HIGHER ORDER MODE PROPERTIES OF SUPERCONDUCTING TWO-SPOKE CAVITIES

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Abstract

Multi-Spoke cavities lack the cylindrical symmetry that many other cavity types have, which leads to a more complex Higher Order Mode (HOM) spectrum. In addition, spoke cavities offer a large velocity acceptance and can be used over a wide velocity range, which means we must perform a detailed analysis of the particle velocity dependence for each mode's R/Q. We present here a study of the HOM properties of two-spoke cavities designed for high-velocity applications. Frequencies, R/Q and field profiles of HOMs have been calculated and are reported.

INTRODUCTION

Superconducting, TEM-class two-spoke cavities, for 352 MHz and 325 MHz, are being considered for high-velocity applications. The diameter of spoke-loaded cavities is on the order of λ/2, which implies lower operating frequencies for a given size and therefore a larger velocity acceptance. Another of the advantages over conventional elliptical TM-class cavities includes increased longitudinal acceptance, lower surface resistance and thus lower heat load allowing for the possibility of 4 K operation [1]. The description and optimization procedure of the 352 MHz cavity has been previously reported [2]; table 1 presents the main characteristic properties of the 325 MHz two-spoke cavities [3].

Table 1: RF parameters for 325 MHz 2-spoke cavity

Parameter	$\beta_0 = 0.82$	$\beta_0 = 1$	**Units**
Frequency fundamental mode	325	325	MHz
R/Q	543	621	Ω
Geometrical factor	167	188	Ω
E_p / E_{acc}	2.49	2.27	
B_p / E_{acc}	5.4	5.32	mT/(MV/m)
B_p / E_p	2.17	2.34	mT/(MV/m)

At E_{acc} = 1 MV/m and reference length = $\beta_0\lambda$

Even for velocity-of-light cavities the fundamental accelerating frequency mode in two-spoke cavities is the lowest order mode. Excitation of higher-order modes (HOMs) by a high-current beam can lead to subsequent beam instabilities and should therefore be damped by appropriate HOM couplers. In this paper, we report a detailed study of longitudinal and transverse HOMs. A CST Microwave Studio picture of the 325 MHz, $\beta_0 = 0.82$ cavity is shown in figure 1.

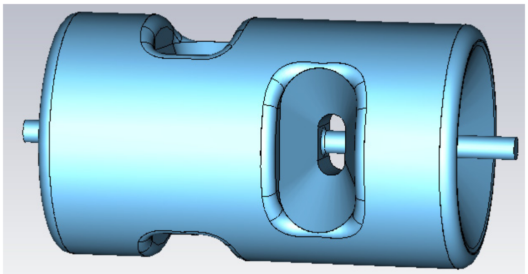

Figure 1: CST Microwave Studio picture of the 325 MHz, $\beta_0 = 0.82$ two-spoke cavity.

HOM PROPERTIES

The mode types can be classified as accelerating, deflecting, or TE-type (meaning a strong H_z field component along the beam axis and negligible transverse electric field). If we define the longitudinal direction (that which the beam propagates in) as z, then one of the spokes is oriented parallel to the x-axis, while the other is oriented parallel to the y-axis. Deflection occurs in both the x and y directions because the fields throughout the cavity are not strictly in either the x- or y-directions but rather contain components in both (see figure 2). This requires us to calculate separate, direction-specific [R/Q] for each deflecting mode.

The longitudinal [R/Q], which applies to accelerating modes where the electric field is along the beam axis, is given by:

$$\left[\frac{R}{Q}\right] = \left.\frac{\left|\int_{-\infty}^{\infty} \vec{E}_z(z,r=0)\, e^{i\left(\frac{\omega z}{\beta c}+\varphi\right)}\right|^2}{\omega U}\right|_{\max\varphi} \tag{1}$$

where ω is the angular frequency of the mode, βc is the particle velocity, and U is the stored energy in the cavity. On the other hand, for deflecting modes, where we have deflection with E_x and H_y in addition to E_y and H_x for the same mode, the transverse $[R/Q]_T$ is calculated by direct integration as,

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$$\left[\frac{R}{Q}\right]_T = \left.\frac{\left|\int_{-\infty}^{\infty}\left[\left(\vec{E}_T(z,r=0)+i(\vec{v}_z\times\vec{B}_T)\right)e^{i\left(\frac{\omega z}{\beta c}+\varphi\right)}\right]\right|^2}{\omega U}\right|_{\max\,\varphi} \quad (2)$$

where T is either x or y (noting that if we are discussing $E_{x(y)}$, then we are also talking about $B_{y(x)}$). The transverse electric field profiles for the fourth mode (M4) in the 352 MHz, $\beta_0 = 0.82$ two-spoke cavity are shown in figure 2, and the electric field profiles of the first three accelerating modes for the 325 MHz, $\beta_0 = 0.82$ two-spoke cavity are shown in figure 3.

The Panofsky-Wenzel theorem [4] can also be used and is given by

$$\left[\frac{R}{Q}\right]_T = \lim_{a\to 0}\left.\frac{\left|\int_{-\infty}^{\infty}\left(\vec{E}_z(z,r=a)-\vec{E}_z(z,r=0)\right)e^{i\left(\frac{\omega z}{\beta c}+\varphi\right)}\right|^2}{(ka)^2\omega U}\right|_{\max\,\varphi} \quad (3)$$

where $k = \frac{\omega}{c}$, a is the offset from the beam axis, which can be in the x- or y-direction, and φ in both (2) and (3) is the phase which is varied to maximize [R/Q].

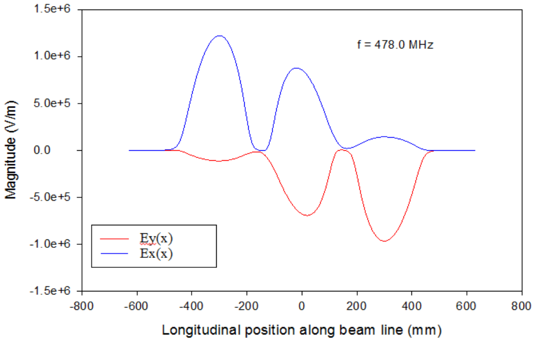

Figure 2: Transverse electric field components along the longitudinal direction for mode 4 (M4) of the 352 MHz, $\beta_0 = 0.82$ cavity.

Frequency Spectrum

The fact that most of the deflecting modes of the two-spoke cavity are two-fold degenerate leads to there being nearly 80 modes between the fundamental mode, f_0, and $3f_0$. Table 2 shows the type and frequency of some of these modes.

The [R/Q] values for the modes in table 2 have been calculated for particles travelling at the design velocity, 0.82c. Figure 4 shows the results. It is clear that the fundamental mode has, by far, the highest value, with the next largest being several times lower.

Table 2: Two-spoke cavity modes for $\beta_0 = 0.82$

Mode type	**352 MHz Cavity Frequency (MHz)**	**325 MHz Cavity Frequency (MHz)**
Accelerating	352, 358.9, 378.5, 501, 527, 663, 686, 714, 719, 775, 803, 819, 838, 866, 873	325, 329.5, 352.5, 465, 493, 613, 635, 682, 756, 770, 796, 806, 823, 824, 836, 840
Deflecting	478*, 530*, 573*, 615*, 664, 673, 677*, 702, 709.*, 729*, 773*, 848*, 878*	443*, 498*, 534*, 567*, 639*, 648*, 672*, 706*, 773*, 786*, 805*, 813*, 841*
TE-Type	663, 672, 702, 735, 791, 820, 837, 850, 889, 931	628, 642,666, 689, 705, 739, 791, 815, 819, 860

*indicates degenerate modes

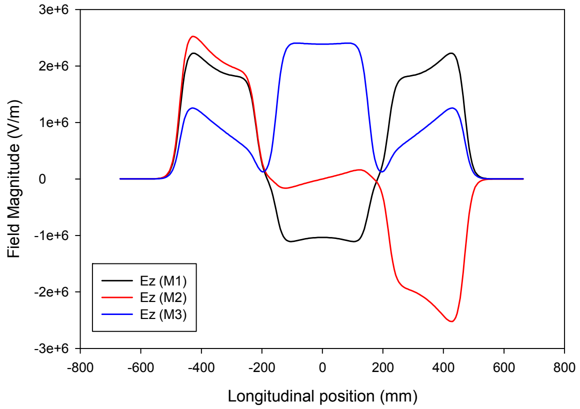

Figure 3: Longitudinal electric field components along the longitudinal direction for the first three accelerating modes of the 325 MHz, $\beta_0 = 0.82$ cavity.

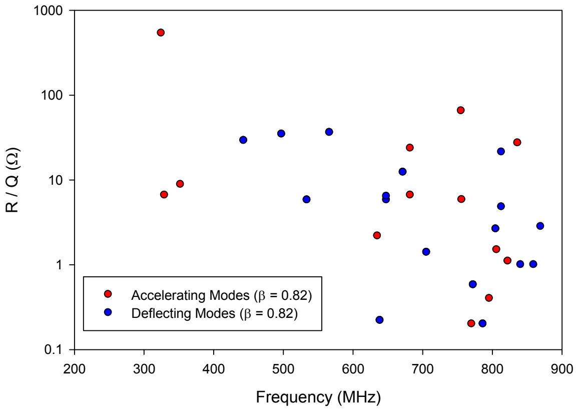

Figure 4: R/Q values for particles at the design velocity, $\beta_0 = 0.82$.

Particle Velocity Dependence

As mentioned previously, spoke cavities are intended to operate over a wide velocity range and we need to analyze the [R/Q] value of HOMs for a large range of β. Furthermore, we need to consider that the phase φ_T that maximizes the mode coupling with beam particles will be velocity-dependent as well. Since the transverse electric and magnetic fields are not necessarily odd or even for two-spoke cavities, the phase φ_T cannot be determined by inspection. Therefore, we are required to calculate it by maximizing the expressions given in (1) and (2). Thus, for each β we have identified a corresponding φ_T that yields a maximum [R/Q] for the relevant mode.

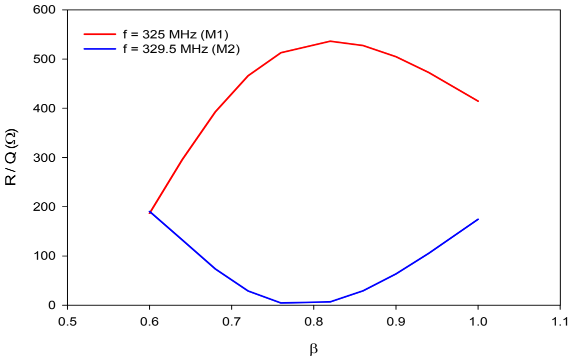

Figure 4: R/Q values for the fundamental mode and the next highest accelerating mode as a function of β.

Figure 4 shows how the [R/Q] of the fundamental and second mode varies for a wide range of particle velocities. This second mode is the only one analysed thus far that has a comparable [R/Q] value for $0.60 \leq \beta \leq 1.0$. In fact, particles travelling at velocities of 0.60c would see the same [R/Q] for the both modes.

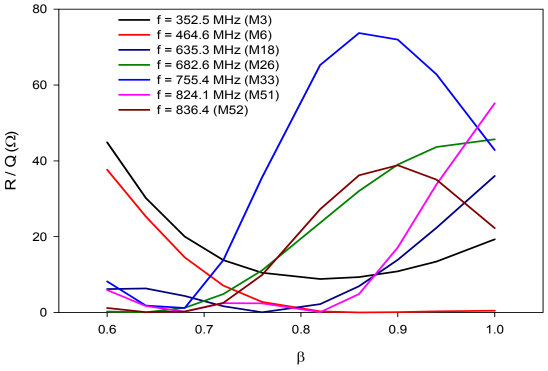

Figure 5: Highest R/Q values for the accelerating modes up to 900 MHz as a function of particle velocity.

Figures 5 and 6 show [R/Q] as a function of β, for accelerating and deflecting HOMs respectively, where the magnitude of [R/Q] is close to or greater than 10% that of the fundamental mode.

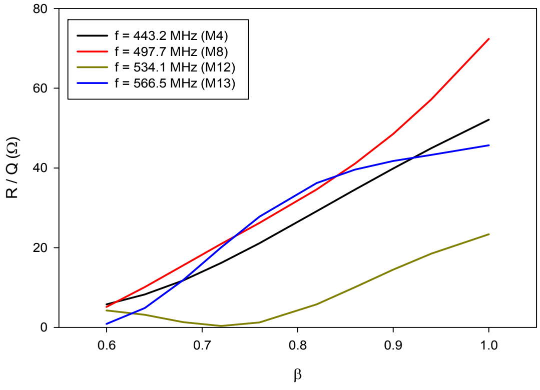

Figure 6: Highest R/Q values for the deflecting modes up to 900 MHz as a function of particle velocity.

CONCLUSION

It has been known that low- and medium-β spoke-loaded cavities have the benefit that the lowest frequency mode is the fundamental accelerating mode. We have shown that this is the case for high-β two-spoke cavities as well.

An initial HOMs study of two-spoke cavities for high-velocity applications is presented. HOMs in the range of f_0 and $3f_0$ have been analysed in terms of their [R/Q] for a large range of β. While a large number of modes exist (some exhibiting two-fold degeneracy due to the geometrical symmetries of the cavity), their individual [R/Q] is substantially smaller than that of the fundamental mode at the design particle velocity. We are currently continuing this investigation for higher frequency modes of both the 325 MHz and 352 MHz cavities optimized for $\beta_0 = 0.82$ and beginning the HOM analysis for $\beta_0 = 1$ cavities at the same frequencies.

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