

# ANALYSIS OF HIGH ORDER MODES IN 1.3 GHz CW SRF ELECTRON LINAC FOR A LIGHT SOURCE

A. Sukhanov\*, A. Vostrikov, V. Yakovlev, Fermilab†, Batavia, IL 60510, USA

## Abstract

Design of a Light Source (LS) based on the continuous wave superconducting RF (CW SRF) electron linac is currently underway. This facility will provide soft coherent X-ray radiation for a broad spectrum of basic research applications. Quality of the X-ray laser radiation is affected by the electron beam parameters such as the stability of the transverse beam position and longitudinal and transverse beam emittances. High order modes (HOMs) excited in the SRF structures by a passing beam may deteriorate the beam quality and affect the beam stability. Deposition of HOM energy in the walls of SRF cavities adds to the heat load of the cryogenic system and leads to the increased cost of building and operation of the linac. In this paper we evaluate effects of HOMs in an LS CW SRF linac based on Tesla-type 9-cell 1.3 GHz cavities. We analyze non-coherent losses and resonance excitation of HOMs. We estimate heat load due to the very high frequency HOMs. We study influence of the HOMs on the transverse beam dynamics.

## INTRODUCTION AND MOTIVATION

A concept of a Next Generation Light Source (NGLS), which provides soft coherent X-ray radiation for a broad spectrum of basic research applications is proposed at LBNL [1]. Another proposal of a similar facility, Liner Collider Light Source (LCLS-II) is advocated by SLAC. The integral part of such a machine is the continuous wave superconducting RF (CW SRF) electron linac feeding sections of free-electron lasers (FELs). The linac design is based on the technology developed for International Liner Collider (ILC) [2]. It utilizes Tesla-type 9-cell 1.3 GHz cavities grouped in cryomodules. The full linac is built of up to 30 cryomodules combined in the injector section and 2 or 3 main sections separated by bunch compressors.

The electron beam with an average current of 0.3 mA is accelerated up to 4 GeV. The bunch length is 30–50  $\mu\text{m}$  and bunch repetition rate is up to 1 MHz. The normalized transverse emittance of the electron beam is 0.6  $\mu\text{m}$  and the transverse position stability is better than 5%.

In order to achieve design parameters of X-ray laser radiation, it is important to preserve parameters of the electron beam while it accelerates through the linac. Longitudinal and transverse beam emittances, stability of the beam transverse position are the most important parameters affecting quality of FEL radiation.

Charged beam bunches interact with the accelerating SRF structures by the radiation of electromagnetic (EM) fields. Radiated EM field can be considered as superposition of excited eigenmodes of SRF cavities. These modes, other than the fundamental cavity mode, are conventionally called high order modes (HOMs). Field of excited HOMs acts back on the beam and may deteriorate quality of the beam. If a very strong HOM is excited it may even affect beam stability and in the worst case break the beam.

Another adverse effect of the excited HOMs is deposition of EM energy in the walls of SRF cavities. The expected heat load in NGLS during normal operation is 120 W per cryomodule at 17 MV/m. The excessive heat load due to HOM excitation leads to increased cost of building and operation of the linac.

Analysis and *control* of the HOMs are important parts of design of SRF linacs. In this paper effects of the HOMs in an LS linac based on Tesla-type 9-cell 1.3 GHz cavities are studied. We estimate incoherent losses and loss factors. Resonance excitation of the monopole HOMs and cryogenic heat load are evaluated. We also estimate losses due to very high frequency HOMs using diffractive model. Excitation of dipole HOMs and their effect on beam stability is studied.

## INCOHERENT LOSSES AND LOSS FACTORS

A bunch of charged particles passing through an RF cavity loses its energy into EM radiation. The radiated field can be characterized by a voltage induced in the cavity, which is proportional to the bunch charge,  $V = 2k_{\text{loss}}q_b$  (see, for example [3]). The radiated EM energy is  $W = k_{\text{loss}}q_b^2$ . The parameter  $k_{\text{loss}}$  is the *loss factor*.

If the lost EM energy dissipates during the period between bunches, or the phase of the EM field in the cavity left by the previous bunches is random w.r.t. to the bunch arrival time, the single bunch losses can be considered as independent of the other beam bunches. This is the case of *incoherent losses*. The average power loss is  $P_{\text{av}} = k_{\text{loss}}q_b I_{\text{av}}$ , where  $I_{\text{av}}$  is the average beam current.

The loss factor depends on the longitudinal bunch size  $\sigma_z$ . This dependence for Tesla-type 9-cell cavity has been calculated by T. Weiland and I. Zagorodnov [4]. The total loss factor for the 50  $\mu\text{m}$  NGLS bunches is 18.3 V/pC. One can estimate the loss factor of the fundamental cavity mode only:  $k_{\text{loss}}^0 = \omega(R/Q)/4 \approx 2$  V/pC. Thus, the major contribution into the total loss factor in NGLS linac is due to the high order modes.

Average incoherent beam power loss in NGLS is 11.5 W per cryomodule. Comparing this number with the expected

\* ais@fnal.gov

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heat load of about 120 W/cryomodule, we conclude that incoherent losses should not be a problem for NGLS linac.

## RESONANCE EXCITATION OF MONOPOLE HOMS

CW bunched beam passing through SRF cavity may coherently excite HOMs with a high loaded quality factor,  $Q_L$ . If HOM is excited close to its resonance frequency, the effect may be significantly higher compared to incoherent losses. Also, in periodic structure of multiple SRF cavities in linac conditions may be realized when HOMs with frequency above beam pipe cut-off frequency [5] are effectively trapped inside cavities [6].

We have developed a model for estimation of resonance excitation of the HOMs and applied this model for the Project X CW SRF linac design [7–10].

The basic features of our model in application to the analysis of HOMs in LS linac are the following. We use a conservative approach and estimate the maximum possible effect. Since LS bunch timing structure is very uniform, no significant effects on longitudinal beam dynamics are expected. Tesla-type 9-cell structure with HOM couplers and absorbers providing  $Q_L < 10^6$  is assumed. We use SuperLANS RF simulation code [11] to calculate cavity spectrum. The worst trapping conditions for the propagating HOMs, when the  $(R/Q)$  values are at their maximum, are found by the variation of the distance between cavities in our model. Random variations of HOM frequencies from cavity to cavity with R.M.S. value  $\sigma_f \sim 1$  MHz are assumed [12]. An idealized beam current spectrum without time and charge jitter is calculated.

### Beam Spectrum

The bunch repetition frequency in NGLS is assumed to be constant and equal to 1 MHz, and the bunch length is  $\delta t = 70$  fs. We suppose that bunches have the uniform charge distribution with the total charge  $q_b = 0.3$  nC.

Calculated beam spectrum is shown in Fig. 1. Spectrum lines are separated by 1 MHz. One can see, that for the frequencies below 1 THz the beam spectrum is quite flat.

### HOM Spectrum

We evaluate HOM spectrum of Tesla-type 9-cell cavity using SuperLANS code. HOMs can be characterized as trapped or propagating by their relation to the beam pipe cut-off frequency. Frequency of the trapped modes is below  $f_{\text{cutoff}}$ , while the propagating modes have frequency above the cut-off frequency.

Propagating modes can form standing waves in the structure of multiple cavities and became effectively trapped. Impedance  $(R/Q)$  of such trapped modes depend on where the standing wave is formed. In order to evaluate  $(R/Q)$  variations with the length of the standing wave, we vary length of the beam pipe between cavities in our simulation.

Spectrum of the monopole HOMs of Tesla-type 9-cell cavity is shown in Fig. 2. We approximate dependence of

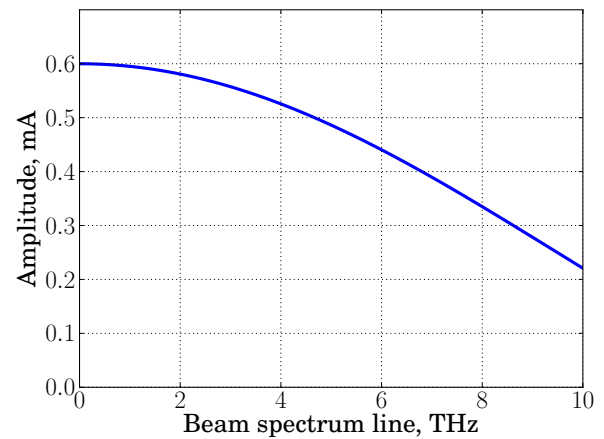


Figure 1: Beam spectrum for NGLS in assumption of average current of 0.3 mA, bunch size of 70 fs and uniform particles distribution within bunches.

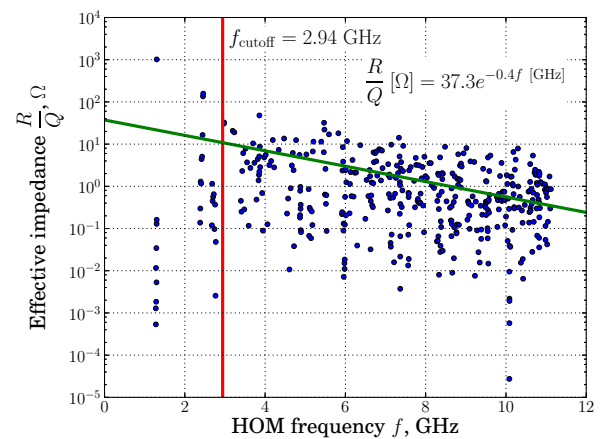


Figure 2: Spectrum of monopole HOMs of Tesla-type 9-cell cavity. Green line shows approximation of data with exponential function. Vertical red line shows location of beam pipe cut-off frequency.

the impedance on the mode frequency by an exponential function  $(R/Q)[\Omega] = 37.7 \exp(-0.4f[\text{GHz}])$ . It is interesting to note, that the density of the monopole modes increases approximately linearly with the frequency, with the slope parameter  $\sim 7$  modes/(GHz)<sup>2</sup>.

### Power Loss Calculation

We simulate 1000 cavities with random variations of the HOM frequency with R.M.S. value  $\sigma_f = 1$  MHz. Distribution of the power loss caused by the excitation of the HOMs with the frequency spread 1 MHz and quality factor  $Q_L = 10^7$  is shown in Fig. 3. The mean power loss is approximately 1 mW per cavity, even when *all* monopole HOMs with the frequency below 11 GHz are considered.

Occasionally, due to random variation of its frequency, a single HOM in one cavity may come close to resonance. In that case power loss may increase up to 100 mW. We esti-

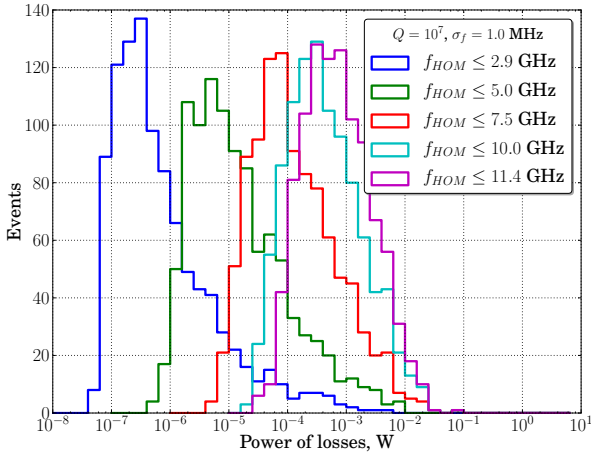


Figure 3: Distribution of power loss due to resonance excitation of HOMs with the mode frequency spread 1 MHz and quality factor  $Q_L = 10^7$ .

mate probability of the high losses caused by a single HOM closest to the resonance. Fig. 4 shows cumulative probability of the HOM power loss for the modes with quality factor  $Q_L = 10^7$ . We evaluate, that for the HOMs with the frequency below 11 GHz and  $Q_L < 10^7$  the probability to have losses above 1 W is less than  $10^{-3}$ .

We conclude, that cryogenic losses due to coherent excitation of monopole HOM are small.

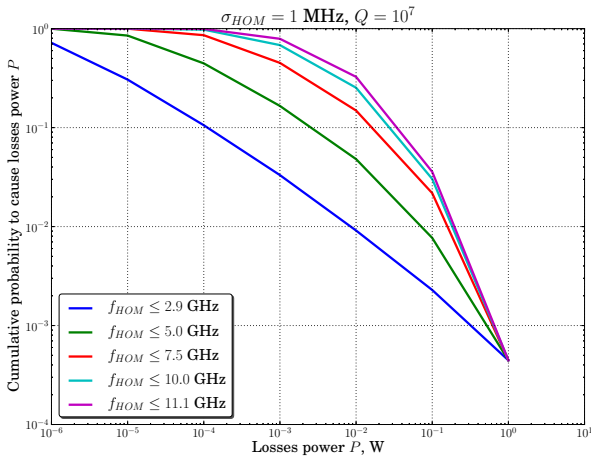


Figure 4: Cumulative probability of HOM power loss for the modes with the mode frequency spread 1 MHz and  $Q_L = 10^7$ .

## VERY HIGH FREQUENCY HOMS

Due to the very short longitudinal bunch size, the LS beam current spectrum extends into THz range. This means, that some sizable fraction of EM energy radiated by bunches is in the frequency region above the energy gap of Cooper pairs in superconducting niobium, which corresponds to 750 GHz at 2 K. Absorption of EM radi-

ation above this frequency in niobium breaks Cooper pairs and increases normal conducting phase, resulting in the increased surface resistance and deterioration of the cavity intrinsic quality factor,  $Q_0$ .

In order to estimate losses into very high frequency modes, we use a diffraction model, following P. Hülsmann *et. al* [13]. According to this model, energy lost in a pill-box cavity of length  $L_{cell}$  by a bunch of longitudinal size  $\sigma_z$  and charge  $q_b$  coming from a beam pipe of radius  $a$  is

$$\Delta E_{1cell} = \frac{q_b^2}{4\pi\epsilon_0 a} \sqrt{\frac{L_{cell}}{2\sigma_z}}, \quad (1)$$

where  $\epsilon_0$  is permittivity of free space. For LS bunches  $q_b = 0.3$  nC,  $\sigma_z = 50$   $\mu$ m, and  $\Delta E_{1cell} \approx 0.7$   $\mu$ J. As bunches travel through consecutive cells in multi-cavity structures (cryomodules) the losses per cell become smaller, as the energy density of the bunch EM field in the cell iris radius decreases due diffraction in previous cells. It has been shown in [13], that after about 200 cells energy lost per 9-cell structure  $\Delta E_{9cell} \approx \Delta E_{1cell}$ .

Average power loss is  $P_{loss} = f_b \Delta E_{9cell} \approx 0.7$  W/cavity (here  $f_b = 1$  MHz is bunch frequency). Only a fraction  $r$  of this losses is radiated in the frequency region above 750 GHz. We can estimate this fraction, assuming the following frequency dependence of the energy density of diffracted field:  $\frac{dE}{d\omega} \sim \frac{e^{-\sigma_z^2 \omega^2 / c^2}}{\sqrt{\omega}}$ . Then

$$r = \int_{\omega_g}^{\infty} \frac{dE}{d\omega} d\omega \Big/ \int_0^{\infty} \frac{dE}{d\omega} d\omega \approx 0.2, \quad (2)$$

where  $\omega_g$  is circular frequency corresponding to 750 GHz. Thus, we expect that the average power loss above the energy gap of Cooper pairs in niobium is less than 0.2 W/cavity for LS beam parameters.

We conclude that power loss into very high frequency HOM should not be a problem in LS CW linac.

## DIPOLE HOMS AND CUMULATIVE EFFECTS

A bunch of charged particles passing through a cavity off the cavity axis excites dipole modes, which interact with the following bunches. This interaction can be expressed in terms of additional transverse momentum of bunches, or *transverse kick*. If excitation of dipole modes is strong it may result in the effective emittance dilution over the time or even beam break-up (BBU) effect.

In this section we evaluate dipole mode excitation in LS linac, simulate transverse motion of bunches and analyze dilution of effective transverse emittance.

### Dipole HOM Spectrum

We calculate dipole HOM frequencies [14] and the effective impedances [15]  $R^{(1)}/Q$  of Tesla-type 9-cell cavity using SuperLANS code. Fig. 5 shows spectrum of the dipole modes. The impedance dependence on the mode frequency can be approximated by an exponential function

$R^{(1)}/Q$  [k $\Omega$ /m<sup>2</sup>] = 23 exp(-0.27f[GHz]). The dipole mode density increases approximately linearly with the slope parameter  $\sim 13$  modes/(GHz)<sup>2</sup>.

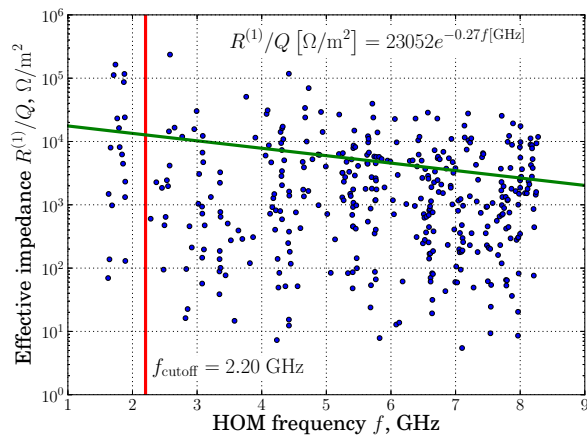


Figure 5: Spectrum of dipole HOMs in Tesla-type 9-cell cavity. Green line shows approximation of data with exponential function. Vertical red line shows location of beam pipe cut-off frequency.

Similarly to the monopole HOMs, the propagating dipole modes may form standing waves in the structure of multiple cavities. In this case propagating HOMs produce the same effect as the trapped ones. That is why both propagating and trapped HOMs should be considered in the analysis.  $R^{(1)}/Q$  of propagating modes depends on where the standing wave formed. We evaluate this dependence by simulation of cavities with varying beam pipe length. We find that only one dipole mode at a time can be effectively trapped. We can safely use 275 k $\Omega$ /m<sup>2</sup> as the upper limit for the impedance of the trapped dipole mode. The impedance of other dipole modes is much smaller and their effect on transverse motion of beam can be neglected.

Separation between frequencies of dipole HOMs is significant compare to the distance between two adjacent lines in the beam spectrum. Thus each line of the beam spectrum can interact with only one dipole HOM.

### Simulation of Transverse Beam Motion

We assume that transverse and longitudinal motions are uncoupled and monopole HOMs are not simulated. LS beam is represented by a continuous train of electron bunches separated by 1  $\mu$ s interval. The bunch charge is 0.3 nC.

We setup simulation of the transverse beam motion as the following. The bunches are accelerated from 350 MeV to 2 GeV. Cavities are grouped into cryomodules with 8 cavities each. Each cavity has accelerating voltage of 20 MV and synchronous phase of  $-12^\circ$ . The cavity length is 1.2 m. The distance between cavities is 0.2 m. The cavities axes are randomly misaligned with standard deviation of 0.5 mm. There is a single quadrupole between cryomodules. Focusing and defocussing quadrupoles follow one af-

ter another. The quadrupole length is 65 cm, magnetic field gradient is 1.5 T/m. The distance between a quadrupole and the first cavity in the following CM is 40 cm. The distance between a quadrupole and the last cavity in the previous CM is 1 m. We consider the HOM frequency spread to be in the range from 1 Hz to 10 MHz. HOM effective impedance is 275 k $\Omega$ /m<sup>2</sup>. HOM quality factor is  $10^5$ .

### Results

In the simulations all bunches enter the linac with identical energy and position. HOMs are not excited before the first bunch passes through the linac. Final position of the center of each bunch is recorded. This information allows to calculate effective transverse emittance dilution. Calculated value is compared to the initial transverse emittance of 0.6 mm-mrad.

HOM frequencies have normal distribution. A range of the frequency spread values is considered. For each case 500 runs of simulations are made. Different positions of HOM frequency with respect to beam spectrum line are considered.

The results for the resonance case (when HOM frequency equals beam spectrum line frequency) are shown in Fig. 6a. There is no emittance dilution if there is no HOM frequency spread, because in this case each bunch passing each cavity when electric field of HOM is zero. Frequency spread introduce emittance dilution. It has maximum between 10 and 100 kHz, which corresponds to the line width:  $\frac{f}{Q} = \frac{2000 \text{ MHz}}{10^5} = 20 \text{ kHz}$ . The maximum value is smaller than 1% of initial emittance.

The results for the case when the HOM frequency is between two beam spectrum lines are shown in Fig. 6b. If the HOM frequency spread is small compare to the distance between the beam spectrum lines, the emittance dilution is very small. The greater the frequency spread, the more probability for HOM to be close to the resonance. The emittance dilution increases with HOM frequency spread, but it is still smaller than 1% of initial emittance.

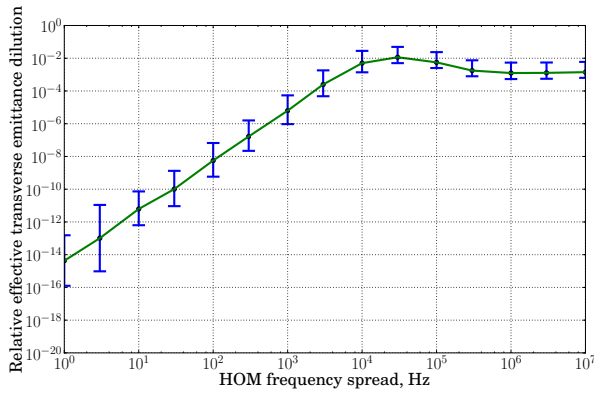
We conclude, that resonance excitation of dipole HOMs seems not to an issue for the LS transverse beam dynamics.

## SUMMARY AND CONCLUSION

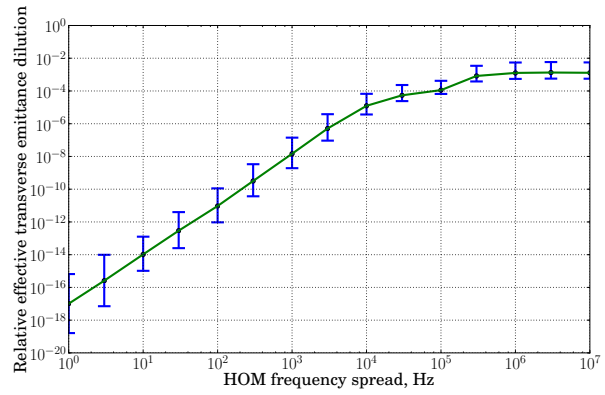
We study high order modes in LS CW electron linac. Analysis of incoherent losses is performed. With the bunch charge 0.3 nC and beam size 50  $\mu$ m incoherent losses are less than 2 W per cavity. Resonance excitation of longitudinal (monopole) HOMs is considered. We estimate that the probability of the power loss above 1 W due to excitation of HOMs with frequency up to 11 GHz is  $10^{-3}$ . Effect of very high frequency HOM excitation analyzed using diffractive model. We find that the mean power loss in very high frequency HOMs is less than 0.2 W/cavity. We study resonance excitation of transverse (dipole) modes and their effect on the transverse beam dynamics. Relative change in transverse emittance is found to be less than 1%.

We conclude, that HOMs should not be a problem in LS CW SRF linac with Tesla-type 9-cell 1.3 GHz cavities.





(a) Mean HOM frequency is 2000.0 MHz.



(b) Mean HOM frequency is 2000.5 MHz.

Figure 6: Relative effective transverse emittance dilution due to dipole HOM as a function of HOM frequency spread for the resonance case and the farthest from the resonance case.

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[4] T. Weiland and I. Zagorodnov, *The Short-range Transverse Wake Function for Tesla Accelerating Structure*, TESLA Report 2003-19, 2003.

[5] For longitudinal monopole modes the cut-off frequency in a cylindrical pipe is defined as  $f_{\text{cutoff}} = 2\pi c \frac{\chi_{01}}{r} \approx 2\pi c \frac{2.4048}{r}$ , where  $\chi_{01}$  is the first root of  $J_0(r)$  — the Bessel function of the first kind of order 0. For Tesla-type 9-cell 1.3 GHz cavity  $f_{\text{cutoff}} = 2.94$  GHz.

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[10] A. Sukhanov, *et al.*, *High Order Modes in Project-X Linac*, HOMCS12, Daresbury, UK, June 25–27, 2012, NIMA (in press) <http://www.sciencedirect.com/science/article/pii/S0168900213010796>

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[12] At least two models of HOM frequency spread based on measurements of HOM frequencies in production cavities exist. One is from Cornell,  $\sigma_f \approx 10.9 \times 10^{-4}(f_{\text{HOM}} -$

$f_0)$ . Another model is from SNS,  $\sigma_f \approx (9.6\text{--}13.4) \times 10^{-4}(f_{\text{HOM}} - f_0)$ . We collect data on HOM frequency spread in Tesla cavities at Fermilab and will build our own model soon.

[13] P. Hülsmann *et al.*, *The Influence of Wakefields on Superconducting Tesla Cavities in FEL-operation*, Workshop on RF Superconductivity, Abano Terme, Italy, 1997.

[14] Note that for the transverse dipole HOMs the cut-off frequency of the beam pipe is defined as the following:

$$f_{\text{cutoff}} = 2\pi c \frac{\chi_{11}}{r} \approx 2\pi c \frac{1.8412}{r},$$

where  $\chi_{11}$  is the first root of  $J_1(r)$  — the Bessel function of the first kind of order 1.

[15] We use the following definition of the dipole mode impedance:

$$\frac{R^{(1)}}{Q} = \frac{\left| \int (\nabla_{\perp} E_z) |_{x=x_0} e^{i\omega z/v} dz \right|^2}{W\omega},$$

where  $\omega$  is the mode circular frequency,  $W$  is stored energy,  $v$  is beam velocity. The integral is taken along the line parallel to the cavity axis at the distance  $x_0$ .