# HOM CALCULATIONS FOR DIFFERENT CAVITIES AND BEAM INDUCED HOM POWER ANALYSIS OF ESS

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*Abstract*

For different design of ESS superconducting cavities, the higher order modes (HOMs) of monopoles, dipoles, quadrupoles and sextupoles are found. Their $R/Q$ values are also calculated at their geometric beta.

Main HOM related issues are the beam instabilities and the HOM induced power especially from TM monopoles. The analysis for the beam induced HOM voltage and power in this paper shows that, if the HOM frequency is a few kHz away from the beam spectrum, it is not a problem.

In order to understand the effects of the beam structure, analytic expressions are developed. With these expressions, the induced HOM voltage and power were calculated by assuming external $Q$ for each HOM. Our analysis confirms that, with the beam structure of ESS and a good cavity design, no special tight tolerances are required for cavity fabrication and no HOM couplers in the cavity beam pipes are planned.

## INTRODUCTION

A time–averaged HOM power spectrum normalized to the cavity's $R/Q$ has been calculated by Sangho Kim for the Spallation Neutron Source (SNS) beam time structure (single pass) [1], and Haipeng Wang's paper 'Beam Induced HOM Power Spectrum in JLab 1MW ERL-FEL' [2]. The analysis of ESS beam time structures is explained in this paper. Since the most dangerous HOM monopole is close to 5th harmonic of micro-bunch frequency for ESS median-beta cavity, the HOM induced voltage and power analysis based on the 5th harmonic of micro-bunch frequency is also showed in this paper.

HOMs are found with the CST MW Studio [3], and HOM properties such as frequencies and $R/Q$ are calculated. We can find the dangerous mode which frequency is close to the beam spectrum from the simulation results. The total HOM induced power can also be calculated by the $R/Q$ values. These works are showed in this paper.

## ESS BEAM& HOM IDUCED VOLTAGE ANALYSIS

### *Time Structure of ESS*

Time structure of ESS (50 mA pulse) is showed in Fig. 1.

HOM induced voltage analysis of ESS based on Sangho Kim's SNS/AP Technical Note No. 10 and Haipeng Wang's paper are showed below.

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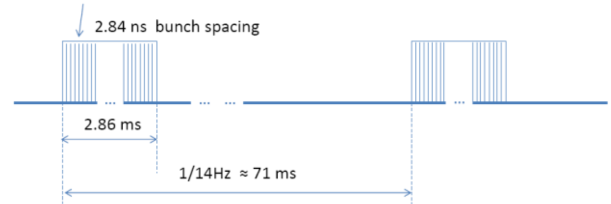


Figure 1: Time structure of ESS beam (bunches at 352.21 MHz).

The parameters used in the calculation are listed as following:

- HOM decay time constant: $T_d = \dfrac{2Q_e}{\omega} = \dfrac{Q_e}{\pi f}$
- SRF cavity's fundamental mode frequency: $f_1 = 704.42*10^6$ Hz
- Total numbers of micro-pulse within one macro-pulse: $N = 1.01*10^6$
- Pulse period between each micro-bunch: $T_b = 1/(352.21\ \text{MHz}) = 2.839*10^{-9}$ s
- Bunch period between each macro-pulse: $T_m = 71*10^{-3}$ s
- Macro-pulse length: $T_{mb} = 2.83*10^{-3}$ s
- Macro-pulse spacing: $T_G = T_m - T_{mb} = 68.17*10^{-3}$ s
- Single bunch charge (approximate to a point charge): $q = 1.42*10^{-10}$ C
- $R/Q$ normalized induced voltage by a point charge: $V_q(\omega) = \dfrac{\omega}{2}\cdot q$
- At the time just after $k$th macro-pulse:

$$A(\mathrm{k}, \mathrm{Q}_e, \omega) = V_q(\omega)\cdot\frac{1\text{-}\exp(\frac{-\mathrm{NT}_b}{T_d}+\mathrm{i}\,\omega\,\mathrm{NT}_b)}{1-\exp(\frac{-\mathrm{T}_b}{T_d}+\mathrm{i}\,\omega\,\mathrm{T}_b)}\cdot\frac{1\text{-}\exp(\frac{-k\,\mathrm{T}_m}{T_d}+\mathrm{i}\,\omega k\,\mathrm{T}_m)}{1-\exp(\frac{-\mathrm{T}_m}{T_d}+\mathrm{i}\,\omega\,\mathrm{T}_m)} \tag{1}$$

- During the gap between $k$th and $(k+1)$th macro-pulse:

$$B(\mathrm{k}, \mathrm{Q}_e, \omega, \mathrm{t}_1) = A\cdot\exp(\frac{-\mathrm{t}_1}{T_d}+\mathrm{i}\,\omega t_1),\ 0<t_1<T_G \tag{2}$$

- At the time just after $n$th micro-pulse in $(k+1)$th macro-pulse:




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$$C(\mathrm{n},\mathrm{k},\mathrm{Q}_e,\omega)=B(\mathrm{k},\mathrm{Q}_e,\omega,\mathrm{T}_G)\cdot\exp[\frac{-(\mathrm{n}-1)\mathrm{T}_b}{T_d}+\mathrm{i}\,\omega(\mathrm{n}-1)\mathrm{T}_b]$$

$$+V_{\mathrm{q}}(\omega)\cdot\frac{1\text{-}\exp(\frac{-\mathrm{n}\,\mathrm{T}_b}{T_d}+\mathrm{i}\,\omega n\,\mathrm{T}_b)}{1-\exp(\frac{-\mathrm{T}_b}{T_d}+\mathrm{i}\,\omega\mathrm{T}_b)} \tag{3}$$

- During the gap between $n$th and $(n+1)$th micro-pulse in $(k+1)$th macro-pulse:

$$D(\mathrm{n},\mathrm{k},\mathrm{Q}_e,\omega,\mathrm{t}_2)=C(\mathrm{n},\mathrm{k},\mathrm{Q}_e,\omega)\cdot\exp(\frac{-\mathrm{t}_2}{T_d}+\mathrm{i}\,\omega t_2)\,,$$

$$0<t_2<T_{\mathrm{b}} \tag{4}$$

### *HOM Induced Voltage and Power*

Plot of envelop of the induced voltage/ $(R/Q)$ at $f$=1761.05 MHz (5*352.21 MHz) 5th harmonic of micro-bunch frequency after 1000 pulses nearly in steady state between the macro pulses in Fig. 2.

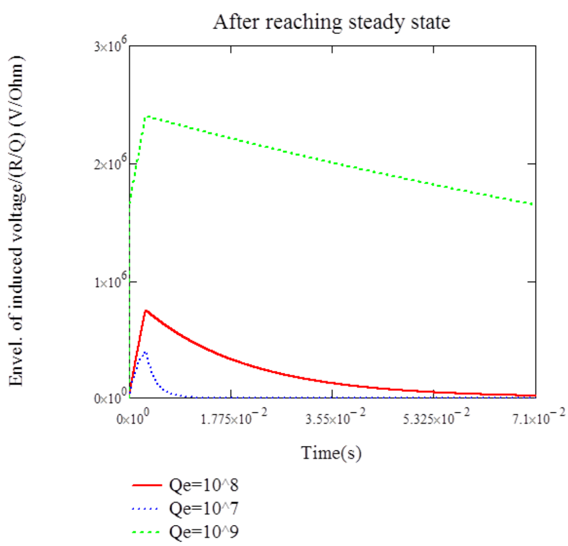


Figure 2: Normalized induced voltage / $(R/Q)$ at $f$= 1.76105 GHz (5*352.21 MHz) between the macro-pulses in steady state at different $Q_{\mathrm{e}}$.

From the result we can see, if $Q_{\mathrm{e}}<10^8$, the damping of the HOM induced voltages at 1761.05 MHz is sufficient. The HOM power dissipated by the beam is:

$$P(\mathrm{Q}_e,\omega,\mathrm{t})=\frac{|V(\mathrm{Q}_e,\omega,\mathrm{t})|^2}{Q_e} \tag{5}$$

Plot of envelop of HOM power/$(R/Q)$ at $f$=1761.05 MHz (5*352.21 MHz) after 1000 pulses nearly in steady state between the macro pulses in Fig. 3.

The time averaged HOM power can be obtained by integrating the $P$ ($Q_{\mathrm{e}}$, $\omega$, $t$) in one period of $T_{\mathrm{m}}$:

$$P_{\mathrm{ave}}(\mathrm{Q}_e,\omega)=\frac{1}{T_m}\int_0^{T_m}P(\mathrm{Q}_e,\omega,\mathrm{t})dt \tag{6}$$

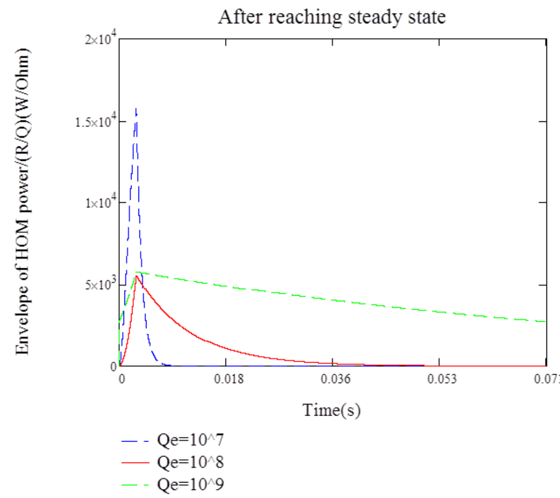


Figure 3: Normalized HOM power at $f$=1.76105 GHz (5*352.21 MHz) between the macro-pulses in steady state at different $Q_{\mathrm{e}}$.

The normalized time-average HOM power is only a function of the $Q_{\mathrm{e}}$ and the frequency, which is the HOM property. With this general behavior of the normalized HOM power as a function of frequency or $Q_{\mathrm{e}}$, the actual HOM powers for each mode is straightforward by applying $R/Q$'s those are found in section 3. Time averaged HOM spectrum at $Q_{\mathrm{e}}=10^8$ is shown in Fig. 4. Time averaged HOM spectrum at different $Q_{\mathrm{e}}$ is shown in Fig. 5.

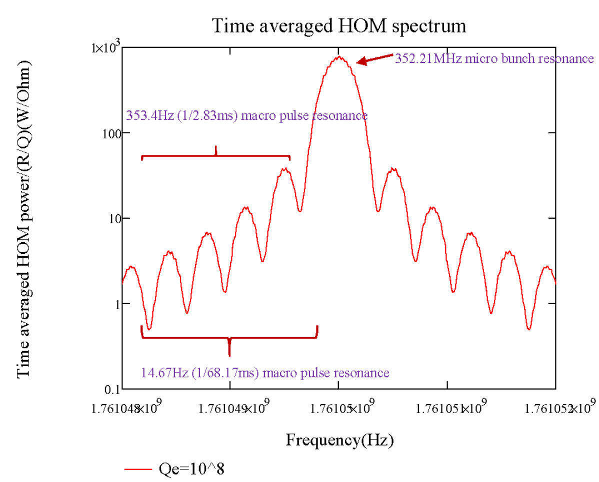


Figure 4: Time average power/ $(R/Q)$ at $Q_{\mathrm{e}}=10^8$ in one period of $T_{\mathrm{m}}$.

If $Q_{\mathrm{e}}=10^9$, the 14 Hz resonance appears since the damping during the macro-pulse is not enough. That means the damping time constant $T_{\mathrm{d}}$ is longer than the macro pulse gap. Time averaged HOM spectrum at $Q_{\mathrm{e}}=10^9$ is shown in Fig. 6.

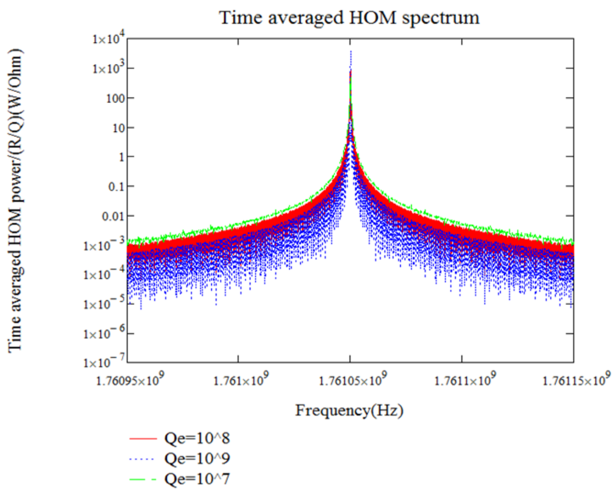


Figure 5: Time average power/ ($R/Q$) at different $Q_e$ in one period of $T_m$ (span: 0.2 MHz).

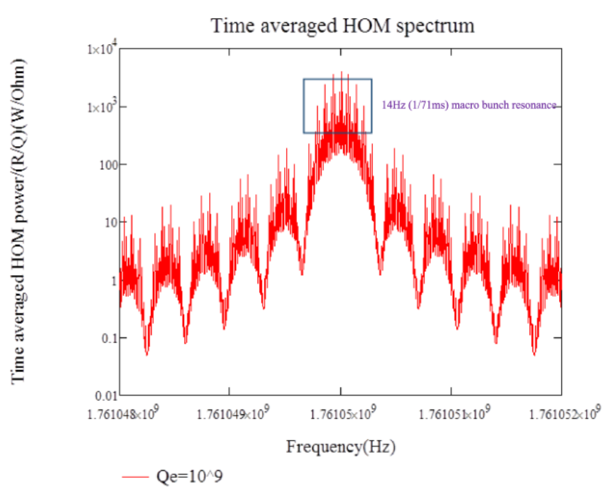


Figure 6: The normalized HOM power from the beam time structure of ESS around 1761.05 MHz at $Q_e = 10^9$.

## *HOM Power Behaviour in the $Q_e$ Space*

From the above calculation we know that when $Q_e > \sim 10^9$, the 14 Hz resonance occurs. So $Q_e$ larger than $10^9$ should be avoid. The frequency span from the every micro-bunch resonance frequency, where the normalized HOM power is higher than 1 W, is about a few kHz at $Q_e \sim 10^9$. Fig. 7 shows the normalized HOM power at different $Q_e$, and the normalized HOM power at different frequencies when $Q_e = 10^9$.

Applying HOM frequencies and $R/Q$'s values together with the HOM power calculated above, then total HOM power can be obtained.

# HOM FINDINGS

## *R/Q Definitions*

$R/Q$ for monopoles is defined as,

$$\frac{R}{Q} = \frac{\left|\int E_z(z)\exp(i\,\omega_n\, z/v)\,dz\right|^2}{\omega_n U}(\Omega) \tag{7}$$

$R/Q$ for dipoles, quadrupoles and sextupoles are defined as,

$$\frac{R}{Q} = \frac{c^2\left|\int \nabla_r E_z(z)\exp(i\,\omega_n\, z/v)\,dz\right|^2}{\omega_n^3 U}(\Omega) \tag{8}$$

$R/Q$'s for the multipoles are evaluated at the position of 1 cm from the axis and their units are Ohm, Ohm/cm$^2$ and Ohm/cm$^4$ for dipole, quadrupole and sextupoles respectively. When the $R/Q$'s at certain radial position is needed, one can have the corresponding $R/Q$ by multiplying square and fourth power of that radial position for quadrupole and sextupole respectively.

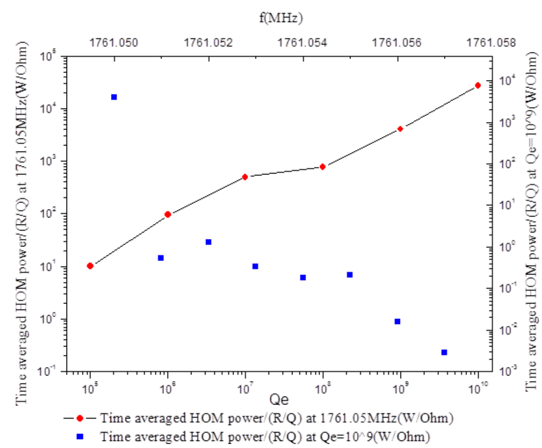


Figure 7: HOM power dependencies on the $Q_e$ at micro-bunch resonance.

## *R/Q Calculations for β= 0.67 Cavity*

HOM's of monopoles, dipoles, quadrupoles and sextupoles are found up to 2.5 GHz for $\beta$=0.67 cavity.

There are two different designs for ESS $\beta$=0.67 cavity. For design 1, the radius of left beam tube is 50 mm and the radius of right beam tube is 68 mm with an adapter to diminish the pipe size to 50 mm. For design 2, the radius of two beam tubes is 68 mm.

Fig. 8 shows the profile of two type cavities. Fig. 9 ~ 12 are plots of the maximum $R/Q$'s for each mode in the cavities. The $R/Q$ values show that:

- There's no monopole mode frequency near the hamonic of micro-bunch frequency for two designs.
- For dipole modes, the $R/Q$ values are different between two designs especially for those modes which frequencies are higher than the beam pipe cut off frequency.
- For quadrupole and sextupole modes, the $R/Q$ values are different between two designs. However, the values are small especially that for sextupole modes. The




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simulations accuracy is also an influence factor for those modes with small *R*/*Q* values.

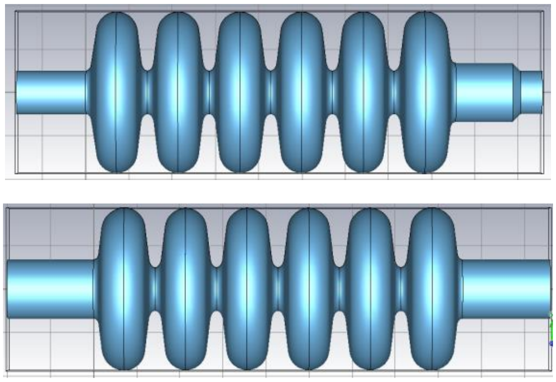

Figure 8: The profiles of two type cavities. (The upper one is design 1with small beam tube. The bottom one is design 2 with large beam tube.)

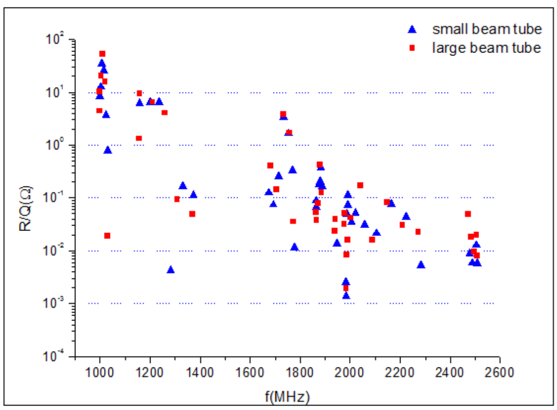


Figure 9: *R*/*Q* for monopole modes.

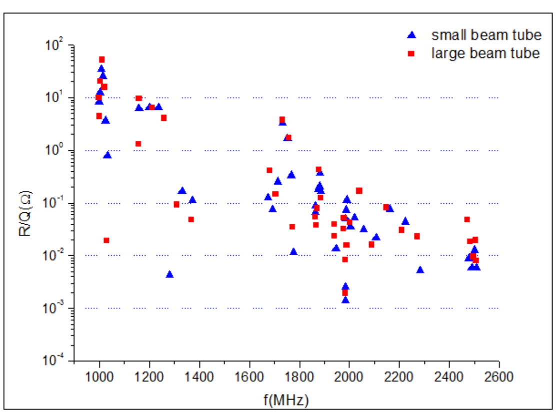


Figure 10: *R*/*Q* for dipole modes.

## *R/Q Calculations for β= 0.86 Cavity*

HOMs of monopoles, dipoles, quadrupoles and sextupoles are found up to 2.2 GHz for $\beta$=0.86 cavity.

There are two different designs for ESS $\beta$=0.86 cavity. For design 1, the radius of left beam tube is 55 mm and the radius of right beam tube is 70 mm in one section. For design 2, the radius of two beam tubes is 70 mm.

Fig. 13 shows the profile of the cavities. Fig. 14 ~ 17 are plots of the *R*/*Q*'s for each mode in the cavity. The *R*/*Q* values show that:

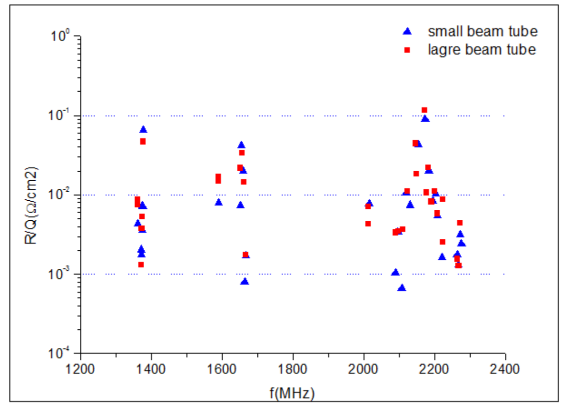


Figure 11: *R*/*Q* for quadrupole modes.

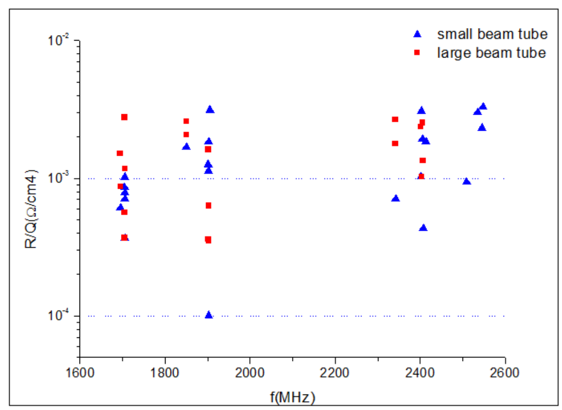


Figure 12: *R*/*Q* for sextupole modes.

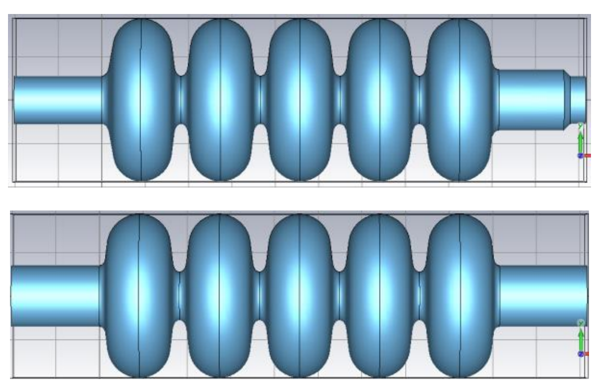

Figure 13: The profile of $\beta$=0.86 cavity. (The up one is design 1. The bottom one is design 2.)

- For monopole modes, the *R*/*Q* values of some modes in design 2 are larger than design 1. Near the 4th hamonic of micro-bunch frequency, there's a mode which frequency is 1413.03 MHz in design 1, while in design 2, the frequency of this mode is 1399.036 MHz.

- For dipole modes, the $R/Q$ values are different between two designs especially for those modes which frequencies are higher than cut off frequency.
- For quadrupole and sextupole modes, the $R/Q$ values are different between two designs. However, the values are small especially for sextupole modes. The simulation accuracy is also an influence factor for those modes with small $R/Q$ values.

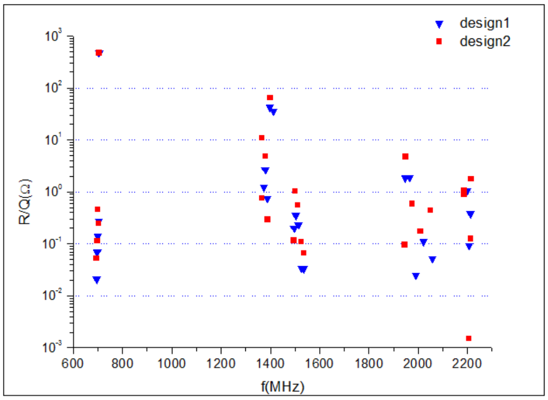


Figure 14: $R/Q$ for monopole modes.

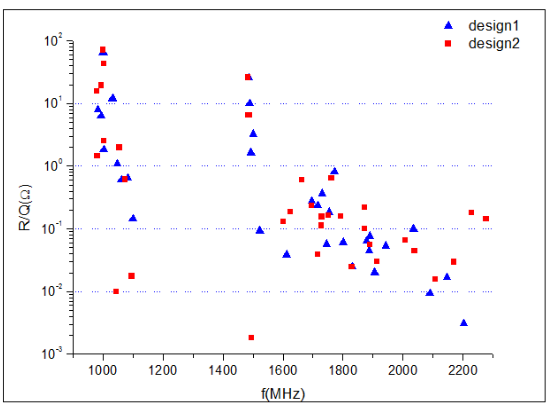


Figure 15: $R/Q$ for dipole modes.

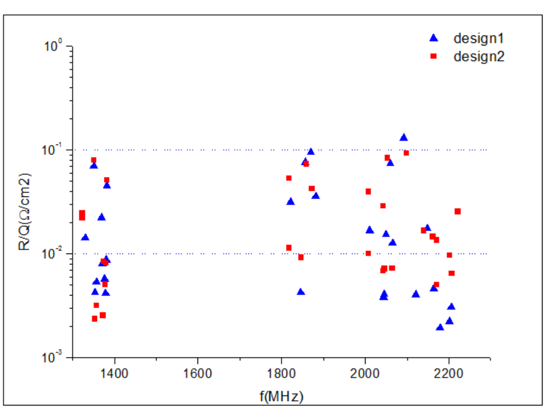


Figure 16: $R/Q$ for quadrupole modes.

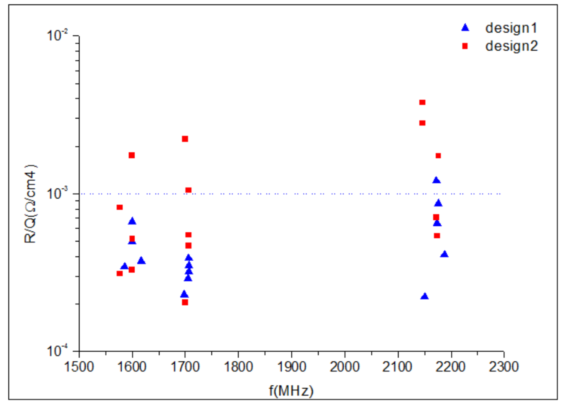


Figure 17: $R/Q$ for sextupole modes.

## CONCLUSIONS

This paper mainly gives the analysis results based on the beam structure of ESS and the different cavity designs. The higher order modes (HOMs) of monopoles, dipoles, quadrupoles and sextupoles are found. Their $R/Q$ values are also calculated for different design cavities. General expressions of the beam structure on the HOM induced voltage and power are given. HOM induced voltages and powers are estimated by assuming different $Q_e$.

The analysis shows that, with the beam structure of ESS and a good cavity design, no special tight tolerances are required for cavity fabrication and no HOM couplers in the cavity beam pipes are planned.

## ACKNOWLEDGMENT

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[3] https://www.cst.com/Products/CSTMWS




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