SYSTEMATIC UNCERTAINTIES IN RF-BASED MEASUREMENT OF SUPERCONDUCTING CAVITY QUALITY FACTORS*

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Abstract

Q_0 determinations based on RF power measurements are subject to at least three potentially large systematic effects that have not been previously appreciated. Instrumental factors that can systematically bias RF based measurements of Q_0 are quantified and steps that can be taken to improve the determination of Q_0 are discussed.

INTRODUCTION

The intrinsic quality factor, Q_0, of a superconducting cavity is an important measure of its performance. If the coupling factor, β, is close to unity, Q_0 can be determined from RF losses in the cavity. If the coupling is far greater than unity, cryogenic heat load measurements must be employed. Only RF measurement techniques will be considered here.

RF-based quality factor measurements commonly compare the power incident on the cavity, P_F, to the power reflected by the cavity, P_R, to determine the cavity coupling factor:

$$\beta^* = \left(\frac{\sqrt{P_F}+\sqrt{P_R}}{\sqrt{P_F}-\sqrt{P_R}}\right)^{\pm 1}.$$

The sign of the exponent in this equation is chosen to be positive (negative) if the cavity is over-coupled (under-coupled). The loaded cavity quality factor, Q_L, can be determined from the decay time, τ, of the stored energy when power to the cavity is shut off.

$$Q_L = \omega\tau.$$

The intrinsic quality factor is related to the cavity coupling and loaded quality factor:

$$Q_0 = (1+\beta^*)Q_L.$$

RF power levels and cavity decay times can typically be determined with an accuracy of a few percent. These uncertainties limit the accuracy of RF based quality factor measurements to 5% or more, even under the best conditions [1,2,3].

Implicit in this approach are three assumptions:

1. The forward and reflected waveforms are perfectly separated during the coupling factor measurement.
2. No power is incident on the cavity during the decay time measurement.
3. The cavity is precisely on resonance during the measurement of the coupling factor.

Each of these three assumptions is violated in practice.

1. The imperfect directivity of the directional coupler used to separate the waveform incident on the cavity from the reflected waveform inevitably introduces some degree of cross-contamination between the signals.
2. Energy emitted into the reflected waveform from the cavity during the decay can re-reflect back from the circulator commonly used to isolate the RF power amplifier as energy incident on the cavity. The re-reflected energy may interfere constructively or destructively with the cavity field. Constructive interference will systematically bias measured decay times to values longer than the true cavity decay time. Destructive interference will systematically bias the measured decay times to shorter values.
3. Energy re-reflected from the circulator will also systematically shift the resonance frequency of the cavity-waveguide system from the true resonance of the cavity leading to systematic biases in the measured coupling factor.

DIRECTIVITY UNCERTAINTIES

Dual directional couplers are commonly used to separate the voltage incident on the cavity from the voltage in the waveform reflected from the cavity. Perfect separation of the forward and reflected waves within the coupler is not possible. Some degree of cross-contamination will always be present. The level of cross-contamination that may be expected is specified by the directivity of the coupler. The directivity of the forward port can be determined from the S-parameters of the coupler.

$$D = 20\, log_{10}\frac{S_{31}}{S_{41}}$$

Poor directivity couplers may have directivities as low as 10 dB. Couplers with directivities of 20 or 30 dB are commonly employed for cavity testing. Ultra-high directivity couplers may have values as high as 60 dB.

While a directivity of 20 dB implies that less than one percent of the power is leaking into the other port, depending on the relative phases of the direct signal and the contamination, interference effects can lead to systematic power mismeasurements of up to ±10 percent. If the contamination adds constructively with the direct signal the measured power may be systematically larger than the true power by up to $10^{D/20}$. If the contamination

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interferes destructively, the measured signal may be systematically lower than the true value by the same factor.

To demonstrate this, a variable-length rigid coaxial airline (trombone) was inserted into the cavity power circuit between the cavity and directional coupler with the cavity tuned off resonance. The complex phasors of the forward and reflected waveforms measured using three different directional couplers [4,5,6] were recorded as the length of the trombone was varied over one full wavelength of the cavity drive signal, seen in Figure 1. The phasor of forward waveform sweeps through an angle of 4π along a circle centred on the true value. The radius of the circle decreases as the directivity of the coupler improves. The power measured with the 20 dB directivity RF Lambda coupler varies by up to ±25% depending on the relative phase of the direct signal and the contamination. The power measured using the 40 dB directivity HP776 coupler only varies by slightly more than 1% under the same conditions.

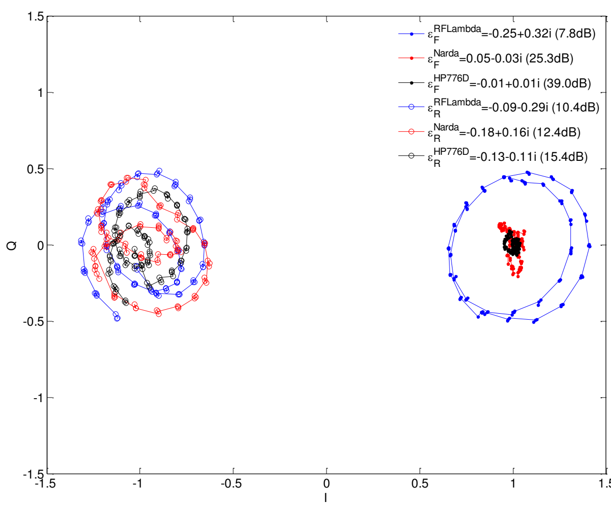

Figure 1: Forward and reflected power from dual directional couplers with as the trombone length is varied over a wavelength.

Mismeasurements of the forward and reflected power due to imperfect directivity lead to a systematic biases in the cavity coupling factor determined from those measurements. The RMS uncertainty in the measured coupling factor due to imperfect directivity will be:

$$\left\langle\left(\frac{\Delta\beta^*}{\beta^*}\right)^2\right\rangle^{\frac{1}{2}}_{Directivity} \approx 10^{-\frac{D}{20}}\sqrt{1+\frac{(\beta^*+\beta^{*-1})^2}{4}}.$$

The fractional systematic uncertainty in β due to imperfect directivity following re-calibration then becomes:

$$\left\langle\left(\frac{\Delta\beta^*}{\beta^*}\right)^2\right\rangle^{\frac{1}{2}}_{Calibrated} = 10^{-\frac{D}{20}}\sqrt{1+\beta^{*2}}$$

RMS uncertainty as a function of coupling factor is plotted in Figure 2 for directivities ranging between 10 and 60.

REFLECTIONS FROM THE CIRCULATOR

Amplifiers in high power RF circuits are commonly protected by ferromagnetic circulators. Circulators are non-linear devices and rarely present a perfect impedance match to the transmission line connecting the load and the cavity. Reflections at the mismatch redirect energy from the reverse waveform back into the waveform incident on the cavity. Specifications for ferromagnetic circulators typically quote Voltage Standing Wave Ratios (VSWR) between 1.20 and 1.50. The magnitude of the reflection coefficient and the VSWR are related as follows:

$$|\Gamma_{Circulator}| = \frac{\mathrm{VSWR}-1}{\mathrm{VSWR}+1}.$$

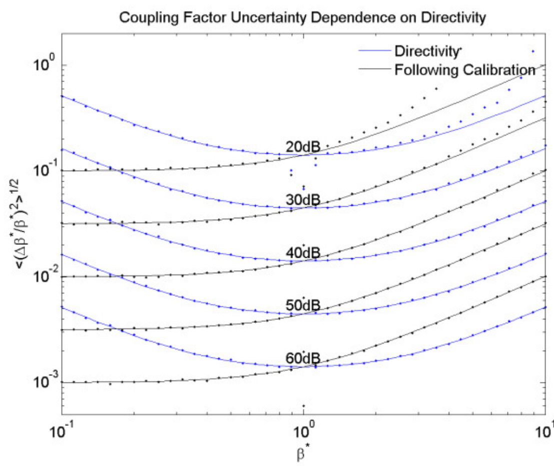

Figure 2: Systematic Uncertainties Associated with Coupler Directivity.

During the decay energy re-reflected from the circulator may interfere constructively or destructively with the cavity field depending on the length of the waveguide that connects the cavity and the circulator, *l*, and the wavenumber, κ, of the RF drive waveform. Constructive interference will systematically bias measured decay times to values longer than the true cavity decay time. Destructive interference will systematically bias the measured decay times to shorter values. The measured decay time of the cavity will by systematically biased from the true cavity decay time as follows:

$$\frac{\Delta\tau}{\tau} = \frac{2}{1+\beta^{*-1}} Re\left(\Gamma_{Circulator}e^{-2i\kappa l}\right)$$

Figure 3 shows how the measured decay time changes as the length of a trombone inserted between the cavity and the circulator and the cavity was varied over one wavelength of the RF drive waveform. As would be expected from the formula above, the measured decay time oscillates through two full sinusoidal cycles around the true cavity decay time as the length of the trombone sweeps over a wavelength.

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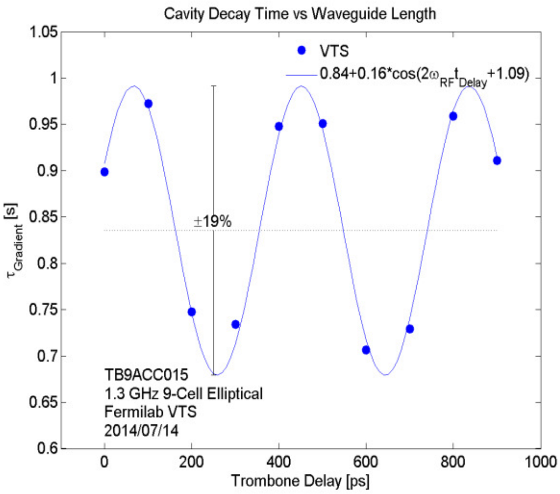

Figure 3: Cavity Decay Time vs Trombone Position.

If no correction is applied to the measured cavity decay time for energy re-reflected from the circulator the systematic bias in the decay will introduce a systematic bias in Q_0.

$$\langle\left(\frac{\Delta\tau}{\tau}\right)^2\rangle^{1/2} = \langle\left(\frac{\Delta Q_0}{Q_0}\right)^2\rangle^{1/2} \approx \frac{\sqrt{2}}{1+\beta^{*-1}}\frac{\mathrm{VSWR}-1}{\mathrm{VSWR}+1}.$$

A circulator with VSWR of 1.30 will induce probable systematic uncertainty in Q_0 of approximately 10% even for an optimally coupled cavity. Figure 4 shows how the expected systematic uncertainty in the cavity decay time varies with β^* and VSWR. For large values of VSWR and β^* the first order equation calculation above (lines) falls below the results of a full simulation (dots).

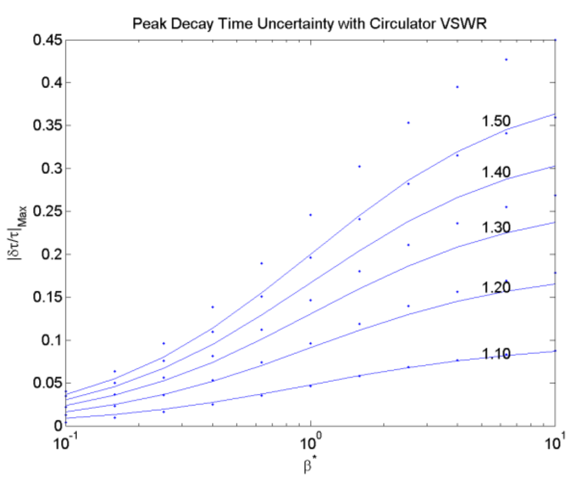

Figure 4: Expected Systematic Uncertainty the Measured Cavity Decay Time with Coupling Factor and VSWR.

RESONANCE FREQUENCY UNCERTAINTIES

Energy re-reflected from the circulator also leads to systematic biases in the measured resonance frequency. The cavity and waveguide together form a coupled resonator system. The resonance frequency of the cavity/waveguide system is systematically offset from the resonance frequency of the cavity alone by a factor that depends on the cavity coupling factor, the reflection co-efficient of the circulator, the wavenumber of the drive signal and the length of the waveguide:

$$\frac{\Delta\delta}{\omega_{1/2}} = -\frac{2}{1+\beta^{*-1}} Im(\Gamma_{Circulator} e^{-2i\kappa l}).$$

If the cavity is not on resonance, the ratio of the reflected to forward power will increase and the coupling factor will be mis-measured. The probable fractional systematic uncertainties in β^* due to uncertainties in the resonance frequency will be:

$$\langle\left(\frac{\Delta\beta^*}{\beta^*}\right)^2\rangle_\delta^{1/2} = \frac{1}{2}\langle\left(\frac{\delta}{\omega_{1/2}}\right)^2\rangle^{1/2}.$$

Figure 5 shows how the expected systematic shift in the cavity resonance frequency varies with β^* and VSWR. For large values of VSWR and β^* the first order equation calculation above (lines) falls below the results of a full simulation (dots).

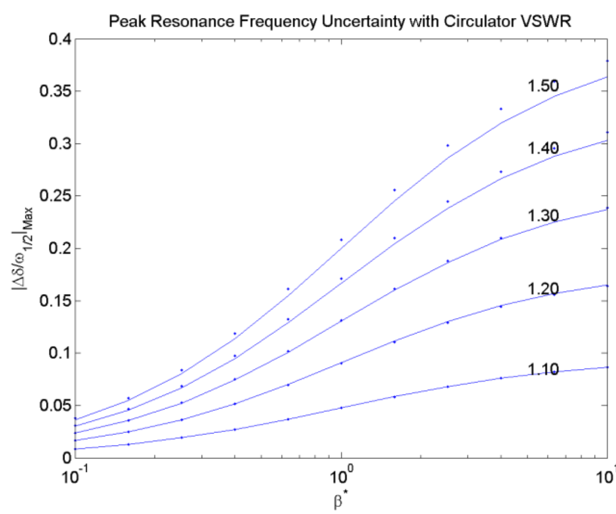

Figure 5: Peak Error in Resonance Due to Circulator Mismatch.

As an illustration Figure 6 compares the magnitude of the probe/forward and probe/reflected transfer functions measured using Fermilab VTS analogue tracking system to independent measurements of the same ratios recorded by an independent digital I/Q system as the analogue PLL phase was systematically varied to sweep the RF drive frequency across the cavity resonance. The resonance sweeps were repeated the length a trombone in the cavity power circuit was varied over a wavelength. The magnitude of each transfer function when plotted against the angle of the transfer function should depend only on the PLL phase and not on the length of the trombone. Each transfer functions recorded by the I/Q system (dots) coalesces along a single curve that depends only on the PLL phase and peaks at zero. In contrast both the magnitude and the peak positions of the transfer functions recorded by the analogue system vary widely as the length of the trombone is changed. Both directivity and circulator effects could lead to such shifts and no attempt to separate the two was made.

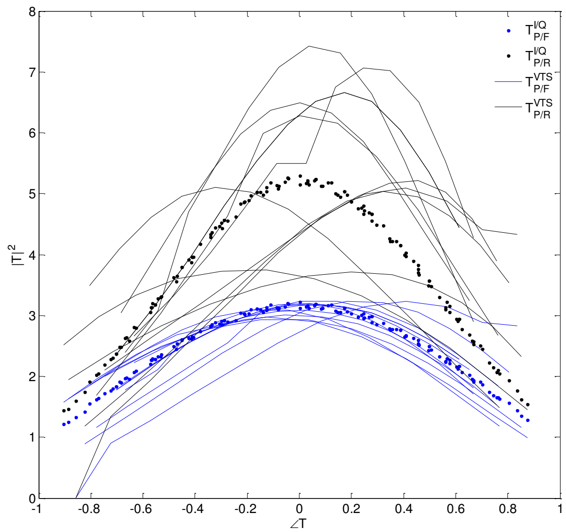

Figure 6: Measured Probe/Forward and Probe/Reflected Transfer Functions as the Length of the Waveguide is Varied.

COMBINED SYSTEMATIC UNCERTAINTIES

Table 1 lists estimates for the probable fractional systematic uncertainty in Q_0 for a coupling factors between 0.1 and 10 assuming a directivity of 30 and a circulator VSWR of 1.30 together with the percentage of single cell cavities tested at each coupling factor between 2007 and 2014. The median coupling factor was β=2.84. The probable systematic uncertainty for cavities tested with this coupling factor is just under 20%. This estimate of the systematic uncertainty in Q_0 measurements is considerably higher than previous estimates (10% at β= 5.0) that did not account for the effects discussed here.

REDUCING SYSTEMATIC UNCERTAINTIES IN QUALITY FACTOR MEASUREMENTS

Systematic uncertainties in Q_0 measurements can be reduced by a variety of measures including:

1. Using a variable power coupler;
2. Using a high-directivity directional coupler;
3. Using digital I/Q system;
4. Using data-based calibration; and
5. Measuring complex transfer functions.

Directivity associated uncertainties depend strongly on β^* and are smallest when β^* is unity. Consistent use of a variable power coupler would allow every measurement to be made while the cavity is optimally coupled and directivity uncertainties are small.

Table 1: Probable Systematic Uncertainties in Q_0 as a Function of β

β	Component				Combined	
	Power Meter Calibration	Directivity D = 30dB	Off-Resonance	Circulator VSWR=1.30	Probable Percentage Error	Percentage of Single Cell Cavities
0.1	18	3	1	2	18	1
0.2	8	3	2	3	10	1
0.3	5	3	2	4	8	2
0.4	4	3	3	5	8	2
0.5	3	4	3	6	8	2
0.6	2	4	3	7	9	2
0.7	1	4	4	8	9	2
0.8	1	4	4	8	10	1
0.9	0	4	4	9	11	1
1.0	0	4	5	9	11	8
2.0	3	7	6	12	16	21
3.0	5	10	7	14	19	18
4.0	7	13	7	15	22	12
5.0	8	16	8	15	25	7
6.0	10	19	8	16	28	5
7.0	12	22	8	16	31	3
8.0	14	25	8	16	34	3
9.0	16	29	8	17	38	2
10.0	18	32	8	17	41	8

Further improvements can be made by using high-directivity directional couplers. Directional couplers with directivities of 40dB are commercially available. High-directivity couplers may cover narrower frequency bands than broadband couplers or may be limited to lower power levels but if accurate measurements are important, a high-directivity coupler should be employed.

Systematic biases in decay time measurements can be reduced to negligible levels by varying the length of a trombone inserted in the cavity power circuit.

In contrast off resonance errors can only be eliminated if both magnitude and phase data is recorded and the cavity is tuned to the peak of the transfer function rather than the peak probe power or the minimum reflected power as is currently common practice.

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CONCLUSION

Q_0 determinations based on RF power measurements are subject to at least three potentially large systematic effects that have not been previously appreciated, directivity, energy re-reflected from the circulator and off-resonance errors. All three of these effects can introduce systematic uncertainties as large as or larger than previous estimates which have focussed exclusively on uncertainties associated with power meter calibration. Uncertainties depend strongly on β^*. If accurate measurements of cavity coupling factors are desired a variable power coupler should be employed.

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