PROPAGATION OF THE HIGH FREQUENCY FIELDS IN THE CHAIN OF THE SUPERCONDUCTING CAVITIES*

A. Novokhatski[#], SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA

Abstract

Combination with the very high repetition rate requires to use the superconducting cavities to accelerate very short bunches for the FEL operation as it planned for LCLS-II. In the cavities these bunches excite very high frequency electromagnetic fields. There are severe concerns, that these fields will remain inside the structure for a long time, bring additional heating or even break up the Cooper pairs. We present results of the simulation of the transient dynamics of wake fields of very short bunches. We show how much of the energy is vanishing through the beam pipes immediately and how much energy is staying in the cavity for a long time.

INTRODUCTION

While passing a cavity a bunch creates electromagnetic fields. A fraction of these fields is staying in the cavity for a very long time (captured modes). After several reflections, another part is leaving the cavity and the rest of the field is chasing the bunch. In a time this field will catch the bunch and take its kinetic energy. The time or the distance, where the bunch is caught, is inversely proportional to the bunch length. It can be very long for a very short bunch.

We may imagine that the spectrum of this part contains mainly high frequency modes. We will try giving the analyses for the TESLA cavity, as it is very important from the point of feasibility of the short bunch acceleration in super conducting cavities.

To give quantitative values, we study the energy distribution of the wake field in a single cell cavity with connected tubes and in multi-cell cavities. In this paper we update the previous study [1]. For wake field calculation we use a computation code NOVO [2]. The algorithm for this code was specially designed for calculation of the wake fields of very short bunches.

ENERGY OF THE WAKE FIELD

To study the fields, which are really acting on particles, it is necessary to split the full field into the wake field, that really acts on the bunch particles and the "self" field, that is moving everywhere together with the bunch, but does not interact with particles (in the relativistic case).

$$E_{full} = E_{bunch} + E_{wake} \quad (1)$$

The energy distribution of the wake field, following the bunch, can be described by the longitudinal energy density $\Lambda(s)$, which is the cross-plane integral over the energy density at a distance s after a bunch center:

$$\Lambda(s) = \int \left(\frac{\varepsilon_0}{2} E_{wake}^2(s) + \frac{\mu_0}{2} H_{wake}^2(s) \right) d\varphi r dr \quad (2)$$

The integral of this density represents the total energy that is following the bunch at a distance s

$$T(s) = \int_{-\infty}^{s} \Lambda(s')ds' \quad (3)$$

Depending on the distance s this integral may include only propagating fields (finite s) or include also trapped modes when s goes to plus infinity. Naturally in this case the integral is approaching the value of the loss factor K_{loss} of a bunch

$$K_{loss} = T(\infty) \quad (4)$$

A SINGLE CELL CAVITY

While decreasing the bunch length, the loss factor is increasing, and more and more high order modes are excited in a cell. The loss factor in one regular cell of the TESLA cavity is shown in Fig. 1 as a function of the bunch length. The integral of energy of the field which is following a bunch is also shown in Fig. 1 for a moment when a bunch left a cell and at distance more than 15 bunch length.

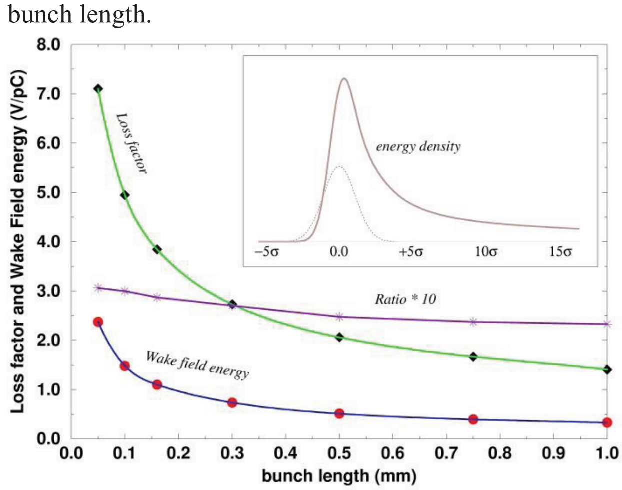

Figure 1: Loss factor and the wake field energy, following the bunch in the tube, after a single cell of the TESLA cavity over the bunch length. Ratio of field energy to the loss factor is multiplied by 10 (to use the same scale).

In this figure, inside a box, one can find the energy density distribution $\Lambda(s)$ in the tube and the bunch charge distribution. At this time there is still some wake fields, which are chasing a bunch but with a slowly vanishing tail. The energy of the following field is defined as $T(s = 15\sigma)$. The ratio of the energy integral to the

*Work supported by DOE-AC02-76SF00515

#novo@slac.stanford.edu

ISBN 978-3-95450-178-6

loss factor is slightly growing up with the bunch length decreasing, coming to the value of 30%. So, one third of the "excited energy" immediately leaves the cell with the bunch.

How far away the wake field will follow a bunch in the tube? One can predict, that at least, up to the distance L, where the field "catches" the bunch

$$L = \frac{a^2}{2\sigma} \quad (5)$$

where a is the radius of the tube, σ is the bunch length. For the regular cell of the TESLA cavity and for the bunch length $\sigma = 0.5$ mm, this distance $l = 1.225$ m, is equal to the length of the 9-cell cavity. For the bunch of $\sigma = 50\,\mu$, the distance is more than one accelerating cryo-module length.

If we calculate the field in a single cell then we can see the periodical time structure of short pulses. The period is equal to the doubled time of passing the cavity. That means that a short bunch excites a short electromagnetic pulse, which contains high frequency modes. These modes are traveling mainly in the longitudinal direction with reflections of the tubes. After a time of 1.5 ns (four reflections) modes of very high frequency disappear and approximately 4 ns later, other non-trapped modes leave the cavity.

In order to analyse the structure of the excited fields inside a cavity we calculated the wake potential of a 0.5 mm bunch. This wake potential is shown on the Fig. 2.

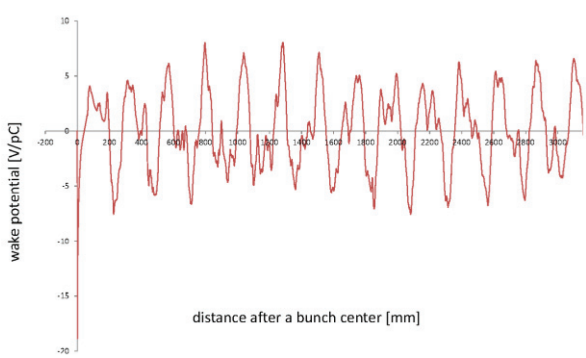

Figure 2: Wake potential of the TESLA 9-cell cavity.

Fourier transform of this wake potential in the form of the energy loss integral is shown in Fig. 3. Sharp steps in the integral correspond to trapped modes. We calculated R/Q for these modes. There are two most important trapped modes. The first one is the fundamental mode. Excitation of this mode means taking energy from a cavity, decreasing the accelerating gradient (beam loading). Also it is responsible for the phase shift and so called "gap transient" if the bunch pattern has gaps. The second mode of approximately 2.3 GHz is really a trapped mode with R/Q=173 Ohm. This mode must be damped by the HOM absorber. In the opposite case it can have high enough amplitude. A beam with a reputation rate of 1 MHz has a tense spectrum and there will be a harmonic with a frequency very close to the frequency of this mode. Resonance excitation may lead to high amplitude of this mode. We can estimate this amplitude. If a loaded Q of this mode is higher than

$$Q_l >> \pi \frac{f_{mode}}{f_{rep}} = 3.14\frac{2300}{1} \approx 7000 \quad (6)$$

then the amplitude at the resonance will be

$$V_{mode} = I\frac{R}{Q}Q_l \quad (7)$$

Assuming that $Q_l \approx 10^8$, then the amplitude will be 5 MV for the beam current of 300 microampere. This amplitude is comparable with accelerating gradient of 16 MV.

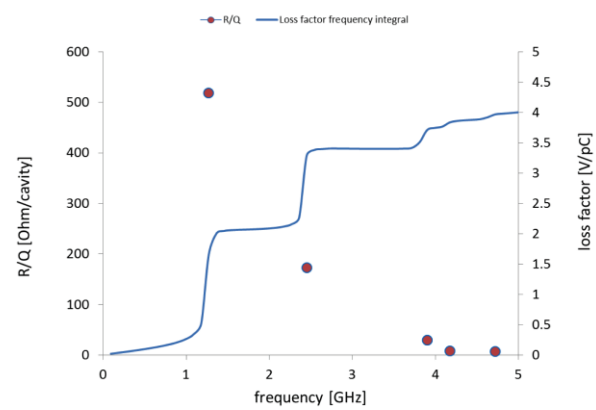

Figure 3: R/Q of trapped symmetrical modes and loss factor frequency integral

MULTI-CELL CAVITY

We may assume that in a multi-cell cavity the amplitude of the wake field increases with the number of cells until the total length of all cells reaches the catch-up distance L (eq. 5). To check this statement we calculated energy integral for different number of regular TESLA cells and for different bunch length. Results of the computation of the energy integral $T(s = 15\sigma)$ are shown in Fig. 4. It can be seen, that the energy of the following field is linearly growing up in first cells. The number of cells, where the field approaches the asymptotic solution, is determined by the "catch up" length L and period of structure D

$$N = L/D = \frac{a^2}{2\sigma D} \quad (8)$$

Values for for the TESLA cavity (9-cell) are shown in Fig. 4 in the left corner for different bunch length.

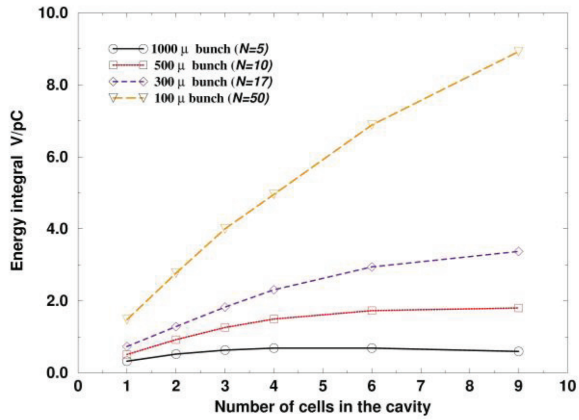

Figure 4: Wake field energy, following the bunch in the TESLA cavity, for a bunch length of 1 mm, 0.5 mm and 0.1mm.

DIRECTED AND REFLECTED FIELDS IN TESLA CAVITY

To study the reflected and the directed fields in the TESLA cavity, the model of excitation of one cell was used. As the full field is separated (eq. 1) and we do calculations only for the wake field, then on the surface of the "excited" cavity the wake field has to take the value of the bunch field with negative sign in accordance with boundary conditions.

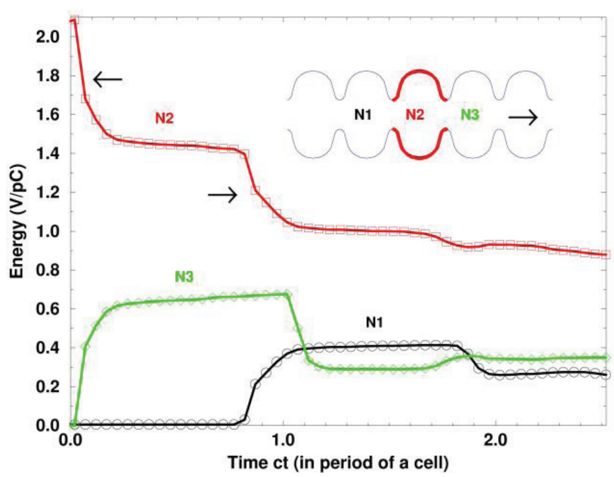

Figure 5: Field energy in cells of TESLA cavity.

The geometry of the "excited" cell (N2) and the cells around (N1, N3) are shown in Fig. 5. Cell N2 is excited by the bunch of 0.5 mm length. The energy in the cells is calculated and presented in time. When the bunch leaves cell N2 the wake field energy is going with it and excites the cell N3; then coming to the end of cell N3, one part of the energy goes to the next cell and the energy in the cell N3 is going down. At the same time, the field, reflected from the iris between cells N2 and N3, crossing the cell N2 and coming to the cell N1. We can estimate the reflected and the directed coefficients compare the cells energies. At first, the transmitted energy is around 30 %, in the next cell it is increasing to 50 %. Later high frequency modes are traveling along the structure almost without reflections. If we know the transmission coefficient of the energy Π, then we can estimate the time dependence of the energy in the "excited" cell by

$$E(t) = E_0(t)(1-\Pi)^{ct/D} \approx E_0(t)e^{-t/\tau} \quad (9)$$

We can also find the energy attenuation time of high frequency modes in one cell

$$\tau = -\frac{D}{c\ln(1-\Pi)} \quad (10)$$

Taking Π=0.3 we get τ = 1 ns. This is in good agreement with the field attenuation time.

TRANSFORMATION OF WAKE FIELD

Coming to the periodic solution, wake fields experience a strong transformation. The amplitude goes down and the shape changes. The wake fields of a 50 µ bunch in cavities with different number of the cells are normalized per one cell and are shown in Fig. 6.

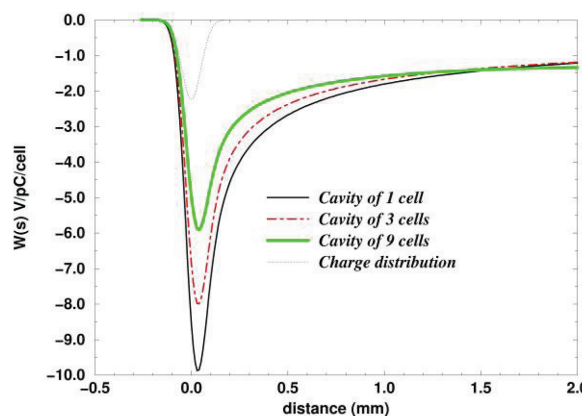

Figure 6: Wakes of a 50 µ bunch in the TESLA regular cavities with a different number of cells.

The wake amplitude becomes twice as small after one TESLA cavity. In the next cavities of the cryo-module, the wake field changes mainly the shape, approaching the integral of the charge distribution.

ACKNOWLEDGMENT

The author would like to thank Mike Sullivan and Yunhai Cai for help and valuable commentsThis work was supported by Department of Energy Contract No. DE-AC02-76SF00515.

REFERENCES

[1] A. Novokhatski, M. Timm, T. Weiland, High Frequency Properties of Wake fields in TESLA Cavities", Proceedings of EPAC'98, p. 1315, (1998).
[2] A. Novokhatski, SLAC-PUB-11556, 2005

ISBN 978-3-95450-178-6