GEOMETRIC OPTICS OF WAKE FIELDS OF VERY SHORT BUNCHES IN SUPERCONDUCTING CAVITIES*

A. Novokhatski#, SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA

Abstract

We study the wake potentials of a very short bunch in a quasi-periodic structure of superconducting cavities. We analyze the pattern of the electric force lines and the shape of a cavity. The behavior of the electric force lines reflects irregularities of the shape structure of a cavity. Simulations were carried for a JLAB 7 cell upgrade cavity with application to future light sources.

INTRODUCTION

Superconducting cavities have been chosen for the RF system in many storage rings. Usually a superconducting cavity consists of one or two cells [1] together with normal conducting HOM dampers, which are placed as close as possible to the cells. The RF voltage of a superconducting cavity is about 1-2 MV. Several superconducting cavities are used to achieve the required total RF voltage for a ring. Cavities with many cells (> 3) were used at high-energy machines like TRISTAN and LEP [2]. Cavities with many cells are usually used in linear accelerators for nuclear physics, FELs or ERL [3]. We propose using these cavities in a storage ring (light source) in order to achieve a very high RF voltage there by strongly compressing the bunch length [4]. If we succeed, we may use the short bunches to drive high-gain soft X-ray FELs using transverse-gradient undulators [5]. The main question is how much current can be put through these cavities.

WAKE FILEDS LINES

A short bunch passing through a cavity excites wake fields, which may stay in the cavity in the form of High Oder Modes (HOMs) or leave the cavity after some time. A train of short bunches may significantly increase the fields inside a cavity, and also the amplitude of some mode, if the bunch spacing frequency is in resonance with this mode. This is very important in a superconducting cavity because the damping time of HOMs is very long. We analyze the structure of longitudinal fields in the upgrade JLAB cavity [6].

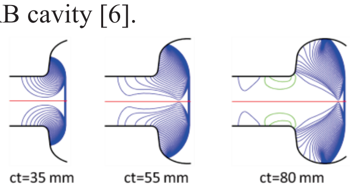

Figure 1: Excitation of an “edge” mode.

For the wake field simulations we use the computer code NOVO [7].

*Work supported by Department of Energy Contract No. DOE-AC03-76SF00515
#novo@slac.stanford.edu

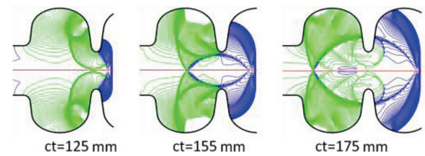

Figure 2: “Edge” excitation at the first iris.

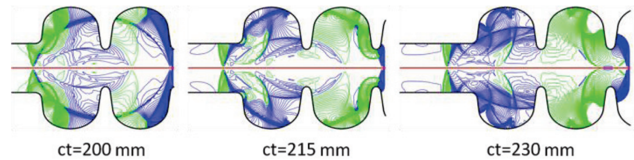

Figure 3: Formation of higher frequency waves.

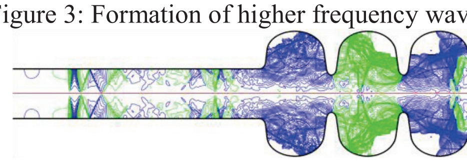

Figure 4: Field leaving a cavity at the time moment equal to 455 mm.

We start with the first cell of a cavity. Fig, 1 shows the distributions of the electric force lines of the wake field excited by a short (0.5 mm) bunch. We choose three different time moments. The first moment corresponds to the position of the bunch equal to one radius of the beam pipe (a=35 mm). At this moment the fields generated at the edge of a cell propagate in a radial direction and reach the axis. The next moment shows how these fields continue to propagate away from axis. The last moment shows how these fields reach the opposite side of the beam pipe and form a wave, which will propagate into the beam pipe. The different colours of the electric force lines correspond to positive (blue) and negative (green) longitudinal directions of the force lines. The blue and green lines in the beam pipe of the last picture of Fig. 1 clearly show one period of the wave. Interference of edge reflected fields produces a wave with a wavelength of approximately two beam pipe radiuses $\lambda=2a$. The correspondent frequency for a beam pipe dimension of a=35 mm f=4.29 GHz. This wave can propagate in the beam pipe because it is shorter than the cut-off wavelength $\lambda_c=1.3\cdot 2a$. The cut-off frequency for a=35 mm, $f_c=3.3$ GHz. We can see this same process when a bunch passes through the first iris, but now the picture is not so clear because of other reflections (Fig. 2)

As the radius of the inner hole of the iris (r=26.5 mm) is smaller than the radius of the beam pipe then the time step is smaller. The time difference between the first and last pictures in Fig. 2 corresponds to 50 mm, almost one wavelength. According to equation (1) the frequency of this mode is 5.66 GHz.

There are also reflected fields, which are propagating in the longitudinal direction. We can see vertical green lines

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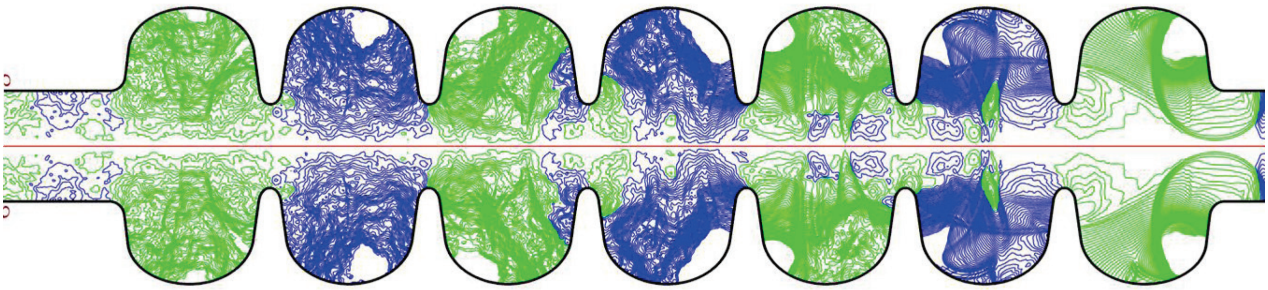

Figure 5: Electric force lines in cells at the time when a bunch (on the write) leaves the JLAB cavity. The blue lines show lines with negative longitudinal components (decelerating forces), the green lines correspond to accelerating forces.

in Fig. 2, which propagate from the right side of the first cell to the left. Interference of these fields with “edge” fields leads to the production of higher frequency waves. Fig. 3 shows the formation of the higher frequency waves. The difference in the radius of the beam pipe and the iris additionally makes the high frequency spectrum wider. It is interesting to see what kind of fields that are leaving the cavity and going into the incoming beam pipe. Fig. 4 shows a time moment equal to 450 mm. At this moment the electron bunch is passing the 5th cell and is not shown in the picture. Qualitatively we can say that the fields, which first enter the incoming beam pipe, have a relatively long wavelength but then following these fields short-wavelength fields exit the cavity. Only trapped stay in the cavity.

The distribution of the electric force lines when the bunch leaves a cavity is shown in Fig. 5. The different color of the lines in neighbouring cells shows that a bunch mainly excites the main, π–mode. Each cell is filled with lines of almost one color and the directions of the lines in the neighbouring cells are opposite. Finally the pattern of the fields coming out of the cavity and following a bunch is shown at Fig. 6. This field consists of mainly two modes, which have waveguide wavelengths of approximately 70 mm and 50 mm.

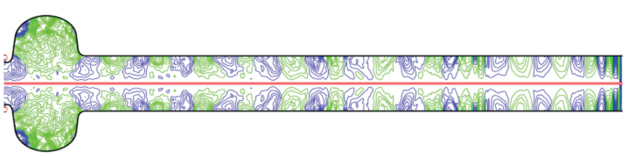

Figure 6: HOMs extraction from a JLAB upgrade cavity: fields following a bunch in a beam pipe.

In a more precise study of the spectrum of the excied fields we calculate a ”long” (20 m) wake field potential of a 2 mm bunch. Half of this potential is shown at Fig.7.

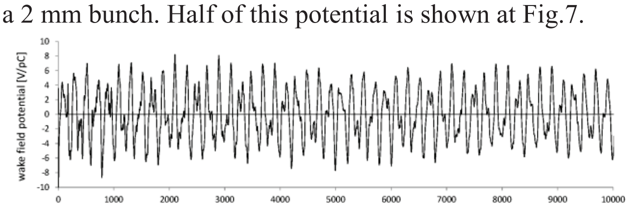

Figure 7: Wake potential of a 2 mm bunch.

The frequency spectrum of the wake field potential (Fig. 8) shows the third longitudinal mode which has a frequency of 4.2 GHz. Also we can see modes around 6 GHz.

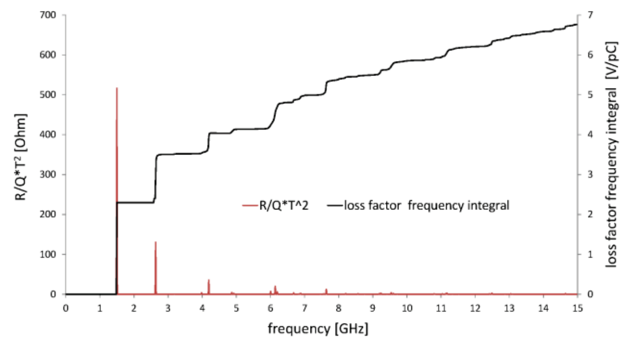

Figure 8: JLAB cavity spectrum and loss frequency integral.

Wake field potentials for a short bunch, which travels along a quasi-periodic structure (a series of cavities and connecting pipes), approach a steady state distribution after some distance.

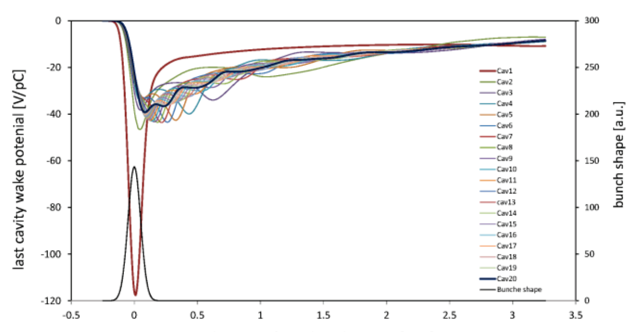

Figure 9: Wake potential of the last cavity of a series of JLAB cavities: from a single cavity (red line) to a series of 20 cavities. The blue line shows the wake potential of the 20th cavity. The distance between cavities is equal to the length of three cells. The bunch length is 50 microns.

We calculate wake potentials of a 50 micron bunch in a series of JLAB cavities. We assume that the length of the connecting pipe or distance between cavities is equal to the length of 3 cells. In this case the period of the quasi-periodic structure is approximately 1 m. We check the wake field potential gained in the last in the series. Fig. 9

shows this wake potential for different number of cavities in a series: from a single cavity to a series of 20 cavities. The wake potential changes very rapidly after the first cavity and then damps down oscillates around a smooth line. The oscillations are come from the complicated structure of the wake fields that are chasing the bunch. Fig. 10 shows the electric force lines of the field in a connecting pipe after a bunch has passed one, two, three and twenty cavities.

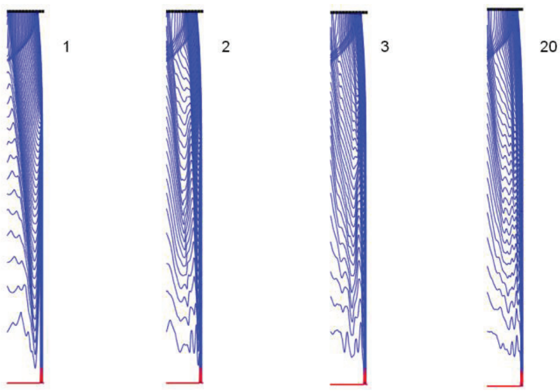

Figure 10: Electric force lines of a bunch field after first, second, third and twentieth cavity.

We also show the bunch energy loss in the last cavity of a train as a function of the number of cavities in a train (Fig.11). We can see saturation after 16 cavities.

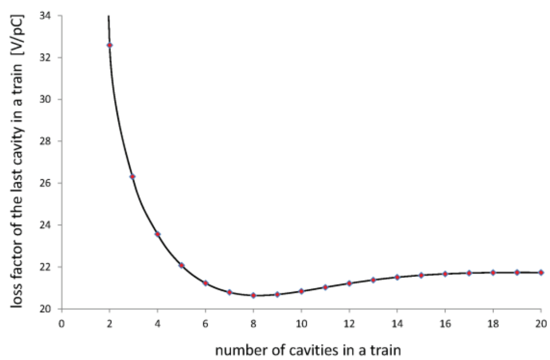

Figure 11: The loss factor of the last cavity in a series.

We may calculate the wake potential per unit length using a Green's function

$$W(s) = \int_{-\infty}^{s} g(s-s')\,q(s')ds' \qquad (3)$$

Following the same approach as in reference [8] we approximate Green's function for the JLAB cavity by a formula

$$g(s) = \frac{Z_0 c}{\pi a^2}\left(1+\alpha\frac{s}{s_0}\right) exp\left(-\sqrt{\frac{s}{s_0}}\right) \qquad (4)$$

The amplitude of the Green's function (a=26.5 mm) is

$$\frac{Z_0 c}{\pi a^2} = 51.2\ V/pC/m$$

Parameter α=-0.1 and s_0=1.4 mm. This approximate Green's function can be used for wake potential and loss factor calculations for bunch lengths down to 4 mm. A comparison plot of a direct calculation from formulas (3)-(4) (solid black line) and the code NOVO (red diamonds) is shown in Fig. 12.

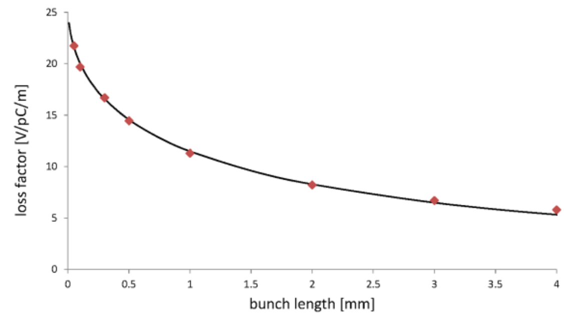

Figure 12: Loss factor of a quasi-periodic accelerating structure of JLAB cavities as a function of bunch length.

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