COMPUTATION OF WAKEFIELDS AND HOM PORT SIGNALS BY MEANS OF REDUCED ORDER MODELS

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Abstract

The investigation of wakefields is an important task in the design and operation of particle accelerators. Computer simulations are a reliable tool to extend the understanding of these effects. This contribution presents an application example of a new method to compute wakefields as well as parameters derived from those fields, such as higher order mode (HOM) port signals. The method is based on a reduced order model of the structure created by as set of 3D eigenmodes, a set of 2D waveguide port modes and the current density of the beam. In contrast to other wakefield computations, the proposed method operates directly on the reduced order model. Therefore, once having established this model, the beam-excited fields can be determined quickly for different beam parameters. As a matter of fact, only a small part of the reduced system has to be recomputed for every sweep point. From these advantages it is obvious, that the method is highly compatible for beam parameter studies. In a proof of principal the effectiveness of the method compared to established methods of wakefield computations in terms of computational time and accuracy is shown.

INTRODUCTION

For the design and operation of particle accelerators, the computation of beam excited fields, also referred to as wakefields, is of crucial importance. Several methods for the computation of wakefields are known and implemented in highly useful software tools like [1]. The investigation of the influence of certain beam parameters on the electromagnetic fields in the structure, as well as parameters derived from those fields, leads to repetitive computations of wakefields in the same structure with varying beam parameters. For real life examples these structures are often very large, which leads to large system matrices and therefore to very long computational times for such parameter studies. Thus, it is useful to apply the coupling scheme proposed in [2], to split the full structure in several smaller segments, describe them by means of the suggested state space equations and concatenate the segments to the full structure. The actual purpose of this paper is the generalization of the method described in [2] such that an ultra-relativistic beam is accounted for in the state space equations. In fact, the bunch of charged particles traversing the structure leads to an additional excitation source.

STATE SPACE MODELS

The straight forward discretization using e.g. the finite integration techniqe (FIT) leads to a large state space system of the structure. This system can be reduced using a set of eigenmodes of the structure. The basic principles of the derivation are taken from [2] and [3].

Creation of the State Space Model

The state space model is derived by a truncated series expansion of the wave equation arising from Maxwell's equations. For the transient electric field $\mathbf{E}(\mathbf{r}, t)$, the following ansatz is chosen:

$$\mathbf{E}(\mathbf{r}, t) \approx \sum_{n=1}^{N} \tilde{\mathbf{E}}_n(\mathbf{r}) x_n(t), \tag{1}$$

with the field distribution $\tilde{\mathbf{E}}_n(\mathbf{r})$ of the n-th 3D eigenmode and a modal, transient weighting factor $x_n(t)$ [2]. The field distributions satisfy the 3D Helmholtz equation on the computational domain $\mathbf{\Omega}$:

$$\Delta\tilde{\mathbf{E}}_\nu(\mathbf{r}) + k_\nu^2 \tilde{\mathbf{E}}_\nu(\mathbf{r}) = 0 \qquad \text{on } \mathbf{\Omega}, \tag{2}$$

and boundary conditions, which state that the waveguide port boundaries are made of perfect magnetic conducting material and the structure wall of perfect electric conducting material. The beam is mathematically described by the charge density of the single bunches:

$$\rho(\mathbf{r}, t) = q \cdot g(\mathbf{r}, t) = q \cdot g_\perp(\mathbf{r}) g_z(\mathbf{r}, t) = \\ q \cdot \delta(x - x_0)\delta(y - y_0) \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(z - ct - z_0)^2}{2\sigma^2}\right), \tag{3}$$

with the shape function of the bunch $g(\mathbf{r}, t)$ and the total charge of the bunch q [4]. The computations presented in the scope of this paper are restricted to single bunch excitations. In most cases it is sufficient to describe the longitudinal shape function $g_\perp(\mathbf{r})$ as Dirac distribution with the center x_0 respectively y_0 and the transversal shape function $g_z(\mathbf{r}, t)$ as Gaussian distribution with the square root of the variance in the direction of propagation, also refered to as the rms bunch length σ and the starting point z_0 [5]. Note that the beam is ultra-relativistic which states that the particles moves with the speed of light without changing its velocity while moving through the structure [4].

In the following derivation a state space model is obtained in terms of an impedance formulation using modal voltages and currents which have certain advantages over the more often used normalized wave amplitudes. The state space system without charges can be described in terms of interaction integrals using the 3D eigenmodes of the structure and the current density at the ports as well as parameters from the eigenmodes as in [2].

When taking charges into account, additionally a similar interaction integral between the 3D eigenmodes $\tilde{\mathbf{E}}_n(\mathbf{r})$ and the bunch current density $\mathbf{J}_{\mathrm{Beam}}(\mathbf{r},t)$ needs to be computed:

$$\iiint_{\Omega} \mathbf{J}_{\mathrm{Beam}}(\mathbf{r},t)\cdot\tilde{\mathbf{E}}_n(\mathbf{r})\,\mathrm{d}\mathbf{r} = \underbrace{\frac{qc}{\sigma\sqrt{2\pi}}\int_0^L \tilde{E}_{n,z}(x_0,y_0,z)\exp\left(-\frac{(z-ct-z_0)^2}{2\sigma^2}\right)\mathrm{d}z,}_{\mathrm{p_{Beam,n}}(t)} \tag{4}$$

with the speed of light in vacuum c and the length of the beam axis L. For the description of the beam a modal incident beam power $\mathrm{p_{Beam,n}}(t)$ is defined. From the interaction integral in equation (4) and the energy stored in the n-th 3D eigenmode W_n, the state space model from [2] is extended in the following manner:

$$\frac{\partial}{\partial t}\mathbf{x}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}_{\mathrm{Port}}\,\mathbf{i}_{\mathrm{Port}}(t) - \frac{1}{2}\underbrace{\begin{pmatrix} 0 \\ \frac{1}{\sqrt{W_1}} \\ \vdots \\ 0 \\ \frac{1}{\sqrt{W_N}} \end{pmatrix}}_{\mathbf{B}_{\mathrm{Beam}}} \underbrace{\begin{pmatrix} p_{\mathrm{Beam},1}(t) \\ p_{\mathrm{Beam},2}(t) \\ \vdots \\ p_{\mathrm{Beam},N}(t) \end{pmatrix}}_{\mathbf{p}_{\mathrm{Beam}}(t)}, \tag{5}$$

$$\mathbf{v}(t) = \mathbf{C}\mathbf{x}(t). \tag{6}$$

For this system of ordinary differential equations a correction term is obtained, in order to account for neglected eigenmodes in the series expansion and by this, improve the accuracy of the system. Therefore, from the full FIT system a number N_{cr} of "correction modes" are computed[1]. The corrected state space model is transformed from an impedance formulation to a scattering formulation by using the port impedances. For given input parameters, current and incident beam power, the ODE system can be solved, since the model is comparably small and can be conveniently computed my means of standard ODE solvers.

Advantages and Disadvantages

With changing beam parameters, such as σ, q, x_0 and y_0, only the excitation term corresponding to the beam $\mathbf{p}_{\mathrm{Beam}}(t)$ has to be recomputed since only this vector depends on these parameters. The incident modal beam power is only a small part of the entire system. While creating and solving the state space system takes longer than the direct evaluation of the port signals with [1], computing solely the incident modal beam power and solving the state space system is faster. This leads to shorter computational time after a certain number of repetitions with different beam parameters.

The size of the model and its accuracy depend highly on the number of 3D eigenmodes which are considered in equation (1). In fact, at least all 3D eigenmodes which have their resonant frequency in the frequency interval of interest have to be employed in the series expansion.

As an additional advantage, investigations in frequency and time domain can be performed with and without beam using the same model. For investigations in frequency domain the tranfer function of the ODE system needs to be computed. Also the full electromagnetic fields in the structure can be computed by means of equation (1), once the transient weighting factors are obtained by solving the ODE system.

APPLICATION EXAMPLE

As application example a simple structure is chosen in form of a cylindrical cavity with an identical beam pipe on every end. To get the field information from the inside, four couplers are clamped on the cavity, as shown in Figure 1.

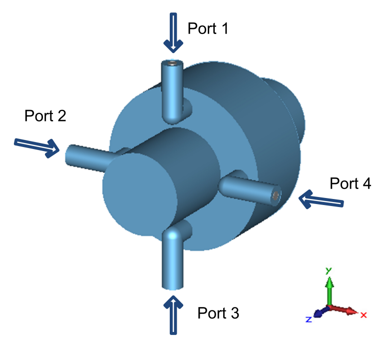

Figure 1: Structure used as application example.

The beam is defined at the center of the beampipes and the cavity. The bunch travels in positive z direction. The presented method is compared to the Wakefield Solver of CST Studio 2012 [1], in both computational time and accuracy. For the computations shown here, a state space model using $N = 30$ 3D eigenmodes and $M = 4$ 2D eigenmodes (since only the TEM port mode at each coupler is considered) is used. The computation of the state space model leads to the equivalent circuit as shown in Figure 2.

Transient Port Signals

For the first basic example a rms-beam length of 35 mm and a charge of 1 nC is chosen. The computed transient port signal at Port 1 is plotted in Figure 3. While the model without correction term (plotted with dashed green line) shows quite poor agreement with the solution computed using CST Microwave Studio (plotted with dashed red line), the solution obtained by the corrected state space model (plotted with blue line) shows a very good agreement with a mean relative error of $7.1101 \cdot 10^{-3}\,W^{-\frac{1}{2}}$.

[1] for further information regarding the correction term see [2]

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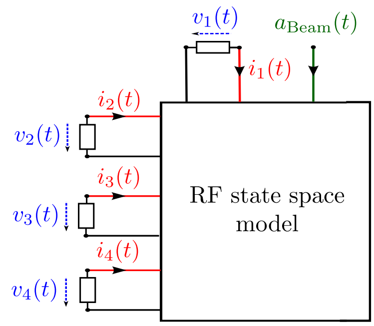

Figure 2: Equivalent circuit with modal voltages (in red), modal currents (in blue) and incident beam power (in green) and the impedances of the couplers as port impedances of 50 Ω.

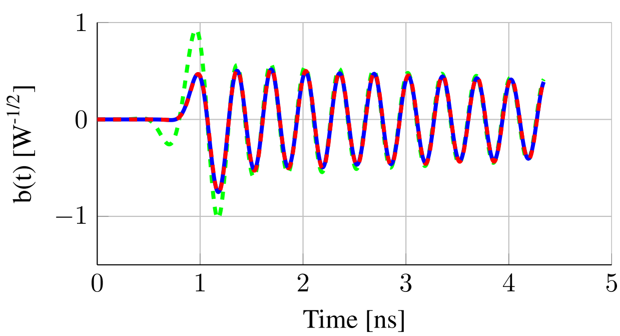

Figure 3: Transient signals at port 1 excited by a beam with $q = 1$ nC and $\sigma = 35$ mm, with no shift from default position.

The method is also verified using different transversal positions x_0 and y_0 of the beam. For the computation shown in Figure 4 a transversal shift of $x_0 = 3.1415\,\mathrm{mm}$ and $y_0 = -14.03\,\mathrm{mm}$ is used, whereas the charge q and the rms beam length σ remain unchanged.

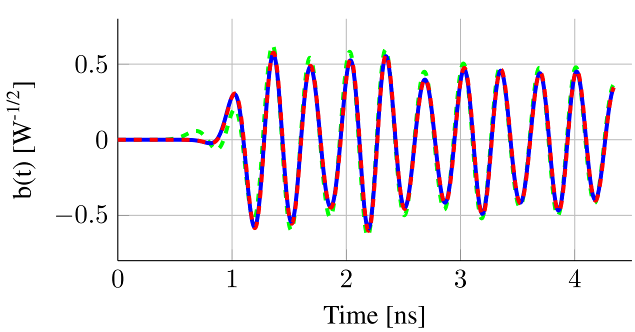

Figure 4: Transient signals at port 3 excited by a beam with $q = 1$ nC and $\sigma = 35$ mm, with shift of $x_0 = 3.1415\,\mathrm{mm}$ and $y_0 = -14.03\,\mathrm{mm}$ from default position.

Here the computation of the state space system without correction shows again a rather poor accuracy, compared to the CST solution. The computation of the corrected state space model shows a very good agreement with a mean relative error of $8.6648 \cdot 10^{-3}\, W^{-\frac{1}{2}}$.

Computational Effort

The computational effort depends highly on the desired accuracy. A very accurate state space model needs a large number of considered 3D eigenmodes which causes the computation to be time demanding. On the other hand too few considered 3D eigenmodes will cause the system to have a bad accuracy. A very important parameter is the spectrum of the beam. For transient investigations of wakefields the number N should be chosen in accordance with the spectrum of the beam. In fact, for shorter bunches more eigenmodes need to be considered for the same accuracy. With a total number of 371,124 mesh-cells, computing the eigenmodes took 22 minutes and 7 seconds using Advanced Krylov Subspace Method and no symmetries of the structure. Obtaining the full state space system from the eigenmodes took 48 minutes and 15 seconds. Recomputation of the incident beam power and solving the ODE system took 68 seconds. The same computation took about 127 seconds with CST Studio. This states that this description becomes useful if more than 70 different transient port signals derived by beam parameters need to be computed. All computations were performed on Intel(R) Xeon(R) CPU E5-1620 0 @ 3.60 GHz with 64 GB RAM running Windows Server 2012.

CONCLUSION

This contribution presents a proof of principle for the presented method. It is usable to achieve shorter computation times for parameter studies having a large number of different beam parameters. In fact, for the actual demonstration example, the described method does not have benefits in terms of computational times. However, it is planned to combine the coupling formalism [2] with the demonstrated methodology to reduce the time for port signal computations of large structures, like complete SRF structures.

REFERENCES

[1] CST GmbH, "CST Microwave Studio 2012," Bad Nauheimer Str. 19, D-64289 Darmstadt, Germany, http://www.cst.com

[2] T. Flisgen, H.-W. Glock and U. van Rienen, "Compact Time-Domain Models of Complex RF Structures Based on the Real Eigenmodes of Segments", IEEE Transactions on Microwave Theory and Techniques, Vol. 61, No. 6, June 2013, pp. 2282 - 2294.

[3] M. Dohlus, H.-W. Glock, D. Hecht and U. van Rienen, "Filling and beam loading in TESLA superstructures", Proc. European Particle Accelerator Conf. Stockholm, 1998, pp. 1248 - 1250.

[4] M. Dohlus, R. Schuhmann and T. Weiland, "Calculation of frequency domain parameters using 3D eigensolutions", International Journal of Numerical Modelling Electronic Networks Devices and Fields 12, 1999, pp. 41 - 68.

[5] T. Weiland and R. Wanzenberg, "Wake fields and impedances", Frontiers of Particle Beams: Intensity Limitations, Springer Berlin Heidelberg, 1992, pp. 39 - 79.