectories between two parallel plats is investigated. Simulations give us fast convergence when we are far from the avalanche threshold. However it is very difficult to get such convergence when we are close to the avalanche threshold. In the present work, we demonstrate how we can cope with this difficulty.

INTRODUCTION

Resonance secondary electron emission or Multipactor can occur when a free electron caught in an RF field, impact with a solid at sufficient energies to release more than one secondary electron at a phase where the RF filed can accelerate these secondary electrons. As this procedure is repeated the MP effect will create an exponential growth of number of electrons [1]. MP is normally an unwanted phenomenon in vacuum operated microwave device. It could cause electric noise, detune microwave cavities, reflecting cavity power, heat the component and in the worst case cause permanent physical damage for the component [2]. In Ref. [3] MP inside a rectangular waveguide was numerically studied with the concept of MP resonance. Vdovicheva *et al.*in[5] employed both analytical approach and numerical simulations to study the MP inside a rectangular waveguide. It was shown when the waveguide height is much smaller than its width; the parallel planes model can be used to stimulate the Multipactor . In this case, conventional resonance theory was used for the MP threshold. The vacuum waveguide part of CESR RF system is the case where paralle plate can be used for simulating MP inside of the waveguide. A new statistical method has been exploited in [4] which theoretical approach was taken to provide exact solution for the arbitrary electrode separations. The initial velocity has the Maxwellian distribution. However, particular calculation was performed in Ref. [5] with the assumption that the velocity distribution depends on the electron ejected angel. They presented the results of numerical simulation for both angular and energy distribution.

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In all works introduced above, the ultimate goal was to investigate conditions under which the MP appears. In the other words, the threshold for the appearance of secondary emission discharge was systematically studied. All these works have one thing in common that is they have kept a short history of the particle trajectory. Now the question is that, have one a fair estimation of the strength of multipacting yield by keeping a short history of the trajectory? In order to answer, this question we start investigating the MP effect by keeping longer histories. knowing the strength of Multipacting yield is essential to decide the effectiveness of the techniques and models used to suppress the MP. Here in this work we report on a convergence problem that we encountered by keeping long histories. It will be shown that a limited number of odd events appears that shifts the average value of the observables dramatically. How to overcome the convergence problem will be discussed as well. In this study, dimensions of the vacuum waveguide part of CESR RF system, and its operational frequency are used as a real numerical example [6].

EQUATION OF MOTION

In this section we write the equation of motion for an electron under the action of high frequency electric field between two metalic parallel plates separated by distance L. Equation of motion, for an electron in the gap is:

$$\ddot{x} = \frac{eU}{m\omega}\sin(\omega t), \quad (1)$$

where the coordinate x, is measured from the surface of one of the electrodes. U is the voltage amplitude across the gap L, $\omega = 2\pi f$, where f is the oscillation frequency and t is the time. We rewrite this equation in the normalized form:

$$\lambda'' = \xi \sin\theta, \quad (2)$$

where $\lambda = \frac{x}{L}$, $\xi = \frac{U}{U_0}$, $U_0 = m\omega^2 L^2/e$ and $\theta = \omega t$. Primes denote derivations with respect to θ while dots indicate derivatives with respect to t.

Electron velocity and trajectory are described by the following formulas respectively [6]:

$$\lambda' = \xi(cos\theta_1 - cos\theta) + \beta_1,$$

$$\lambda = \xi(\theta - \theta_1)cos\theta_1 + \xi(sin\theta_1 - sin\theta) + \beta_1(\theta - \theta_1) \quad (3)$$

where θ_1 is the phase at which the electron enters the gap, and $\beta_1 = \overline{v}/\omega L$ is the dimensionless normal component of the initial velocity of the secondary electron.

Equation 1implies

$$1 = \xi(\theta_2 - \theta_1)cos\theta_1 + \xi(sin\theta_1 - sin\theta_2) + \beta_1(\theta_2 - \theta_1) \quad (4)$$

where θ_2 is the phase at which the electron reaches the second electrodes at $\lambda = 1$. The condition for the electron to "resonantly" cross the gap is that the transit time should be equal to an odd multiple of half periods of the RF field. That means $\theta_2 - \theta_1 = (2n-1)\pi$. Here n is called the order of MP. From (4) we can get:

$$\xi = \frac{1 - (2n-1)\pi\beta_1}{(2n-1)\pi\beta_1 + 2sin\theta_1} \quad (5)$$

The value of β_1 in (5) is given by: $\beta_1 = \frac{\overline{v}_\perp}{\omega L} = \frac{2}{3}\frac{\overline{v}}{\omega L} = \frac{2}{3}\sqrt{\frac{2\overline{U}_s}{U_0}}$, where $\overline{U}_s$ is the voltage corresponding to the mean velocity of the secondary electrons.

MONTE CARLO METHOD

It is emphasized that in the actual situation, the initial ejected velocity and angular of an electron and initial RF phase are random quantities. To illustrate the effect of random emission velocity on the Multipactor discharge, we assume it follows Maxwellian distribution with mean value corresponding to 2ev. The angular distribution of the secondary is assumed to be proportional to $\cos\varphi$, where φ is the angle with the normal to the surface, and the dependence of the secondary emission yield is given by Vaughan's formula [1]

$$\sigma = \sigma_m . \left[\epsilon \, exp(1-\epsilon)\right]^{\nu(\epsilon)} \quad (6)$$

where $\epsilon = W_i/W_m$, $\nu = 0.62$ if $\epsilon < 1$, $\nu = 0.25$ if $\epsilon > 1$. W_i denotes impact electron energy, σ_m is the maximum value of the secondary emission yield, and W_m is the impact energy corresponding to this maximum. Because of complexity of the electron motion in plane parallel plate, the problem was studied by numerical simulations [5]. In our simulation, we consider MP process can be also formed by those secondary electron that return back to their birth plate. Typically, Monte Carlo method is one of the numerical simulations of the Multipacting process. This method is applied to study Multipactor within parallel plate model. Nevertheless, even the use of a Monte Carlo code requires tremendous computing time when it is necessary to carry out the simulation within a wide range of parameter. Basically, numerical simulations are very often carried out with a reduced number of electron trajectories and reliability of the simulation results depends on the method chosen for this reduction. Considering short history of the electrons, e.g. 15 gap crossings, is one way of reducing the number of electron trajectories in which the calculation of the threshold value of secondary electron emission is obtained. In this case, the initial number of electron required for getting reliable result will be relatively decreased. However, it may be questionable why 15 gap crossings are sufficient. The answer might be, for calculation of threshold value, the rate of number of initial electrons in the system is important. The latter case can be clearly defined with even 10-15 gap crossings. In this work, the average yield of secondary electrons per trial electron is called Multipacting yield. The word average means that it is convergent statistically. In all simulations the RF frequency and the gap size are taken 500 MHz and $4''$ respectively, the optimal impact energy and optimal secondary emission yield are $W_m = 400\ eV$ and $\sigma_m = 2.2$ respectively. Multipacting yield, MY, for each run of the code is calculated and recorded.

RESULTS OF NUMERICAL SIMULATION

In our results, each run of n trial electrons yields a mean value of the total secondary electrons which are produced within defined crossing gap. Let us assume, 15 gap crossings for the particle in our simulation. In the following Figs.1-8 will be presented in which power=178 KW is considered as the RF power waveguide.

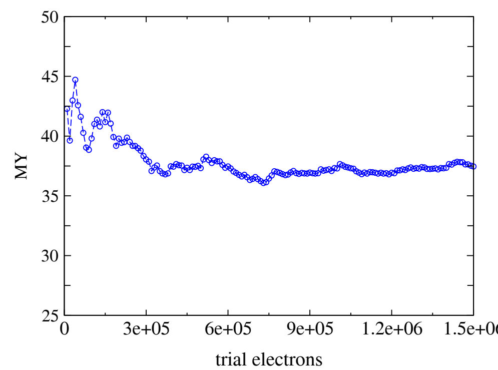

Figure 1: the MY values are shown for different runs of $n = 15 \times 10^5$ trial electrons, where 15 gap crossings are considered

We can see in Fig.1, MY over $n = 30 - 350 \ \times 10^4$ of trial electrons is roughly constant so it has been achieved one clue of getting convergent results. That means MY in $n = 30 - 350 \ \times 10^4$ is a good estimation of Multipacting yield.

Another way to know that those trial electrons are sufficient to achieve desired accuracy is to make different measurements useing a different sequences of random numbers. The left plot of Fig.1 also implies 3×10^5 of trial electrons can give us the desired accuracy. To get a more complete picture of MP process, it is necessary to have a look at the particles for a longer history. In this respect, e.g. 30 gap crossings for the particles are assumed.

As it is seen Fig.3, some rare events are observed in our simulated Multipacting yield for some special trail electrons. We see special trial electron can produce huge amount of MY after 30 gap crossings and its contribution has a significant weight on the total production.

this has been observed once after every few thousand trial electrons. That means the probability of these events

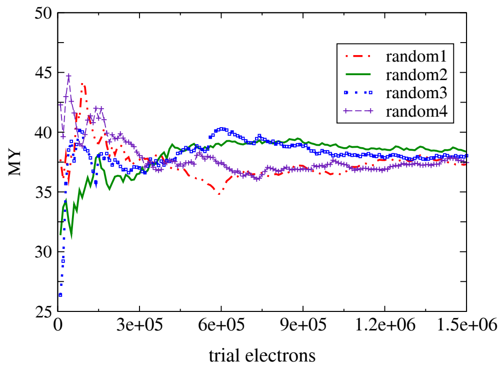

Figure 2: The MY values are shown for different measurements uses a sequence random number with $n = 15 \times 10^5$ trial electrons, where 15 gap crossings are considered.

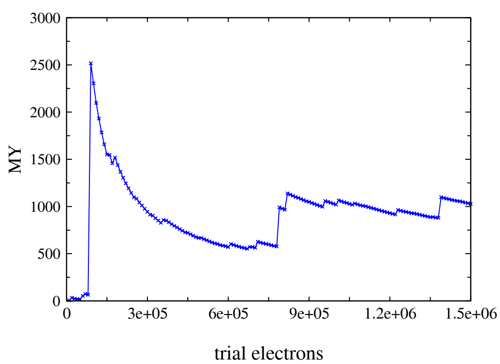

Figure 3: the MY values are shown for different runs of $n = 15 \times 10^5$ trial electrons where 30 gap crossings are considered.

are very low but they carry out large value of MY. In the other word, our results are dominated by these rare events. As we can see this kind of rare events makes some difficulties in fast convergence of Monte Carlo simulation. This is because of being near to the avalanche threshold. Fig.4 shows the probability of each value of secondary electrons when 15×10^5 trial electrons and 30 gap crossings are chosen. As we can see e.g, the secondary electron with 10^7 value is only once observed and the probability close to 1 indicates that values of most secondary electron are between 1000 - 10000. To realize the contribution of these rare events in total value of MY, first of all we will show the MY value by considering all probabilities then we exclude those events which their probability are less than 1E-6 and then exclude those events that their probability are less than 1E-5 and continue this process until 1E-1. The Fig.5 presents, considering all probabilities, MY value is 387 if we exclude the events with the probability less than 1E-6, MY value becomes 22 and continuously it goes down. we can also present MY with considering only indicated probability. It is shown in Figs.7-8 for 3o and 15 gap crossings.

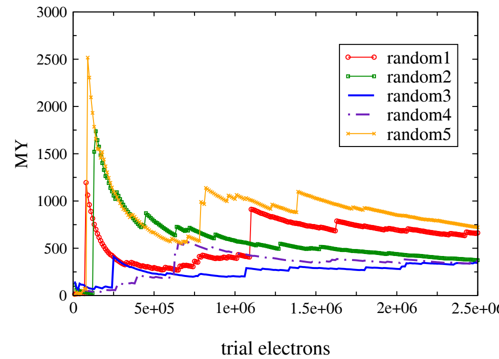

Figure 4: MY values is shown for different measurements use a sequence random number with $n = 25 \times 10^5$ trial electrons where 30 gap crossings are considered.

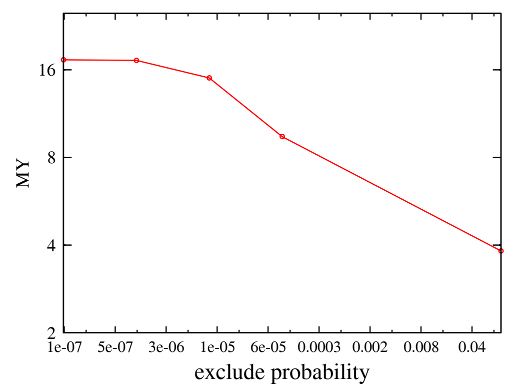

Figure 5: the evaluation of MY values with excluding indicate probability is shown, where 15 gap crossings are considered.

To better understand the role of these rare events, fig.6 shows the probability plot in which we consider only 15 gap crossings when we have convergent results. It is seen in Fig.6 even by excluding those events with low probability e.g, 1E-6, 1E-7, the total result, MY, will not change. So when 30 gap crossings are considered, those events which have low probability produce some big jumps in our results. In fact, obviously, it is necessary to take into account all events to have a fair estimation of Multipacting yield. Therefore we have to know more details about the reason of existence of these rare events.

”Plausible” argument to explain the existence of the kind of statistical noise is the definition of MP. In the other words, MP is narrow phenomenon which can happen at a special RF phase and special conditions. If one concentrate on the characteristic of these points, it can be found that resonance conditions are located in these lucky points, the points with large value of MY. Hence it is important to recognize that observing such kind of big jumps is due to intrinsic of MP property. Typically, it is expected that one can get meaningful statistical convergent results with sufficient trial electrons. But in our case, even by making additional run of trial electrons we couldn’t get the convergent

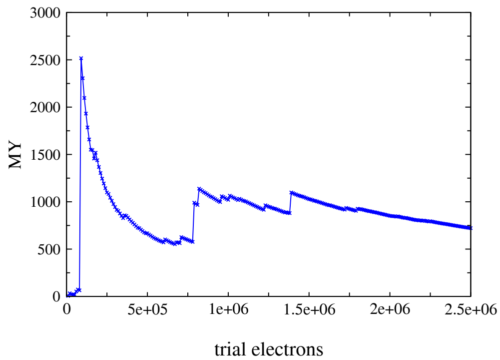

Figure 6: MY values are shown by increasing the number of trial electrons until $n = 25 \times 10^5$ trial electrons where 30 gap crossings are considered.

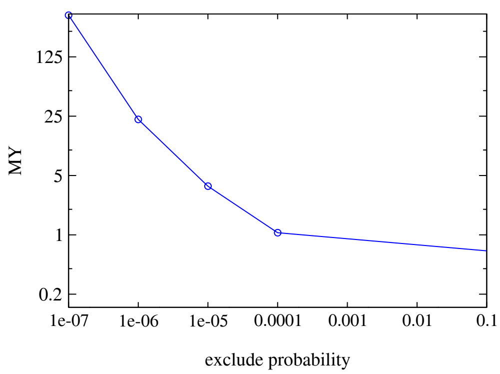

Figure 7: the evaluation of MY values with excluding indicate probability is shown, where 30 gap crossings are considered.

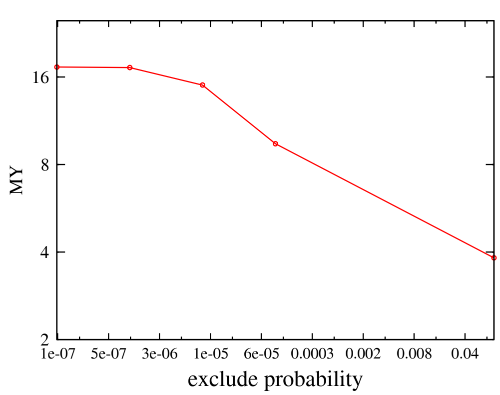

Figure 8: the evaluation of MY values with excluding indicate probability is shown, where 15 gap crossings are considered.

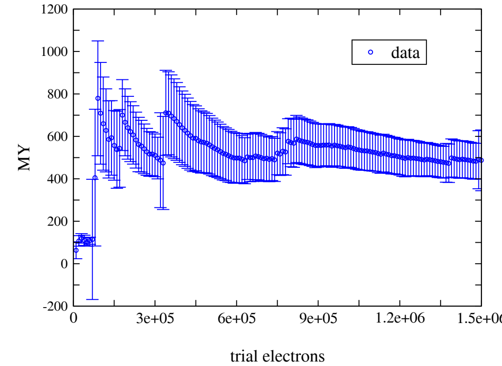

Figure 9: Y values where they are achieved by taking an average over different values of eight different measurements with their error calculation.

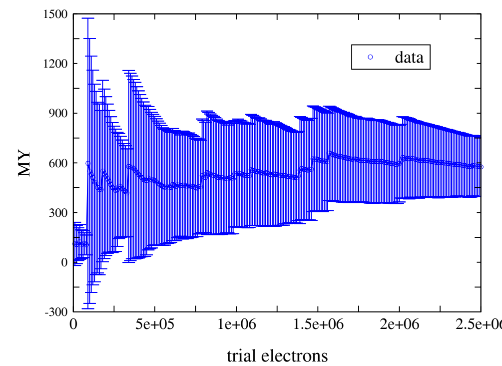

Figure 10: MY values where they are achieved by taken an average over different values of twelve different measurements with their error calculation.

result, Fig.7. Even with taking different measurements using the different sequence of random, we observe statistical noise for each measurment, Fig.8.

Here the question is What would be MY in reality? Is it the one with the heights priority? Or is it basically correct to exclude the rare events from total results?

From the results of different measurements, we obtain different MY values for each run of n trial electrons. If we take an average over these different values of MY, for each run of n trial electrons. As we can see, relatively, the rare events decreases. Reducing rare events, effectively, is arguably the most important point that is offered by this method. To obtain an estimate for the error, we make a calculation of standard deviation of each n trial electrons from the average value which given through our method. The solution that we offer is to take an average value of different seeds for each group of trial electrons as shown in Fig. 26. We expect that we can reduce the error as small as we wish by either increasing the number of trial electrons or by increasing our measurements and thereby reducing the standard deviation. Here we did both method mentioned above, but errors become larger than previous results,Fig.

18-27. This happens because of more observing rare events in this case. Then for getting reliable results we should take more different seeds depending on how much accurate we need. This method is intuitive and gives us a reasonable estimation of the MY.

CONCLUSION

Recently, several analytical models has been presented in order to suppress the Multipactor effect. To verify these models through simulation code, it is necessary to know an estimation of the magnitude of strength of Mulipacor within the system we are interested in . The main purpose of this work, in contrast to most previous studies, is to achieve a fair estimate of the strength of two sided Multipactor within rectangular waveguide. In order to obtain reliable prediction, long history of particle motion in our simulation is proposed to achieve not only the strength of Multipacting yield, but also for getting more complete picture of Multipactor process. This approach causes some problems in getting convergent results where the rare events are observed and they make dramatic effect on other results. The solution that we offer is averaging over different measurements for each run of n trial electrons. In this way, as the results confirm, the effect of rare events weakens.

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