One Point Multipacting Levels Determined without Electron Tracking

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Abstract

We present a simple criterion which allows to determine if a cavity or its auxiliaries present the risk of having multipacting – and at which field level – without numerical electron tracking but simply from the knowledge of the magnetic field and the electric gradients at the cavity surface.

The use of these data avoids long searches with specially designed tracking programs thus gives an easy to handle means to find possible dangers. Also one gets an idea if a small modification can avoid multipacting, difficult to judge from tracking results. Furthermore – as it is intended – one can link those data to a tracking program which scans automatically the whole cavity surface for these conditions. This gives hints where to look at which field level, thus decreasing largely the probability to overlook a possible level. Also one can estimate fields and get hints in complicated 3D designs (couplers,...) normally asking for long 3D field program runs.

In the present paper we derive the general theoretical basis but restrict the data to the case of one point multipacting on (rather) flat surfaces, needing only the knowledge of the magnetic field $B_{z,o}$ and the electric surface gradient $\partial E_x/\partial y$. It is intended to complete this data collection by the cases of stronger curvature, two point multipacting around an electric field zero and in edges.

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1. Introduction

Many RF cavities are plagued by multipacting and this situation is especially dangerous in superconducting cavities since it leads often to a quench. There have been several studies for multipacting [1] [2] [3] [4] [5] using computer programs to track electrons in the cavity RF field and to search for multipacting tracks. All these studies were done for a particular cavity geometry and a new cavity design asks always for a new study. Also such a study has to be very thorough to get an acceptable confidence level since it is never a mathematical proof if one does not find any level.

The basic idea of the present paper is to attack the problem from the opposite direction, thus not to design a cavity, calculate its fields and search for multipacting but to establish a (complete) catalog of all possible field configurations having multipacting of a predefined type (see later) and then take any cavity one likes, calculate or estimate its surface fields and look into this catalog, which is much easier for the user once the catalog exists.

Evidently the program sketched above is much too ambitious to be realized exactly since 'all' field configurations cannot be catalogued and statistical quantities as starting energies are involved. However, multipacting is a rather local situation as shown with the tracking programs, thus we can reduce 'all' field configurations to a rather limited number of parameters allowing to establish a catalog of reasonable size and thus to approach sufficiently close the target described above.

In the present paper we present a first step of this designed program, the one point multipacting levels for rather flat surfaces. It is intended to complete the catalog in a first step for more curved surfaces. In a second step also two point multipacting as observed in [6] [5] − where the electrons move in a half−integer cycle over an electric zero crossing − can be treated. Finally two point multipacting in angles could be catalogued.

It is not necessary to read the following theoretical part to use the data, one can immediately continue with the chapter 'Practical Application', all information should be self explaining without reading the intermediate chapters.

2. Treatment of the Problem

We know the field map in the supposed multipacting area close to the surface and this map is defined only inside the cavity volume. For reasons of the practical treatment of the problem we assume that we have also extended continuously this map outside the real cavity in a small volume around the supposed location. Multipacting can be defined in this context by a triple condition:

- Neglecting the cavity walls there has to exist a track, solution of the equation of motion in the local RF field (extended even outside the real cavity volume) starting and ending at the same point with a time of flight of an integer number of RF oscillations (called 'mathematical' track)
- This track has not to hurt the physical cavity surface before the supposed impact at the starting location ('physical' track). This condition is of course not literally fulfilled when treating two point multipacting but in this case we look for two half tracks ending at the initial start location where those half tracks do not hurt the surface before the supposed impact.
- The impact energy has to be in a range where the average multiplication factor for electrons is higher than unit (multipacting track). Of course, this last condition is rather vague since it depends strongly on the state and cleanliness of the surface. Therefore we do not consider this condition directly in our catalog but we will give the impact energy of the electrons and it is up to the user to decide for his cavity material and surface cleanliness if he is safe − despite the existence of a 'physical' track − or not.

2.1 The Global Equation of Motion and its Normalization

Multipacting tracks have always rather low kinetic electron energies so that one can neglect all relativistic effects. The equation of motion of an electron in the harmonic RF field is then given by

(1) $$m\cdot\partial^2 r/\partial t^2 = -e\cdot B(r) \times \partial r/\partial t\cdot\cos(\omega t) + e\cdot E(r)\cdot\sin(\omega t)$$

where r represents the vector (x,y,z) and B(r),E(r) the general field map. We can normalize this equation in measuring time in RF phase ϕ and distances in units of the inverse wave vector $\lambda/2\pi = c/\omega$, replacing r by the normalized variable ρ thus

(2a) $$\partial f/\partial t = \omega \cdot \partial f/\partial\phi \qquad \text{for any function f}$$

(2b) $$r = \lambda\cdot\rho/(2\pi) = c/\omega \cdot \rho$$

yielding the normalized equation of motion

(3) $$\rho'' = e/(m\cdot\omega)\cdot\{-B(\rho) \times \rho'\cdot\cos(\phi) + E(\rho)\cdot\sin(\phi)/c\}$$

where ′ means derivative with respect to ϕ.

From this equation one sees directly a – well known – fact: If one finds a 'physical' multipacting track in a cavity of frequency f_0 at a certain field $E_{acc,o}$ one will find a **similar track** in any scaled cavity where E_{acc} is scaled with the same factor as the frequencies, thus a cavity with $\mathbf{f_1 = \mu\cdot f_0}$ **at** $\mathbf{E_{acc,1} = \mu\cdot E_{acc,o}}$ **for any scaling factor** μ.

For our purpose we can draw an even more important conclusion. Since multipacting is determined by a rather local field configuration we have not necessarily to compare scaled cavities but we have only to compare scaled local situations of globally perhaps completely different cavities. Therefore we can eliminate the parameter 'frequency' from our catalog in using 'normalized fields' i.e. we use

(4a) $$\gamma(\rho) = e/(m\cdot\omega)\cdot B(\rho)$$

(4b) $$\varepsilon(\rho) = e/(m\cdot\omega\cdot c)\cdot E(\rho)$$

and (for later use) electric field gradients resp. velocities are expressed as

(4c) $$\partial\varepsilon/\partial\rho = e/(m\cdot\omega^2)\partial E/\partial r$$

(4d) $$v = \partial r/\partial t = c \cdot \partial\rho/\partial\phi \qquad \text{(i.e. in units of the velocity of light)}$$

yielding the normalized equation of motion

(5) $$\rho'' = -\gamma(\rho) \times \rho' \cos(\phi) + \varepsilon(\rho) \sin(\phi)$$

where the parameter frequency does not appear any more, thus the catalog is reduced by one parameter.

2.2 The Local Equation of Motion

The multipacting tracks have always a small extension with respect to the cavity size, thus we can assume that the cavity surface is sufficiently flat in this range and that the fields can be developed locally. Of course with this assumption we exclude possible multipacting situations where electrons might move over an edge of the cavity, which has to be looked for separately. We define the start of the (possible) multipacting track to $\rho = 0$ and develop the electromagnetic forces to first order. Since the electric field represents a zero order force and the magnetic field already a first order force, we use the constant value B(0) and express E in first order.

We choose the coordinates in the following way: B(0) defines the z-axis, the x-axis is perpendicular to the cavity surface at the start location and the y-axis along the cavity surface such that x,y and z form a right handed coordinate system. Due to the boundary conditions and the kinematics we have only three essential field components, B_z, E_x and E_y with $E_y(0)=0$ and the problem has only 2 dimensions in this way, thus $r=(x,y)$ respectively $\rho=(X,Y)$. Therefore we have five parameters: the magnetic field and four electric gradients in (x,y).

These gradients can be obtained from a field calculation program but generally not immediately from the printed output. However, we shall express these gradients by easily obtainable quantities as the surface fields E_x and the curvature radii of the cavity surface.

Maxwell's equations in the vacuum give two constraints for the gradients

(6a) $$\mathrm{curl}(E)_z = \partial E_y/\partial x - \partial E_x/\partial y = -\partial B/\partial t = \omega\cdot B$$

(6b) $$\mathrm{div}(E) = \partial E_x/\partial x + \partial E_y/\partial y + \partial E_z/\partial z = 0$$

(x,y,z usual spatial coordinates), thus due to (6a) the quantity $\partial E_y/\partial x$ can be expressed by the magnetic field and the directly obtainable gradient of the surface electric field $\partial E_x/\partial y$. If we are not at a zero crossing of the perpendicular electric field (a case only used for the two point multipacting, thus excluded here) the electric field can be approximated as coaxial, thus if we have a cavity surface curvature radius R_{xz} in the (x,z) plane we can write in first order

(6c) $$\partial E_y/\partial y = E_{x,o}/R_{xz}$$

Similarly we obtain $\partial E_z/\partial z = E_{x,o}/R_{xy}$ and thus from (6b)

(6d) $$\partial E_x/\partial x = -E_{x,o}\cdot(1/R_{xy} + 1/R_{xz})$$

Due to the definition of axis the electron will stay in the (x,y) plane and thus the direct contribution of $\partial E_z/\partial z = E_{x,o}/R_{xy}$ is irrelevant for the equation of motion. In normalized form – transformation (4c) – we can write

(7a) $$e/(m\cdot\omega^2)\partial E_x/\partial x = \partial\varepsilon_x/\partial X = \alpha_x$$

(7b) $$e/(m\cdot\omega^2)\partial E_y/\partial y = \partial\varepsilon_y/\partial Y = \alpha_y$$

(7c) $$e/(m\cdot\omega^2)\partial E_x/\partial y = \partial\varepsilon_x/\partial Y = \beta \qquad (= \partial\varepsilon_y/\partial X + \gamma)$$

(X,Y normalized coordinates) introducing the gradient parameters α_x, α_y and β. Defining finally the 2x2 matrices

(8a) $$B = \begin{pmatrix} 0 & \gamma \\ -\gamma & 0 \end{pmatrix} \qquad A = \begin{pmatrix} \alpha_x & \beta \\ (\beta+\gamma) & \alpha_y \end{pmatrix}$$

and the vector

(8b) $$\varepsilon_0 = \begin{pmatrix} \varepsilon_{0x} \\ 0 \end{pmatrix}$$

allows to write finally the local normalized equation of motion

(9a) $$\rho'' = B\cdot\cos(\phi)\,\rho' + A\cdot\sin(\phi)\,\rho + \varepsilon_0\cdot\sin(\phi)$$

2.3 'Solution' of the Differential Equation

Equation (9a) is an inhomogeneous **linear** differential equation of second order with periodic coefficients. Such an equation can be transformed to an equation of first order by

$$\text{(9b)} \qquad q' = M(\phi)\, q \; + \; p\cdot\sin(\phi)$$

using the 4−vector $q = (\rho,\rho')$ with the 4x4 matrix $M(\phi)$ and the 4−vector p defined by

$$\text{(9c)} \qquad M(\phi) = \begin{pmatrix} B\cdot\cos(\phi) & A\cdot\sin(\phi) \\ I & 0 \end{pmatrix} \qquad\qquad p = \begin{pmatrix} \varepsilon_0\cdot\sin(\phi) \\ 0 \end{pmatrix}$$

Already the one dimensional case − linked to Mathieu functions − yields infinite series with coefficients defined by a continued fraction, thus there is no hope to find a simply structured explicit solution. Therefore the straight forward integration by the computer was used to solve the equation of motion. However, despite this discouraging fact one can get several conclusions from the structure of the differential equation.

First, we see that ε_0 is the inhomogeneous driving term, thus if we find a (one point) multipacting track for a given ε_0, there exists an infinity of scaled tracks for all other $\varepsilon_0 > 0$, simply the impact energy changes. Therefore the value of ε_0 is not essential to determine a 'physical' track and can thus be excluded from the catalogue's necessary parameters, leaving only 4 essential parameters, α_x, α_y, β and γ. (ε_0 is in fact considered later in determining the impact energy!) Therefore we have to establish the possible relations between α_x, α_y, β and γ to obtain a 'physical' multipacting track.

Further information can be got from the structure of equation (9a). In using any regular constant 2x2 matrix S, the second order equation (9a) can be rewritten for a different function $\rho^* = S\rho$

$$\text{(10)} \qquad S\rho'' = SBS^{-1}\cdot\cos(\phi)\, S\rho' \; + \; SAS^{-1}\cdot\sin(\phi)\, S\rho \; + \; S\varepsilon_0\cdot\sin(\phi)$$

$$\text{(10')} \qquad \rho^{*\prime\prime} = B^*\cdot\cos(\phi)\, \rho^{*\prime} \; + \; A^*\cdot\sin(\phi)\, \rho^* \; + \; \varepsilon_0^*\cdot\sin(\phi)$$

Since $\rho = (0,0)$ at the start and at the end of the track, the new function $\rho^* = S\rho$ has the same property, thus will also be a closed track. However, not each transformation matrix S leads to a situation of real physics again since the matrices $A^* = SAS^{-1}$ and $B^* = SBS^{-1}$ and the vector $\varepsilon_0^* = S\varepsilon_0$ have to have a structure as defined in (8). This restricts to only four matrices S: $S_1 = I$ (2x2 identity), $S_2 = -I$, $S_3 = Q$ and $S_4 = -Q$ with

$$\text{(11)} \qquad Q = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

We have to pay attention to get the correct phase definition, the accelerating surface field E_0 has to be defined positively fixing all signs of the other components. If we would invert all field components we would obtain another 'mathematical' track, but this track would 'go off into the wall' immediately, thus would be no 'physical' track.

Evidently the new tracks ρ_2 and ρ_4 would 'go off' to the negative x−direction thus be only mathematical tracks. However, ρ_3 is a track where the x−component is the same as for the original one but y is inverted and of course the impact energies are equal. (This is also physically evident: the magnetic field and the transversal electric field are inverted, thus all transversal acceleration changes its sign) The new matrices A^* and B^* contain the new parameters $\alpha_x^* = \alpha_x$, $\alpha_y^* = \alpha_y$, $\beta^* = -\beta$ and $\gamma^* = -\gamma$. Thus we can conclude:

- If $(+\alpha_x, +\alpha_y, +\beta, +\gamma)$ defines a multipacting track $(+\alpha_x, +\alpha_y, -\beta, -\gamma)$ defines the mirror image track with the same impact energy. Thus we can restrict the catalog to values $\beta \geq 0$, **the cases $\beta < 0$ can be found in inverting γ and using $|\beta|$.**

2.4 The Determination of 'Physical' Tracks

Without entering into details, the method used is the following:

The central part is a routine which tracks electrons in the locally parametrized RF field but without considering possible cavity walls. The tracking is done for a predefined number of (integer!) RF cycles and the distance of the endpoint of this track to the starting point is the quantity s_0 which will be made to zero – if possible – in modifying the local field parameters. Once such a 'mathematical' track is determined, the track is checked for a precocious impact on the real cavity walls before the assumed end point of the track. If this is the case, this track is eliminated from the catalog, otherwise it is recorded.

To include the field gradients $\alpha_{x,y}$ we fix the ratios $\alpha_{x,y}/\beta$ independent of the actual field in the cavity (this is possible since $\beta=0$ leads to $\gamma=0$ as closed track with zero impact energy, thus we can always assume $\beta \neq 0$). Then we choose a value β (or γ) and have to find – if it exists – the corresponding γ (or β) for a closed track.

There is another parameter not mentioned up till now, the starting RF phase ϕ_0, thus one has to vary also ϕ_0 for each γ (β) under test. Unfortunately there exist local minima of the distance s_0 with respect to ϕ_0 and β which are not the absolute minima. Therefore the searching procedure for $s_0=0$ might get trapped in such a local minimum and get the wrong conclusion, that no closed track exists. Therefore two different methods are used to determine the minimum of this distance. One method checks for any tested γ **all phase angles ϕ_0 from 0 to 180°** (contracting the interval around the found absolute minimum) and one is sure, that this is really the absolute minimum. Then a slightly different β (γ) is tested – again all phase angles ϕ_0 are checked – and the result compared with the previous test. If we get trapped in this way in a local minimum with $s_0>0$ **one can be sure that this is also the absolute minimum and that there does not exist any closed track between the starting point in γ for this examination and the local minimum found!** If we find a minimum with s_0 smaller a preset small limit we have (very probably) determined a closed track.

To confirm this result the second method starts working, using a linear approximation which converges very quickly (generally $s_0 < 10^{-15}$ in REAL*8) in a small range to the precise zero value – if it exists. Then we will modify β (and with it $\alpha_{x,y}$) – or γ – by a small amount. Generally starting from the last values the linear method – which is much faster than the global method – finds the next minimum for a slightly modified condition easily and we continue in this way. All tracks are checked of course for a precocious impact before being taken into the catalog. It is possible that we exhaust this way the whole range of β given to examine, in this case a new run is started to continue at the old endpoint in β.

At some moment we will arrive at a point where the linear method runs into trouble (or there is a precocious impact). This might be due to a very non linear behavior of the tracks in this region and one cannot conclude immediately that there are no closed tracks any more. To confirm the hypothesis, the (very CPU time consuming) global test is done and if it fails also, the fact is established! **If we do not find a closed track in this way, we can be sure that it does really not exist.**

This method is done for different number of RF cycles and the results presented as graphs in the (γ,β) plane, parametrized by α_x, α_y.

2.5 The Starting and Impact Energy

Electrons knocked free by the impact of another electron do not start with zero energy but have a certain starting energy U of a few eV. This energy corresponds to a starting velocity which can change trajectories in some cases by a not completely negligible amount. Therefore we have taken also this starting energy into consideration in assuming that the starting velocity is perpendicular to the surface. This is evidently not exactly the case, one could construct a 'double' closed track where the starting velocity is once slightly to the left once slightly to the right but the probability that things fit becomes lower the more statistical quantities have to match. Therefore we use the statistical average – perpendicular emission – and can be sure that the real spread due to the angular distribution is small.

If we have a closed track, we can double the driving electric field $E_{x,o}$ and the starting velocity, getting the same scaled track again. Thus we have to express the starting velocity in relation to the electric field $E_{x,o}$ expressed by the normalized quantity ε_0. Evidently the electric field has to remain the dominant force, if the starting velocity becomes dominant, the fluctuations will become so large that no multipacting is possible any more. On the other hand multipacting tracks are always small compared to the wave length, thus ε_0 is less than say 10^{-2} (corresponding e.g. to a surface field of 320 kV/m at 3 GHz or 37 kV/m at 350 MHz). The normalized starting velocity – expressed in units of c – has to be even smaller than ε_0 not to become dominant. If we use e.g. 2 eV, a value where data fitted well with experimental findings [6] the normalized starting velocity is about $3\cdot10^{-3}$, thus starting energies much higher are not compatible with multipacting any more. Therefore we will produce data for starting energies between 0 and 20 eV corresponding to normalized velocities of up to $9\cdot10^{-3}$, thus

(12) $$\eta_0 = v_{norm,o}/\varepsilon_0 = \omega\sqrt{2\cdot U\cdot m}/(e\cdot E_{x,o})$$

up to about 1 – a value already very improbable for real multipacting.

The program will give at the end for each track the impact velocity normalized to the initial electric field, since both are proportional as shown above. Therefore the 'impact velocity' η_i obeys also relation (12) and in inverting it one can determine the actual impact energy U_i

(13) $$U_i = (\eta_i\cdot e\cdot E_{x,o}/\omega)^2/(2\cdot m)$$

3. Practical Application

In this paper we have developed the theoretical basis for the general method which will be exhausted in future. Actually we restrict the practical application to cases of one point multipacting on not too much curved surfaces such that α_x and α_y can be considered to be zero, i.e.

(14) $$e\cdot E_{surf}/(m\cdot\omega^2\cdot R_{curv}) << 1$$

3.1 Representation of the data

The basic data for one point multipacting on (rather) flat surfaces are pairs of normalized field quantities (β,γ) resulting in closed tracks with a time of flight of an integer RF cycle. To avoid that the user has to scan the catalog for all different field levels expressed by E_{acc}, the presentation is done in using first only ratios of parameters which will tell if there is somewhere a closed track. In a second step the precise value of E_{acc} will be determined.

If one rises E_{acc} from zero on, the normalized magnetic field γ and the normalized gradient β will move along a straight line – starting at the origin – in the (γ,β) plane and evidently where this line hits such a graph, we will have a closed track. Thus the next step would be to fix for the user's cavity at any field level one point (γ_0,β_0) in this plot and draw the straight line through this point and the origin. If this line does not cross any of the graphs in the plot, there will be no multipacting, if it crosses somewhere, there exists a 'physical' track and the value of γ^* (or β^*) of this crossing defines the field level. Once these data established one has to look into the numerical tables to determine the impact energy.

3.2 The User Guide Recipe

- Choose any reference field level E_{acc} (e.g. 1 MV/m). For the surface point to be examined determine the magnetic field B, the surface field E_0, the gradient $\partial E_x/\partial y$ on the surface at this reference field level. The polarity is defined such that E_0 pointing **inside** the cavity is positive (defining the x−axis), and E_{0x}, B_z and the axis y along the cavity surface have to be a right handed system!

These quantities are easily obtainable with a pocket calculator (if one does not want to use a program checking the whole surface automatically) from the printout of E and H on the surface of e.g. URMEL [7] or SUPERFISH [8.]

- Transform these quantities with the cavity resonance frequency f into normalized parameters using

(15)
$$\varepsilon_0 = 9.344\cdot10^{-2} \cdot E[MV/m] \,/\, f[GHz]$$
$$\gamma_0 = 2.803\cdot10^{-3} \cdot B[G] \,/\, f[GHz]$$
$$\beta_0 = 4.461\cdot10^{-3} \cdot \partial E_x/\partial y\,[MV/m^2] \,/\, f^2[GHz]$$

- If β_0 is negative, invert the sign of γ_0 and use $|\beta_0|$

In this case the two tracks (β_0,γ_0) and $(-\beta_0,-\gamma_0)$ are mutual mirror images with respect to E_0 having the same impact energy, thus are in our context completely equivalent.

- In the plot draw the straight line through origin and (β_0,γ_0). If this line does not cross a graph, there is no closed track for any field level. If it crosses any graph, then there exist a closed track and the field level corresponds to γ^* resp. β^* of the crossing point. The type of multipacting is given by the crossed graph.
- Calculate the real surface field $E_{0,m}$ at the field level determined above. There exist several 'parallel' curves for different starting energies. Assume a reasonable value − e.g. 2 eV − and determine the normalized starting velocity per ε_0 with

  (16a) $$\eta_0 = 2.117\cdot10^{-2} \cdot f[GHz] \,/\, E_0[MV/m] \cdot \sqrt{U[eV]}$$

  If the found crossing concerns a graph corresponding the calculated η_0, the closed track with reasonable starting energy exists.
- Take the numerical tables and look up the normalized impact velocity per ε_0, η, for the given conditions at the field level determined above. Determine the real impact energy using

  (16b) $$U_i = 2.231\cdot10^{+3} \cdot (\eta_i\, E_{x,o}[MV/m] \,/\, f\,[GHz])^2$$

  It is up to the user now to judge if this impact energy allows multipacting in his case of cavity material and surface cleanness

3.3 An Example

Let us assume a cavity at 0.5 GHz having at a nominal field of 1 MV/m (at a flat location to be examined) an electric surface field (E_0) of 50 kV/m, along the surface the electric field has a gradient ($\partial Ex/\partial y$) of 5.5 MV/m² and a magnetic field (B_Z) of 30 G (be careful with signs!) Using relations (15) we obtain the normalized fields and gradients

$$\varepsilon_0 = 9.344 \cdot 10^{-3} \qquad \gamma = 0.1682 \qquad \beta = 0.0981$$

If we carry this point (β,γ) into the graph (see figure 3) and draw the straight line connecting this point with the origin we see that this line crosses the graphs of first order multipacting, e.g. the graph of $v_0/\varepsilon_0 = 0.1$ at about $\beta = 0.255$ and $\gamma = 0.437$.

Since $\beta = 0.0981$ ($\gamma = 0.1682$) correspond to 1 MV/m, we have to scale the original values by a factor 2.6 to obtain the crossing values, thus the condition for a physical closed track is given at 2.6 MV/m. The surface field at 2.6 MV/m is 2.6·50kV/m = 130 kV/m and the normalized starting velocity corresponds to (equation (16b)) 1.5 eV, a physically reasonable value.

If we look now in the numerical table of 1 RF cycle and $v_{imp}/\varepsilon_0 = 0.1$ we find for $\beta = 0.255$ for the normalized impact velocity a value of about 3.5 which can be transformed (equation (16b)) to an impact energy of 1.8 keV, a value where multipacting should be possible.

Therefore we have demonstrated that the examined location has a high probability to have multipacting at a nominal field level of 2.6 MV/m. Since the straight line crosses also graphs for other starting energies which are also not out of range, we can even estimate the width of the band around 2.6 MV/m in which multipacting is probable.

4. Numerical Table (including Impact Energy)

1 RF cycles vst/eps$_0$ = 0.

β	γ	ϕ_0	v_{imp}/ε_0
1.0000E−02	3.97356158E−01	+6.97952230E+01	7.737E−01
5.0000E−02	4.73948999E−01	+4.97353683E+01	1.640E+00
1.0000E−01	4.84859904E−01	+3.58363879E+01	2.271E+00
1.5000E−01	4.77812061E−01	+2.57198231E+01	2.735E+00
2.0000E−01	4.64664988E−01	+1.75099435E+01	3.110E+00
2.5000E−01	4.49408995E−01	+1.04723761E+01	3.427E+00
3.0000E−01	4.34039713E−01	+4.22598384E+00	3.700E+00
3.4000E−01	4.22530512E−01	+3.58142036E−01	3.893E+00

+

1 RF cycles vst/eps$_0$ = 0.1

β	γ	ϕ_0	v_{imp}/ε_0
1.0000E−02	4.13401376E−01	+7.23695157E+01	7.885E−01
1.0000E−01	4.84565803E−01	+3.81744886E+01	2.343E+00
2.0000E−01	4.56567324E−01	+2.00671432E+01	3.231E+00
3.0000E−01	4.17152858E−01	+7.14807553E+00	3.881E+00
4.0000E−01	3.79290956E−01	−3.33497035E+00	4.405E+00
5.0000E−01	3.48646477E−01	−1.24325798E+01	4.827E+00
6.0000E−01	3.29260443E−01	−2.06689339E+01	5.132E+00
7.0000E−01	3.25508471E−01	−2.83829608E+01	5.284E+00

1 RF cycles vst/eps$_0$ = 0.2

β	γ	ϕ_0	v_{imp}/ε_0
1.0000E−02	4.29448228E−01	+7.45143972E+01	8.167E−01

1.0000E−01	4.85198220E−01	+4.02268650E+01	2.420E+00
2.0000E−01	4.50764735E−01	+2.22740934E+01	3.354E+00
3.0000E−01	4.04694217E−01	+9.60496644E+00	4.061E+00
4.0000E−01	3.58830531E−01	−5.82726140E−01	4.663E+00
5.0000E−01	3.18080404E−01	−9.34279027E+00	5.194E+00
6.0000E−01	2.85308311E−01	−1.71680143E+01	5.647E+00
7.0000E−01	2.62426245E−01	−2.43169436E+01	5.996E+00
8.0000E−01	2.51109692E−01	−3.09564927E+01	6.202E+00
9.0000E−01	2.53990586E−01	−3.72517757E+01	6.216E+00

1 RF cycles vst/eps$_0$ = 0.3

β	γ	ϕ_0	v_{imp}/ε_0
1.0000E−02	4.45015915E−01	+7.62821078E+01	8.572E−01
1.0000E−01	4.86448796E−01	+4.20394614E+01	2.501E+00
2.0000E−01	4.46575492E−01	+2.41984617E+01	3.480E+00
3.0000E−01	3.95245966E−01	+1.17036533E+01	4.241E+00
4.0000E−01	3.43482191E−01	+1.70336448E+00	4.916E+00
5.0000E−01	2.95906001E−01	−6.87241037E+00	5.547E+00
6.0000E−01	2.55108599E−01	−1.45128459E+01	6.136E+00
7.0000E−01	2.22476025E−01	−2.14537883E+01	6.663E+00
8.0000E−01	1.98539320E−01	−2.78160934E+01	7.093E+00
9.0000E−01	1.83380023E−01	−3.36783912E+01	7.388E+00
1.0000E+00	1.77184224E−01	−3.91180286E+01	7.505E+00
1.1000E+00	1.81092060E−01	−4.42410671E+01	7.395E+00
1.2000E+00	1.99277670E−01	−4.92398715E+01	6.988E+00

1 RF cycles vst/eps$_0$ = 0.5

β	γ	ϕ_0	v_{imp}/ε_0
1.0000E−02	4.73490380E−01	+7.89149964E+01	9.686E−01
1.0000E−01	4.90021474E−01	+4.50855694E+01	2.675E+00
2.0000E−01	4.41368251E−01	+2.73921483E+01	3.739E+00
3.0000E−01	3.82149447E−01	+1.51102048E+01	4.600E+00
4.0000E−01	3.22126369E−01	+5.30796863E+00	5.410E+00
5.0000E−01	2.65755783E−01	−3.12040952E+00	6.226E+00
6.0000E−01	2.15702534E−01	−1.06718361E+01	7.061E+00
7.0000E−01	1.73382627E−01	−1.75672217E+01	7.905E+00
8.0000E−01	1.39111457E−01	−2.38958249E+01	8.727E+00
9.0000E−01	1.12356580E−01	−2.96947798E+01	9.492E+00
1.0000E+00	9.21067047E−02	−3.49912573E+01	1.016E+01
1.1000E+00	7.72171997E−02	−3.98189454E+01	1.069E+01
1.2000E+00	6.66394844E−02	−4.42202149E+01	1.106E+01
1.3000E+00	5.95351662E−02	−4.82432224E+01	1.123E+01
1.4000E+00	5.53249765E−02	−5.19392130E+01	1.118E+01

2 RF cycles vst/eps$_0$ = 0.

β	γ	ϕ_0	v_{imp}/ε_0
4.9910E−11	1.00000000E−02	+8.99819856E+01	6.284E−04
1.6151E−07	5.00000000E−02	+8.95424123E+01	1.599E−02
5.7334E−06	1.00000000E−01	+8.80711546E+01	6.766E−02
5.2555E−05	1.50000000E−01	+8.52230444E+01	1.685E−01
2.9989E−04	2.00000000E−01	+8.00634706E+01	3.526E−01
1.4844E−03	2.50000000E−01	+6.98818998E+01	7.126E−01
8.3501E−03	3.00000000E−01	+4.28175545E+01	1.544E+00

3 RF cycles vst/eps$_0$ = 0.

β	γ	ϕ_0	v_{imp}/ε_0
5.0026E−11	1.00000000E−02	+8.99729574E+01	9.434E−04

1.6854E−07	5.00000000E−02	+8.92988187E+01	2.451E−02
6.9398E−06	1.00000000E−01	+8.68150307E+01	1.117E−01
8.9620E−05	1.50000000E−01	+8.05961596E+01	3.306E−01
1.2043E−03	2.00000000E−01	+5.85723146E+01	1.065E+00

5. Figure captions

Figure 1: Plot of multipacting graphs in the (β,γ) plane. The upper curves correspond to 1 RF cycle, the topmost to a starting energy equivalent of 0., followed by 0.1, 0.2, 0.3 and 0.5. (The tilted line crossing those curves indicates the points where the starting phase becomes negative, thus the electrons are thrown against a decelerating electric field at the beginning and survive long enough to see accelerating phases again, thus the conditions are tight and those closed tracks are probably very unstable in real nature). The curves for 2 and 3 RF cycles for 0 starting energy are indicated left, a positive starting energy destroys the possibility for a closed orbit for the higher orders.

Figure 2 : Example of a closed track for 1 RF cycle

Figure 3: Example of a determined multipacting level: The point 'x' marks the relation of (γ,β) at a certain point of the cavity surface for a given reference field. The points '+' on the graphs define where multipacting is possible for a given starting energy, depending on the graph.

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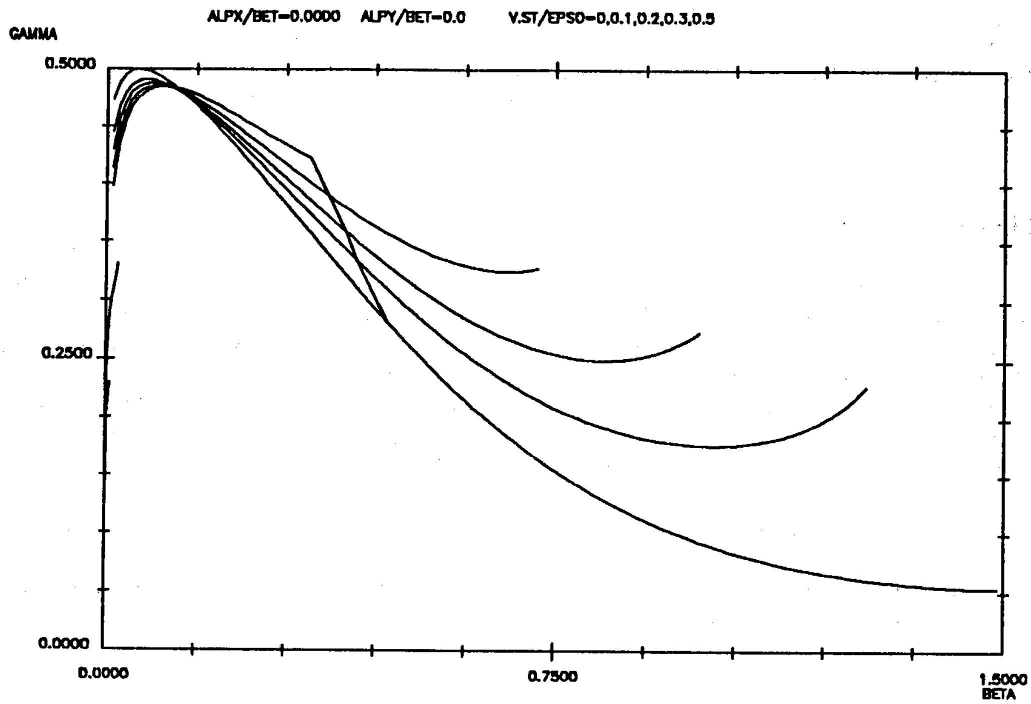

Figure 1

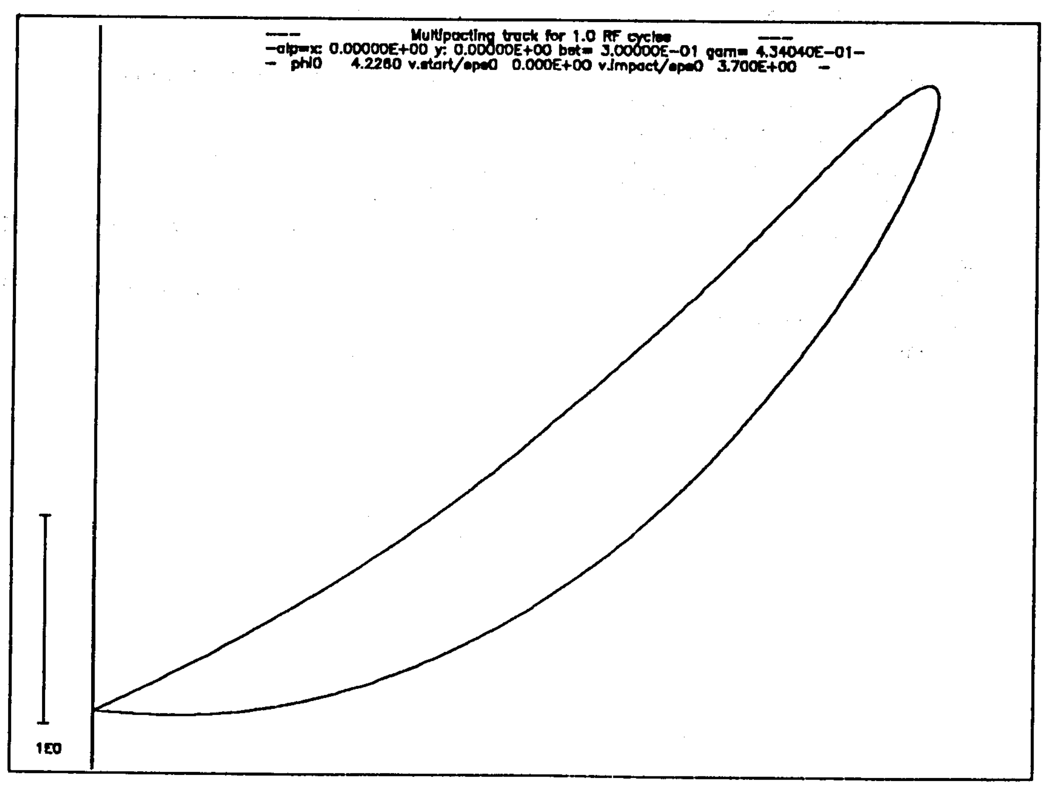

Figure 2

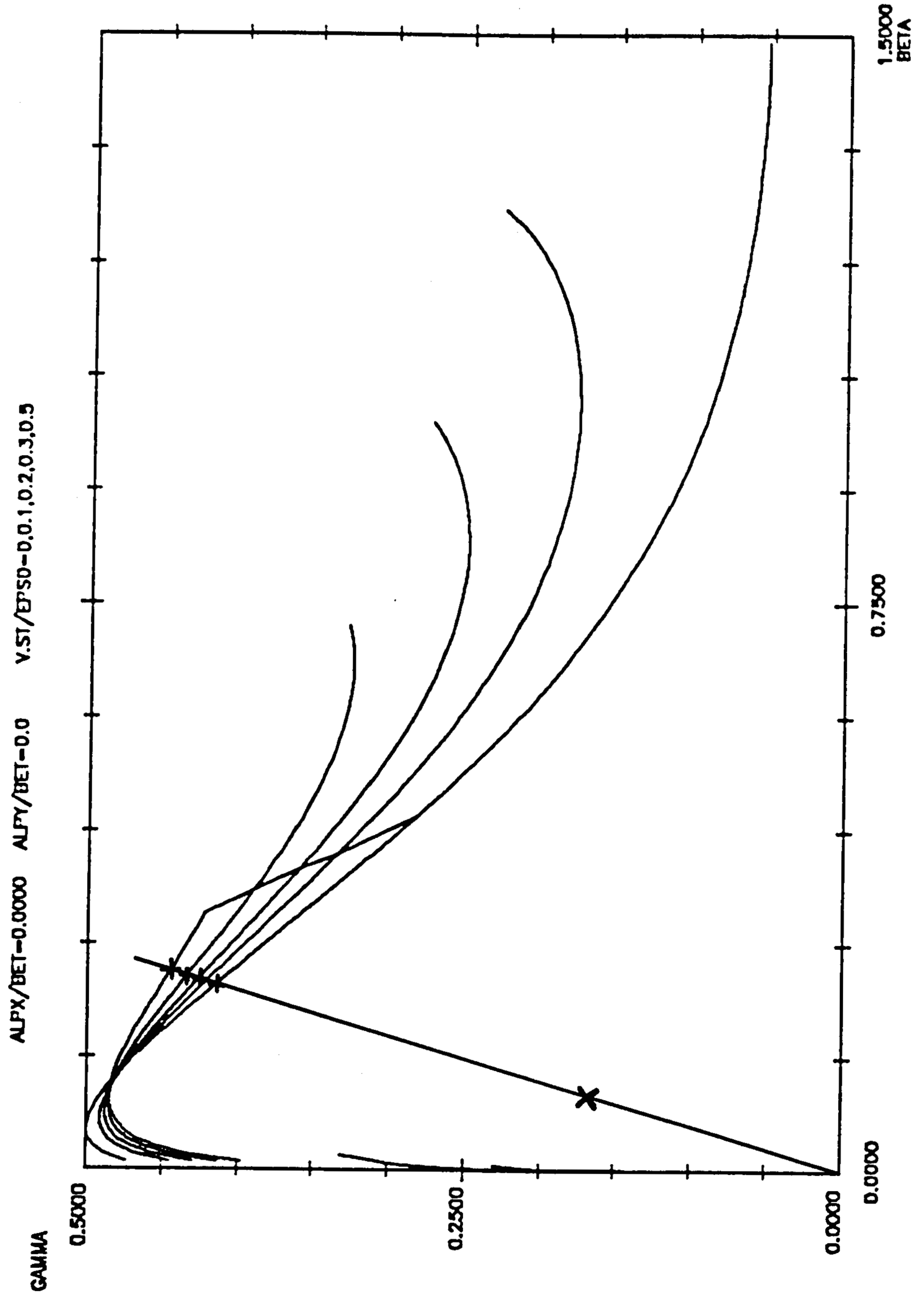
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Figure 3