MAGNETIC DEPENDENCE OF THE ENERGY GAP: A GOOD MODEL TO FIT Q SLOPE OF LOW BETA CAVITIES

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Abstract

The reasons why the intrinsic quality factor (noted Qo) of a superconducting cavity drops with the accelerating field (noted Eacc) are still not well understood. In an effort to explain this phenomenon, mainly for high beta cavities, many models have been developed in the community but few of them could fit experimental data whatever the material treatment or surface conditioning.

In the specific case of low beta cavities made of bulk Niobium (i.e Spiral 2 Quarter Wave Resonator), a model based on a magnetic field dependence of the energy gap has been developed to fit experimental data. The evolutions of the model input parameters depending on the cavity treatment or test conditions are consistent with the changes described in the literature. The model will be described step by step and specific examples will be given.

INTRODUCTION

The aim of this paper is not to explain what is at the origin of the medium or high field Q-slope (MFQS and HFQS), but is to give some feedback on some observations and studies made during the tests of some low beta (low frequency) cavities like Spiral2 β=0.12 Quarter-wave resonator (QWR) [1].

The behaviour of low-frequency cavities (in opposition with high-beta elliptical cavities) is relatively different regarding MFQS and HFQS and thus deserves to be pointed out as, above all, these are very less studied. Tackling the problem from 2 different sides (high frequency side and low frequency side) could bring some more light and help to better understand the origin(s) of the Q-slope.

This paper will first introduce the model used to fit experimental data of QWR and will highlight a point that is often forgotten when we start considering a magnetic field dependence of the surface resistance. The paper will then present different example of fits done.

THE MODEL

After describing the correction done on the relation between the quality factor and the surface resistance, the model will be developed.

Formula Corrections

As already mentioned in [2], applying a model with a magnetic field dependence on the well known formula (1) expressing the quality factor (Qo) versus the surface resistance (Rs) leads to significant errors as a strong assumption is made to factorize Rs term out of the integral (See Figure 2).

$$Q_0 = \frac{G}{R_s} \qquad G = 2\cdot\pi\cdot F\cdot\frac{\iiint_V \frac{1}{2\cdot\mu_0}\cdot B^2\cdot dV}{\iint_S \frac{1}{2}\cdot\left(\frac{B}{\mu_0}\right)^2\cdot dS} \tag{1}$$

With G the geometric factor derived from RF simulations and F the frequency of the cavity.

What is suggested here is not a corrected analytical expression but a solution requiring discretization work (2). The strategy is to divide the cavity surface (S) into several zones where we can assume that the magnetic field is constant.

$$\iint_S Rs(B)\cdot B^2 \approx \sum_i Rs_i(B_i)\cdot B_i^2\cdot X_i\cdot S \tag{2}$$

$$X_i = \frac{S_i}{S} = \frac{N\cdot S_{element}}{S}$$

$S_{element}$ corresponds to the area of an element, S_i and N respectively the area and the number of element where the magnetic field is between B_i and B_{i+1}.

The 3D simulation code HFSS* has been used to generate the data file giving the surface magnetic field on each surface element. One has to make sure that all elements have approximately the same size. A routine has been written to build the histogram (Figure 1) of N versus B_i and to filter some singular elements where the field was excessively high due to computation error.

The following formula gives a good approximation of the quality factor versus magnetic field whatever the model used for the surface resistance.

$$Q_0 = G\cdot\frac{\sum_i B_i^2\cdot X_i}{\sum_i Rs_i(B_i)\cdot B_i^2\cdot X_i} \tag{3}$$

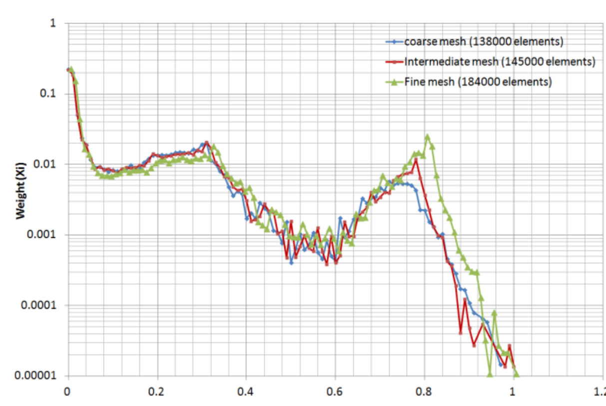

Figure 1: Histogram of the weights ($X_i=S_i/S$) versus B_i for a Spiral2 QWR β=0.12.

One can appreciate, on figure 2, the error made in the model if no correction is implemented in formula 1. As the field dependence of the model is very important, the

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error increases a lot and reaches 100% at medium field (50 mT) and about 600% at high field (90 mT).

A convergence study has been done. Defining more than 100 zones is not necessary (See Figure 2).

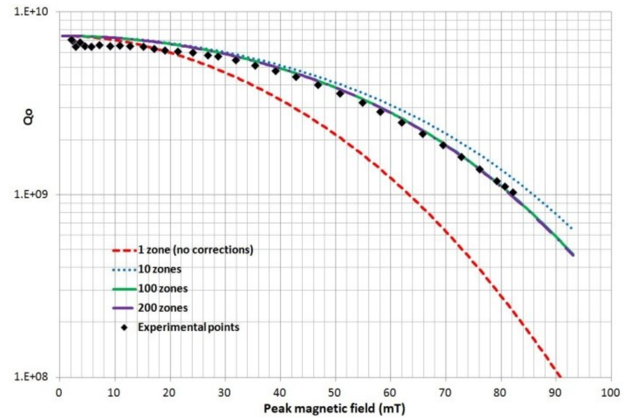

Figure 2: Error made on the evaluation of the Qo versus the peak magnetic field depending on the number of zones defined in the model.

RF Surface Resistance Model

By definition, the RF surface resistance of a superconductor is the sum of 2 terms:

- R_{BCS}, the resistance coming from the BCS theory has been derived by Halbritter [3] and can be simplified as:

$$R_{BCS} = \frac{A(\lambda,\xi,l,...)\cdot F^2}{T}\cdot \exp\left(\frac{-\Delta(T=0)}{k_B\cdot T}\right) \tag{4}$$

With A a constant depending of many parameters of the superconductor like London penetration depth, coherence length, mean free path,..., T the temperature of the superconductor, $\Delta(0)$ the energy gap at T=0K and K_B the Boltzmann constant.

What is proposed here is to add a dependence of the energy gap with the RF surface magnetic field. This approach has already been considered several time [4] but has been put aside because this dependence has only been proved for thin films of a type I superconductor subject to a DC magnetic field.

As defined in the Ginsburg-Landau theory, the thermodynamic critical field Bc can be written:

$$\frac{Fn - Fs(B=0)}{V} = \frac{Bc(T)^2}{2\cdot\mu_0} \tag{5}$$

Where Fs(0) and Fn are respectively the free energies in the superconducting state with no field and the normal state and V the volume of the superconductor. When the magnetic energy zeroes the difference of free energy between the normal and the superconducting state, the latter is not stable anymore and the transition occurs. If we consider this formula true whatever the field, we can write, using equation (5):

$$Fn - Fs(H) = \left(Fn - Fs(H=0)\right)\cdot\left(1-\left(\frac{B}{Bc(T)}\right)^2\right) \tag{6}$$

As the energy gap is directly linked to the difference of free energies [3], we add the following correction to the expression of the energy gap:

$$\Delta(T,B) = \Delta(T,B=0)\cdot\left(1-\left(\frac{B}{Bc(T)}\right)^2\right) \tag{7}$$

This formula will be inserted in equation (4) to model the Q-slope.

- R_{res}, the residual resistance is the additional resistance due to the imperfections, impurities and trapped vortices in the superconductor. Contrary to the BCS resistance which vanishes when the temperature approaches zero, the residual one doesn't depend on the temperature. The model we use, is nevertheless introducing a indirect temperature dependence through the critical field Bc. It can be expressed by:

$$R_{res} = R_{res0}\cdot\left(1+\gamma\cdot\left(\frac{B}{Bc(T)}\right)^2\right) \tag{8}$$

With Rres0 the residual resistance at B=0 and γ the fitting factor.

The γ factor has been defined many times to model the Q-slope of elliptical cavities [5] or low beta cavities [2]. This model is, in general, mainly applied on the BCS resistance. We will consider it part of the residual resistance as we will see that its contribution is negligible for "good cavities" (See Figures 3, 4 and 5).

Taking into consideration equations (3), (4) and (8), we can fit Q curves with the following formula:

$$\begin{aligned} R_S &= R_{BCS}(T,B) + R_{res}(T,B) \\ &= \frac{A(\lambda,\xi,l,...)\cdot\omega^2}{T}\cdot\exp\left(\frac{-\Delta(T=0,B=0)}{k_B\cdot T}\cdot\left(1-\left(\beta\cdot\frac{B}{Bc(T)}\right)^2\right)\right) \\ &+ R_{res0}\cdot\left(1+\left(\gamma\cdot\frac{B}{Bc(T)}\right)^2\right) \end{aligned} \tag{9}$$

With R_{res0}, β and γ the three free fitting factors. β and γ can only be positive.

THE RESULTS

The model has been tested and compared to the results obtained on a Spiral2 QWR (MB09) made of bulk RRR=250 Niobium and resonating at 88 MHz. This cavity had very good performances and had no field emission. The cavity has been prepared following the standard procedure. The cavity has been etched (BCP) of at least 150 microns and high pressure rinsed. A first test has been done. The cavity has been then baked at 120°C during 48h. Several tests have been then performed at 4.24K, 2.44K and 2.1K.

Comparison at Different Temperature

A good way to evaluate a model is to test the cavity at different temperatures. As no interventions on the cavity are required between each temperature step (meaning no venting, no temperature cycling, no changes on the RF surface), only the BCS resistance (See equation (4)) and the critical field (See equation 10) are changing.

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Bc is given by the following formula [3]:

$$Bc(T) = Bc(0) \cdot \left(1 - \left(\frac{T}{Tc} \right)^2 \right) \tag{10}$$

We can see on the next three figures that none of the three free fitting parameters (R_{res0}, β and γ) defined in the model need to be changed to perfectly adjust the fit.

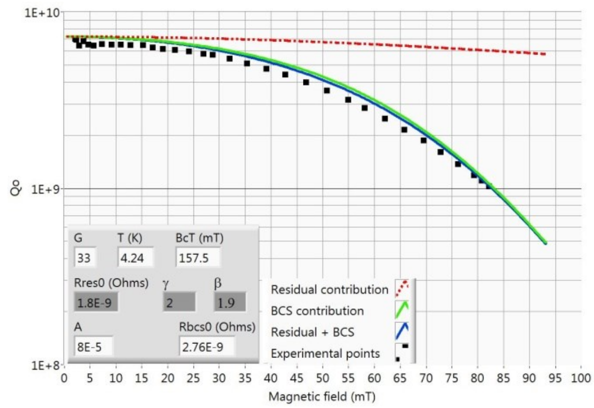

Figure 3: Comparison of the model and experimental data of Spiral2 MB09 QWR tested at 4.24K after baking. The geometrical factor G=33, Bc=157.5 mT and R_{BCS0}=2.76nΩ are given. Rres0, β and γ have been respectively set to 1.8 nΩ, 2 and 1.9 to fit the data.

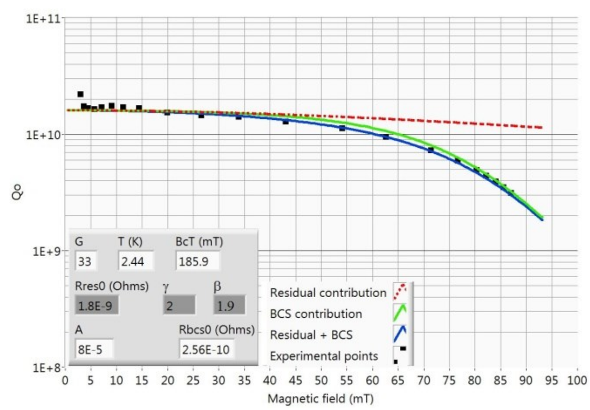

Figure 4: Model and experimental data of Spiral2 MB09 QWR tested at 2.44 K after baking.

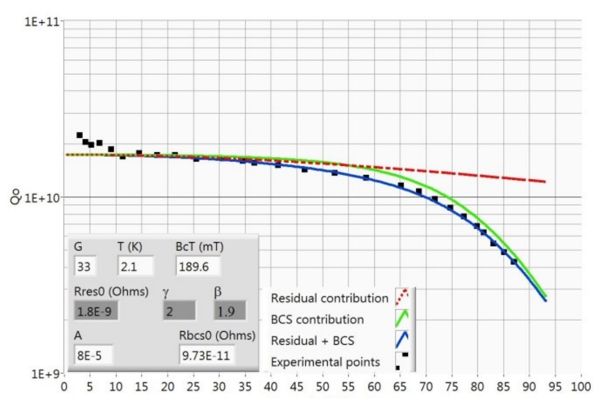

Figure 5: Model and experimental data of Spiral2 MB09 QWR tested at 2.1 K after baking.

This cavity has been tested before baking (figure 6), showing that only the γ parameter is changing significantly. R_{res0} is slightly decreased and R_{BCS0} increases of about 33%. This observation is in accordance with [4].

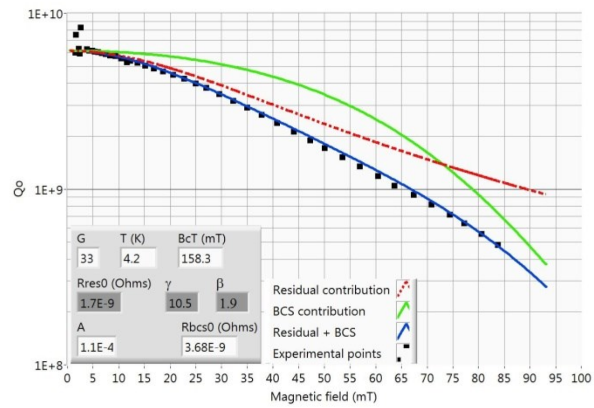

Figure 6: Comparison of the model and experimental data of Spiral2 MB09 QWR tested at 4.24K before baking.

CONCLUSION

The model developed here is fitting very precisely experimental data obtained with Spiral2 QWR. Other experiments will be done in order to fit data of cavities with Q-disease and to test an unbaked cavity at different temperature. This will confirm whether or not the quadratic correction applies on the BCS or residual resistance as the latter would be predominant at 2.1K.

This study will also be extended to other low-beta structures and high-beta cavities like Spoke and elliptical cavities.

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