THEORETICAL FIELD LIMIT AND CAVITY SURFACE CONDITIONS: NANO-SCALE TOPOGRAPHY AND SUB-MILLIMETER PIT*

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Abstract

The recent two theoretical papers [1] and [2] are briefly introduced. The former [1] addresses the superheating field (B_s) suppression due to nano-defects distributing almost continuously on the cavity surface. We introduce a model of the nano-defect. An analytical formula for B_s suppression factor is derived in the framework of the London theory. By using the formula, suppression factors of bulk or multilayer superconductors and those after various surface processing technologies can be evaluated. The latter [2] addresses the magnetic field enhancement (MFE) at the sub-millimeter pit on the surface of cavity, which is thought to cause quench. There exists the famous well-type pit model, but many of pits are not well-type but have gentle slopes. Impacts of the slope angle on MFE have not been well understood. We introduce a model that can describe a pit with an arbitrary slope angle. A formula to evaluate the MFE factor is derived. A pit with a gentle slope angle yields a much smaller MFE factor than the well-type pit. The formula can be applied to the calculation of MFE factors of real pits with arbitrary slope angles.

INTRODUCTION

The fundamental limit of the peak magnetic-field $B_{\rm pk}$ of the superconducting (SC) radio-frequency (RF) cavity is thought to be imposed by the superheating field, B_s, at which the Bean-Livingston barrier for penetration of vortices disappears. According to studies on surface topographies of SCRF materials, nano-scale defects almost continuously exists and distribute with a much higher density than micrometer or sub-millimeter scale defects. B_s is reduced at each nano-defect, and thus the limit of $B_{\rm pk}$ of a real cavity would be imposed not by B_s but by an effective superheating field $\widetilde{B_s} = \eta B_s$, where η is a suppression factor that contains effects of nano-defects. Ref. [1] studies the field limit of SCRF cavity made of a type II SC with a large GL parameter, taking effects of nano-defects into account. An analytical formula for η is derived in the framework of the London theory. Its results can be applied not only to a bulk SC but also to a multilayer SC [4]. The half of the present contribution is devoted to introducing results of Ref. [1].

Actually, rather macroscopic defects, such as pits associated with cavity fabrication processes, which cause the thermal magnetic breakdown, is (will be) the major obstacle to achieving a high $B_{\rm pk}$ by the present Nb cavity (future cavities made from alternative materials). The magnetic-field enhancement (MFE) effect is the key to understand the thermal magnetic breakdown at a pit. The surface magnetic-field in the vicinity of a pit is generally written as $H(\mathbf{r}) = \beta(\mathbf{r}) H_0$ with $0 \le \beta(\mathbf{r}) \le \beta_{\rm M}$, where $\mathbf{r}$ is a position, H_0 is the surface magnetic-field far from the pit, and $\beta(\mathbf{r})$ is a coefficient introduced to reflect an effect of a pit geometry. $\beta(\mathbf{r})$ reaches its maximum value, $\beta_{\rm M}$, at an edge of a pit. If the enhanced field, $\beta_{\rm M} H_0$, is large enough, the edge becomes normal conducting due to thermal and magnetic effects, which triggers a thermal runaway. A difference of edge shape affects the $\beta_{\rm M}$-factor and thus the breakdown field. To reveal the relation between the $\beta_{\rm M}$-factor and the geometry of pit is the first step to understand the quench due to a pit. While many of pits found on cavity surfaces are not well-type but have gentle slopes, pits with general slope angles had not been studied. Impacts of the slope angle on the $\beta_{\rm M}$-factor had not been well understood. Ref. [2] studies the model of pit that can describe a pit with arbitrary slope angle, edge radius, and pit width. A formula to evaluate the MFE factor is derived. A pit with a gentle slope angle yields a much smaller MFE factor than the well-type pit model. Application of the formula to evaluations of MFE factors of real pits is also shown. In the last half of this contribution, the results of Ref. [2] are introduced.

NANO-DEFECTS AND SUPERHEATING FIELD SUPPRESSIONS

In this section, results of Reference [1] are briefly introduced.

A nano-defect is modeled by a groove with a depth smaller than the penetration depth (see Fig. 1). The surface of SC is parallel to the xz plane. The groove and the applied magnetic-field are parallel to the z-axis. The geometry of the groove is specified by the depth, δ, and the angle, $\pi\alpha$ $(1 < \alpha < 2)$. The slope angle is then given by $\theta = \pi(\alpha - 1)/2$. The material is a type II SC with a large GL parameter, and its coherence length and penetration depth are given by ξ and λ $(\gg \xi)$, respectively. Furthermore, the assumption $\xi \ll \delta$ is necessary for treating the model in the framework of the London theory.

The detailed derivation process of the superheating field suppression factor due to the nano defect is found in

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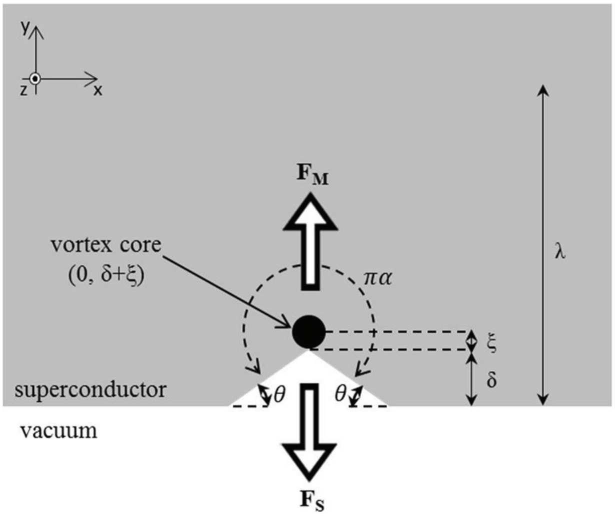

Figure 1: Triangular groove with a depth smaller than the penetration depth as a model of nano-defect [1]. [Originally published in Progress of Theoretical and Experimental Physics **2015**, 063G01 (2015). Published by Oxford University Press.]

Ref. [1]. The result is

$$\widetilde{B}_s = \eta \, B_s \,, \tag{1}$$

$$\eta = \frac{1}{\alpha}\left(\frac{\Gamma(\frac{\alpha}{2})\Gamma(\frac{3-\alpha}{2})\alpha \sin\frac{\pi(\alpha-1)}{2}}{\sqrt{\pi}}\frac{\xi}{\delta}\right)^{\frac{\alpha-1}{\alpha}}, \tag{2}$$

where B_s is the superheating field of the semi-infinite SC with the ideal flat surface, η is the suppression factor, which depends on a nano-defect geometry, and $\widetilde{B}_s$ is the suppressed superheating field. The corresponding formula for the $\mathcal{S}$ layer of the multilayer coating can also be written in the same form as the above,

$$\widetilde{B}_s^{(\mathcal{S}\text{ layer})} = \eta B_s^{(\mathcal{S}\text{ layer})} \,, \tag{3}$$

where η is given by Eq. (2), and $B_s^{(\mathcal{S}\text{ layer})}$ equals the effectively enhanced superheating field of the $\mathcal{S}$ layer of the multilayer SC given by $B_v^{(S)}$ in Ref. [3]. Detailed explanation on an application to the multilayer coating is presented in this conference [4], where we see that the evaluation of η of the $\mathcal{S}$ layer of multilayer coating is necessary to optimize the thickness of the $\mathcal{S}$ layer [4].

Figure 2 shows a contour plot of η. As the depth δ and the angle α increase, η decreases. Combining Eq. (2) or Fig. 2 with a surface topographic study, a suppression factor of material can be evaluated. In Ref. [1], the formula of η was applied to the dirty Nb processed by EP as an example. In much the same way as for EPed dirty Nb, suppression factors of surfaces of other bulk SC and multilayer coating, and those after various surface processing technologies can also be evaluated.

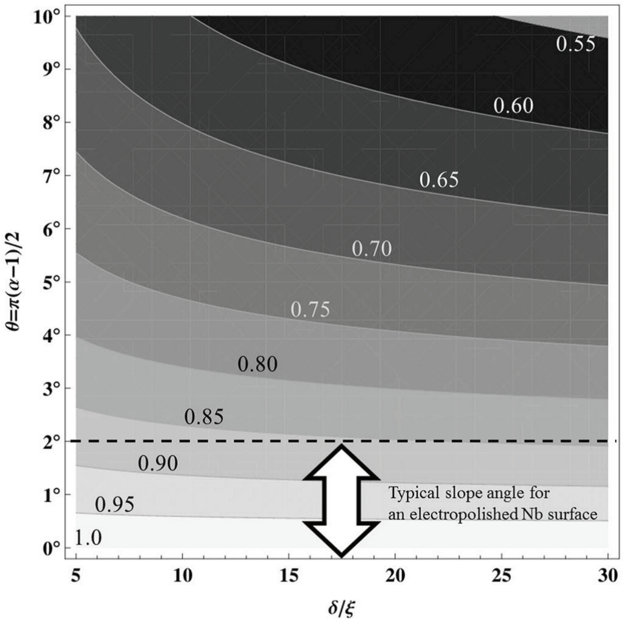

Figure 2: Contour plots of η [1]. The abscissa represents the depth δ in a unit of ξ, and the ordinate represents the slope angle $\theta = \pi(\alpha - 1)/2$. The region below the horizontal dashed line corresponds to typical slope-angles of the surface of electropolished Nb. [Originally published in Progress of Theoretical and Experimental Physics **2015**, 063G01 (2015). Published by Oxford University Press.]

MAGNETIC FIELD ENHANCEMENT AT A PIT

In this section, results of Reference [2] are briefly introduced.

The magnetic field is enhanced at edges perpendicular to the direction of the surface magnetic-field, which correspond to A and C in Fig. 3. Detailed shapes of other parts of edges are not essential. Thus the MFE of pits is well approximated by a groove perpendicular to the direction of the surface magnetic-field. Furthermore, since the magnetic field attenuates toward the bottom, the bottom shape is not essential for describing the MFE. We developed a two-dimensional model shown in Fig. 4. Its geometry is parametrized by r_e, R and α, where r_e is the radius of edge, R is half the width of pit aperture, and $\pi\alpha$ $(0 < \alpha < 1/2)$ is the slope angle. Note that a pit with $\alpha \to 1/2$ corresponds to the well-type pit with an infinite depth.

The detailed derivation process of the $\beta_{\rm M}$ factor due to the pit is found in Ref. [2]. The result is

$$\beta_{\rm M} \simeq \left(\frac{e}{\pi}\frac{2^\alpha}{\alpha}p\right)^{\frac{\alpha}{1+\alpha}}\left(\frac{r_e}{R}\right)^{-\frac{\alpha}{1+\alpha}} . \tag{4}$$

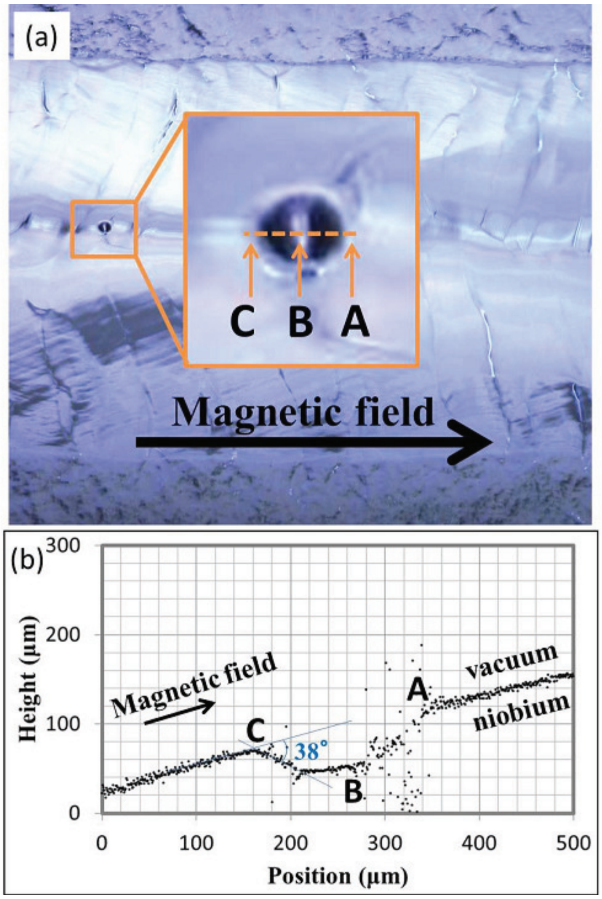

Figure 3: An optical image and a profile of quench-inducing pit found on an equator-weld [2]. The cavity performance was limited by a quench at $E_{\rm acc} = 31\,{\rm MV/m}$.

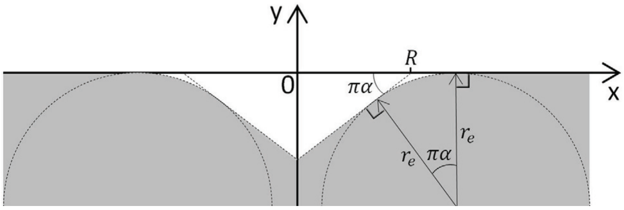

Figure 4: Model of pit [2]. Gray and white regions represent the SC in the Meissner state and the vacuum, respectively. r_e is the radius of edge, R is half the width of pit aperture, and $\pi\alpha$ ($0 < \alpha < 1/2$) is the slope angle. The arcs of edges are smoothly connected to the slopes and the flat surfaces. [Originally published in Progress of Theoretical and Experimental Physics **2015**, 073G01 (2015). Published by Oxford University Press.]

The constant p is the solution of the following equations for p and Δ:

$$\Delta^{1+\alpha} p = \frac{\pi \alpha e^{\alpha}}{2^{\alpha}} \frac{r_e}{R}\,, \tag{5}$$

$$(1+\alpha\Delta) p = \frac{1-2\alpha}{{}_2F_1(a,b;c;\zeta)} \left(\frac{1}{\cos\pi\alpha} - \frac{r_e}{R} \frac{1-\cos\pi\alpha}{\sin\pi\alpha} \right), \tag{6}$$

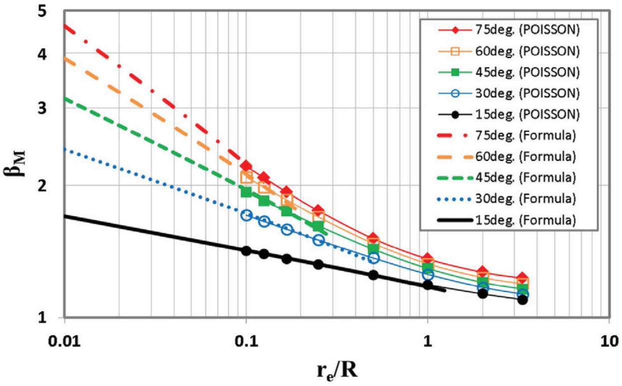

Figure 5: $\beta_{\rm M}$-factors evaluated by using the formula, Eq. (4), and those obtained by the simulation code, POISSON, are shown as functions of r_e/R for slope angles, 15°, 30°, 45°, 60°, 75° [2]. [Originally published in Progress of Theoretical and Experimental Physics **2015**, 073G01 (2015). Published by Oxford University Press.]

where ${}_2F_1(a,b;c;\zeta)$ is the Gaussian hypergeometric function with $a \equiv -\alpha$, $b \equiv 1/2-\alpha$, $c \equiv 1+b$ and $\zeta \equiv 1-\Delta$.

Figure 5 shows $\beta_{\rm M}$ calculated by the formula Eq. (4) as functions of r_e/R. The analytical calculations agree very well with the simulation results (see Ref. [2] for the simulation method that utilizes the "electrostatics" simulation code, POISSON). Not only the edge radius r_e/R, but also the slope angle $\pi\alpha$ has a substantial impact on $\beta_{\rm M}$. The larger the slope angle is, the larger the $\beta_{\rm M}$ is. A pit with a slope angle $\simeq 75^\circ$ and a sharp edge $r_e/R = 0.01$ can reach $\beta_{\rm M} > 4$. Pits with the smaller slope angles, however, can not yield such a large value: when the slope angle is 15°, even if the pit has a very sharp edge $r_e/R = 0.01$, we obtain $\beta_{\rm M} < 2$. Then we see that real pits, which often have gentle slopes, yield much smaller $\beta_{\rm M}$ than the well-type pit.

$\beta_{\rm M}$ factors of real pits found on cavities can be evaluated by extracting the model parameters from pit profiles and substituting them into the formula Eq. (4) or Fig. 5. An example of evaluation of $\beta_{\rm M}$ of a real pit is found in Ref. [2]).

More detailed discussions and explanations are given in Ref. [2].

SUMMARY

The recent two theoretical studies on cavity surface conditions [1, 2] were briefly introduced.

Nano-defects and Superheating Field Suppressions

- Ref. [1] addresses the superheating field suppression due to nano-defects distributing almost continuously on the cavity surface.

- A model of the nano-defect is given by Fig. 1. The gray region represents the SC. The depth δ and the angle α (or θ) are the parameters of the model.

- We evaluated the superheating field suppresion factor η. Eq. (2) is the analytical formula for η. Fig. 2 shows the contour plot of η as functions of the depth δ and the angle θ.

- By using Eq. (2) or Fig. 2, suppression factors of bulk or multilayer superconductors and those after various surface processing technologies can be evaluated, if data of surface topographic studies are available.
- An application to the electropolished dirty Nb is found in Ref. [1]. Applications to the multilayer coating are found in Ref. [4], which is practically important because η shifts the optimum thickness of the SC layer of the multilayer coating.

Magnetic Field Enhancement at a Pit

- Reference[2] addresses the magnetic field enhancement at a pit on the surface of cavity.
- Figure3 shows a typical pit found on the cavity inner surface. While many of pits found on cavity surfaces are not well-type but have gentle slopes like this, pits with general slope angles had not been studied. Impacts of the slope angle on the $\beta_{\rm M}$-factor had not been well understood.
- We introduced a model that can describe a pit with an arbitrary slope angle shown in Fig. 4. Two dimensional model is enough. The geometry of the present model is parametrized by r_e, R and α, where r_e is the radius of edge, R is half the width of pit aperture, and $\pi\alpha$ $(0 < \alpha < 1/2)$ is the slope angle.
- To evaluate the enhancement factor, $\beta_{\rm M}$, an analytical method based on the conformal mapping was developed. Eq. (4) is the formula to evaluate $\beta_{\rm M}$.
- A new simulation method was also developed, which utilizes two-dimensional “electrostatics". Detailed explanations are found in Ref. [2].
- $\beta_{\rm M}$ calculated by the formula Eq. (4) is shown in Fig. 5, which agree well with the simulation results. We see that not only the edge radius r_e/R, but also the slope angle $\pi\alpha$ has a substantial impact on $\beta_{\rm M}$.
- $\beta_{\rm M}$ factors of real pits found on cavities can be evaluated by extracting the model parameters from pit profiles and substituting them into the formula Eq. (4) or Fig. 5. An example of evaluation of $\beta_{\rm M}$ of a real pit is found in Ref. [2].

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