SOME DESIGN ANALYSIS ON THE LOW-BETA MULTI-SPOKE CAVITIES*

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Abstract

The spoke cavities have achieved some promising gradients worldwide, which make them good candidates for accelerating low beta proton and ions. Although, there is still a space to further optimize them, especially for multi-spoke cavities, the design and optimization are re-considered based on the total capital and operational efficiencies over a given beta range. An initial result of the 3D Electromagnetic (EM) design optimization by the CST MWS code for a double-spoke, velocity β ~0.5 cavity is reported. An equivalent circuit for the double-spoke cavity is also developed.

GOAL FOR EM OPTIMIZATION

Accelerating Efficiency of the Low-β Cavities

Currently when a cavity is being designed, people usually focus on the performance at β_0 which refer to the max transient time factor (TTF). When the cavity design is finished, the overall acceleration within a given β range is calculated by integrating the EM field numerically. It will be interesting if we can find some methods to optimize the acceleration efficiency in a given β range into consideration at the EM design phase.

It is well known that, by omitting the β change within a cavity, the TTF can be calculated as [1]:

$$\mathrm{TTF}(\beta) = \frac{\int_{-\infty}^{\infty} E(x)\cos(\omega x/\beta c)dx}{\int_{-\infty}^{\infty} |E(x)|dx} \tag{1}$$

where cosine is for the even field profile. Then the increment of β for one cavity is found to be:

$$\Delta\beta = \frac{U_0}{m_0 c^2}\frac{TTF(\beta)}{\beta\gamma^3} \tag{2}$$

where $U_0 = qV_0\cos\varphi$, $V_0 = \int_{-\infty}^{\infty}|E(x)|dx$, and φ is the phase of field when the particle is in the middle of the cavity.

Now assuming one cavity shape is used from β_1 to β_2, if longitudinal stability is not an issue, we can drive all the cavities on crest (i.e. φ=0), then the number of cavities to be used is

$$Nca(\beta_1,\beta_2) = \frac{m_0c^2}{U_0}\int_{\beta_1}^{\beta_2}\frac{\beta\gamma^3}{TTF(\beta)}d\beta \tag{3}$$

So we can define the accelerating efficiency of this cavity from β_1 to β_2 as:

$$Eff(\beta_1,\beta_2) = \frac{\Delta E_k}{U_0 Nca} = \frac{\gamma_2-\gamma_1}{\int_{\beta_1}^{\beta_2}\frac{\beta\gamma^3}{TTF(\beta)}d\beta} \tag{4}$$

The efficiency can be seen as an averaged TTF in the β range. The result of Eq.4 typically has a less than 1% error comparing with particle tracking results.

Project Specified Goals

General challenges and considerations to low and medium β cavity design have been well described [2]: Ep, Bp, G*R/Q and HOM are usually most concerned for EM design. Here we restrict ourselves to fundamental mode and discuss the goals for more specified cases.

- Pulsed machine:

For pulsed machines, dynamic heat loading may not be important, so gradient and peak surface field that will affect the capital cost, will be set as the design goal. The published test results at 2K of overall 13 spoke cavities show that [[3]-[4]]: Bp achieved 80-130 mT, and Ep achieved 30-80 MV/m, while 5 of them had field emission around Ep=30MV/m and all were processed away expect for one that had a large surface defect. These typical levels of surface field and FE have also been reported for QWR [4].

So, if an accelerator is proposed within the state of art, then a goal of Ep<30MV/m and Bp<80 mT is reasonable; if for some reason, an accelerator is proposed to push the limit of low-beta cavities, then a goal of Ep<60-80MV/m and Bp<120mT could be considered, in other words, lower Bp/Ep ratio is preferred.

Since the number of cavities and their total length are:

$$\begin{cases} Nca(\beta_1,\beta_2) = \dfrac{\Delta E_k}{q\cos\varphi}\dfrac{Ep/\overline{Va}}{Ep} \\ Ltotal(\beta_1,\beta_2) = \dfrac{\Delta E_k}{q\cos\varphi}\dfrac{Ep/\overline{Ea}}{Ep} \end{cases} \tag{5}$$

where $\overline{Va} = V_0 Eff(\beta_1,\beta_2)$, $\overline{Ea} = \overline{Va}/Lend$, and Lend is the length of the cavity from one end to the other end. So, the optimization goal should be: $Ep/\overline{Va}$ or $Ep/\overline{Ea}$ as low as possible, while keeping Bp/Ep ratio less than a chosen value.

- CW machine:

For CW machine cryogenic loss is worth taken into consideration. The number of cavities is possibly determined by the trade-off of between capital and

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operational costs, then the total dynamic heat loading is found to be:

$$Pdyn = \frac{R_s \Delta E_k^2}{(q\cos\varphi)^2} \frac{1}{Nca} \frac{1}{G\overline{(R/Q)}} \quad (6)$$

where $\overline{(R/Q)} = \frac{\overline{Va^2}}{\omega U}$, G is cavity's geometry factor

So the optimization goal should be: to let $G\overline{(R/Q)}$ as large as possible, while keeping Ep/Bp ratio less than some chosen value.

Example with a Linac Profile

A linac composed of three families of cavity is taken as an example: QWR at 170MHz from β= 0.1 to β1, HWR or single spoke at 340MHz from β1 toβ2, and Duo-spoke at 340MHz from β2 to 0.6.Assuming all the cavities are driven on crest, and length of one cavity is Nβλ/2 where N is the number of gaps. For one type of cavity used within a β range, it is found that it is a good choice to set the β_0 in the middle of the β range for sine as well as square-wave field profile, while it is preferred to set β_0 ~10% larger for a realistic field profile, as shown in Fig. 1.

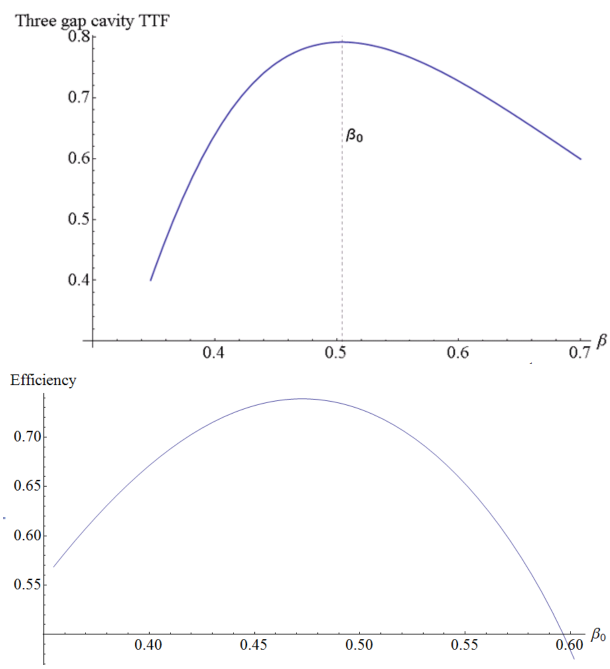

Figure 1: An example of the cavity efficiency optimization.

By further assuming the linac is operated at Ep=30MV/m, applying the typical Ep/Ea ratio and length of the cavities, the V_0 are 13MV*β_0, 9MV*β_0, and 11MV*β_0, for QWR@170MHz, single-spoke@340MHz, and Duo-spoke@340MHz respectively. β1, β2 can be chosen to minimize number of cavities used, which is shown in Figure 2.

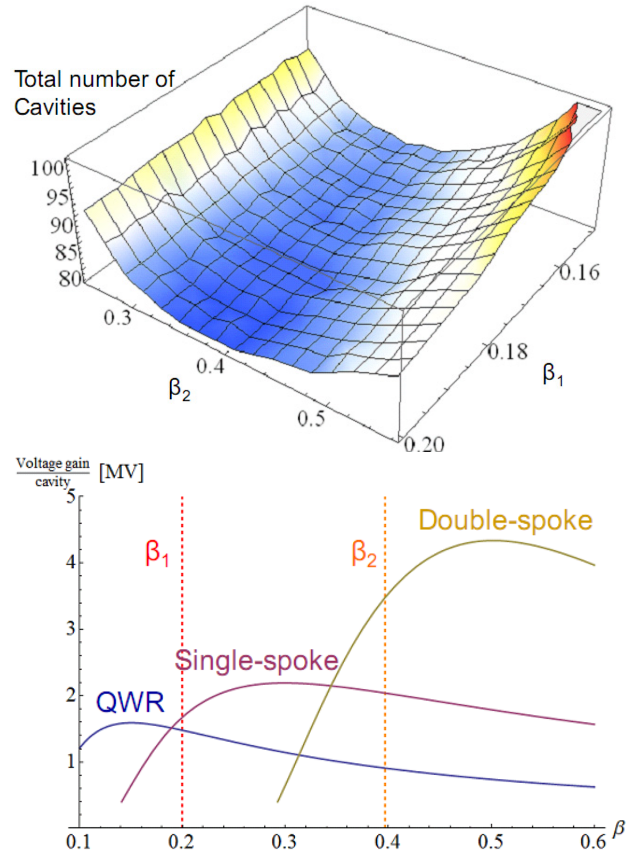

Figure 2: An example of the linac design choice.

EM DESIGN OF MULTI-SPOKE CAVITY

Initial Result of the Duo-spoke Cavity

A Duo-spoke cavity at 352MHz with βmax ~ 0.5 is chosen as our first prototype shape, which is shown in Figure 3. The impacts of the central and base part of spoke as well as vessel radius and iris-to-iris length have been described in [5], and our calculation confirmed these conclusions. One complement is that for multi-spoke structure, the shape of end-cover contributes more to the cavity performance since it affects the field flatness, e.g. the field amplitude of end gap increases when height of end cone increases, which typically increases TTF and R/Q. If the total length of the accelerator is concerned, height of the end cone will contribute even more to the Ep/Ea and Bp/Ea, and the optimized shape is not necessarily with perfectly flat field profile.

The same algorithm for multi-parameter optimization as in [5] is used, within the frame of parameter sweep of CST2011 MWS, in addition with a macro to tune the cavity frequency automatically and process result of each run. Bp/Ep<1.5 mT/(MV/m) and minimum Ep/Va is chosen as design goal. Since the accuracy of surface field is typically 5-10%, which is not satisfying, we did not optimize them too much. The initial results are listed in Table 1, and all the parameters include TTF_{max}; cavity length is 750 mm. Further optimizations with more accurate surface fields may be done by HFSS or Omega3P.

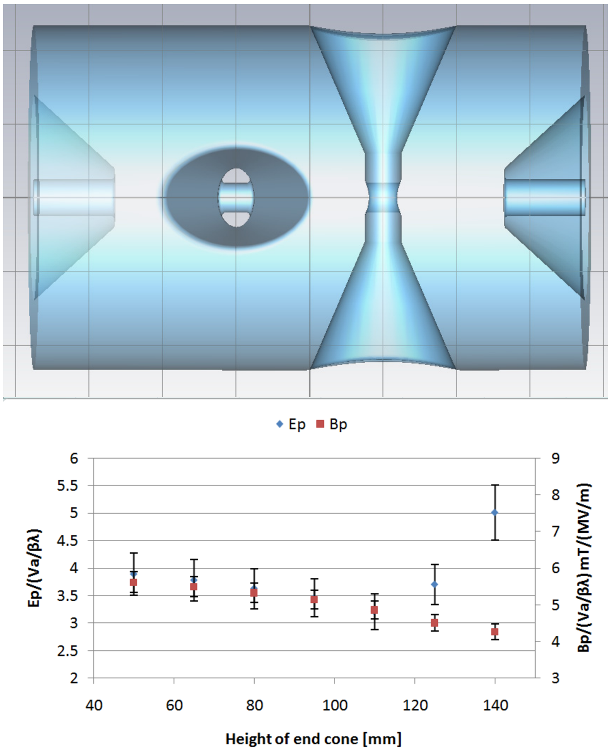

Figure 3: Duo-spoke cavity.

Table 1: RF parameters of Duo-spoke cavity

Parameter	Value
Ep/(Va/Lcavity)	5.6
Bp/(Va/Lcavity)	8.5 [mT/(MV/m)]
G*R/Q	5.86e4 [Ω^2]

EQUIVALENT CIRCUIT MODEL

An equivalent circuit model for Duo-spoke cavity is built. The capacitor between spokes, end covers, and vessel wall, and the inductance of spokes and end covers are modelled, which is shown in Figure 4. By the field flatness and symmetry, we have:

$$\frac{\mathrm{Vmid}}{\mathrm{Vend}} = 2\left(\frac{\omega^2\,\mathrm{Ce\,Le}}{1-\omega^2\,\mathrm{Cwe\,Le)}} - 1\right) = \frac{2\,\omega^2\,\mathrm{Lm\,Ce}}{\omega^2\,\mathrm{Lm}\,(2\,\mathrm{Cm}+\mathrm{Cwm})-1}$$

for π mode and π/3 mode, and

$$\left(\frac{\omega^2\,\mathrm{Lm\,Ce}}{1-\omega^2\,\mathrm{Lm\,Cwm}} + \frac{\omega^2\,\mathrm{Le\,Ce}}{1-\omega^2\,\mathrm{Le\,Cwe}}\right) = 1$$

for 2π/3 mode, resonance frequencies can be derived; and it has been confirmed that electric energy equals to the magnetic energy stored in the loop with the roots solved. Moreover, it is found that a flat π mode exists with Le(2Ce+Cwe)=Lm(2Ce+2Cm+Cwm).

We can also reverse the process: if we have numerically calculated three resonance angular frequency Ω1, Ω2, Ω3, and voltage ratio of middle and end gap for π mode and π/3 mode (set as ra1 and ra3), and the R/Q of any of the three modes (e.g. rq1 for π mode), then all the circuit parameters can be solved. We have done part of the analysis with Mathematica, and we are still working on it.

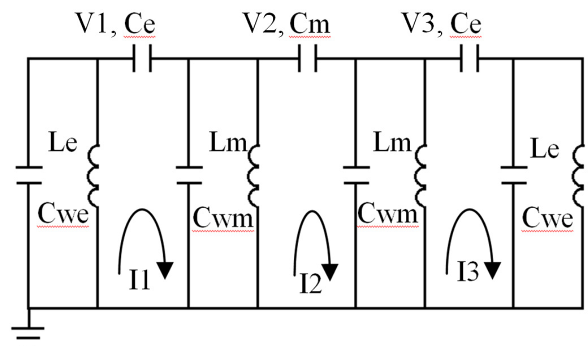

Figure 4: Equivalent circuit model.

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