Surface Resistance of a Superconductor

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Abstract

In this paper, we discuss the variation of the order parameter Ω of a superconductor as a function of temperature deduced from RF measurements of the surface resistance made on superconducting cavities. It appears that the classical two fluid model fails in matching experimental results but that the empirical law

$$\Omega = 1 - \left(\frac{T}{T_c}\right)^3$$

can be held as a fairly good approximation.

Further, we report the effect of impurities and RRR on the gap parameter as well as on the intrinsic surface resistance at T=0.

Introduction

The understanding of RF losses in superconducting cavities is very important because cryogenic power will be one of the major limitations for future superconducting particle accelerators. It turns out that RF measurement is very sensitive to the intrinsic properties of a superconductor namely its order parameter. We first show how using an improved two fluid model one can deduce from the surface resistance the order parameter Ω. Then knowing Ω, we can compute the frequency shift of a cavity as a function of temperature and compare it to experimental measurements. Finally, we point out that impurities and RRR modify Ω giving the experimental variation for niobium at 1.5GHz.

The surface resistance

Let us consider a gaz of electrons containing N_s electrons in the superconducting state and N_n in the normal one. We can define the order parameter by

$$\Omega = \frac{N_s}{N_n + N_s} = \frac{N_s}{N}$$

The total current density is the sum of the normal current density which follows Ohm's law $\mathbf{J_n} = \sigma_n \mathbf{E} = \frac{N_n e^2 \tau}{m} \mathbf{E}$ (*) and the superfluid current given by London's equation [Ref. 1]

$$\mathbf{J_s} = -\frac{N_s e^2 \mu}{m} \mathbf{A}$$

where $\mathbf{A}$ is the potential vector.

In an alternating field of pulsation ω, we can write using Maxwell's equations

$$rot\, \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} = -\mathbf{i}\, \mu\omega\, \mathbf{H}$$

So $rot\, \mathbf{J_s} = -\frac{N_s e^2 \mu}{m} rot\, \mathbf{A} = -\mathbf{i}\, \frac{N_s e^2}{m\omega} rot\, \mathbf{E}$ implies $\mathbf{J_s} = -\mathbf{i}\, \frac{N_s e^2}{m\omega} \mathbf{E}$

and the total current is

$$\mathbf{J} = \mathbf{J_n} + \mathbf{J_s} = -\mathbf{i}\, \frac{Ne^2}{m\omega} \left[\frac{N_s}{N} + \mathbf{i}\, \omega\tau\, \frac{N_n}{N} \right] \mathbf{E}$$

Defining $\lambda = \sqrt{\frac{m}{Ne^2\mu}}$ which, as we shall see hereafter, is the penetration depth at T=0, we get

$$\mathbf{J} = -\frac{\mathbf{i}}{\omega\mu\lambda^2} \left[\Omega + \mathbf{i}\, \omega\tau\, (1-\Omega) \right] \mathbf{E} = -\frac{\mathbf{i}}{\omega\mu\lambda^2}\, \chi\, \mathbf{E}$$

$\chi = \Omega + \mathbf{i}\, \omega\tau\, (1-\Omega)$ can be understood as a complex order parameter.

Considering that the displacement current is small, $\mathbf{E}$ will be the solution of the simple differential equation

$$\Delta \mathbf{E} = \frac{\chi}{\lambda^2} \mathbf{E} \qquad (1)$$

As the power loss per unit area is

$$P = \frac{1}{2} \int_0^\infty \mathcal{R}e\, (\mathbf{E}.\mathbf{J}^*)\, dz$$

the surface resistance is defined by $P = \frac{1}{2} R_s H_0^2$ where $H_0 = H_y(z=0)$ is the magnetic field at the surface of the conductor.

(*) An exact description must take into account Bolzmann's equation instead of Ohm's law but for the sake of clarity in this paper, we omit the anomalous behaviour as all the following could be deduced in a similar manner..

Defining the surface impedance by $Z = \frac{E_x(z=0)}{H_y(z=0)}$, it is straightforward to show that $R_s = \mathcal{R}e(Z)$ with $Y = \frac{i\omega\mu\lambda}{Z}$ obeying the differential equation

$$\lambda^2 \frac{\partial^2 Y}{\partial z^2} + Y^2 = \chi \tag{2}$$

In the case where χ does not vary in the conductor, the trivial solution $Y = \sqrt{\chi}$ leads to

$$R_s = \mathcal{R}e\left(\frac{i\omega\mu\lambda}{\sqrt{\chi}}\right) \tag{3}$$

Equation (3) is the important relation used for the computation of the surface resistance. Inversely, measuring R_s, one can deduce χ and thus the real order parameter Ω. This can also be extended to the case of a uniform layer of a conductor labeled 1 and thickness h upon another conductor labeled 2. The corresponding equation becomes

$$R_s = \mathcal{R}e\left\{\frac{i\omega\mu\lambda_1}{\sqrt{\chi_1}}\left[\frac{1-\left(\frac{1-x}{1+x}\right)e^{-\frac{2h}{\lambda_1}\sqrt{\chi_1}}}{1+\left(\frac{1-x}{1+x}\right)e^{-\frac{2h}{\lambda_1}\sqrt{\chi_1}}}\right]\right\}$$

with $x = \sqrt{\frac{\chi_1}{\chi_2}}\,\frac{\lambda_2}{\lambda_1}$. This relation has been used to calculate the surface resistance of niobium cavities showing the 100K disease where a uniform layer of another superconductor (presumably Nb hydride) was formed on the Nb surface [Ref. 2].

Measurements

We have analysed measurements taken mainly at the GECS laboratory at Saclay [Ref. 3], on superconducting niobium cavities. From the experimental curve $R_s(T)$ we have calculated $\Omega(T)$ using equation (3). In figure 1, we plotted $\log(1-\Omega)$ as a function of $\log(T/T_c)$. This kind of plot is quite convenient as it provides directly the power variation of (N_n/N). We clearly see that a classical two fluid model which predicts a law [Ref. 4] $\Omega = 1 - t^4$ $(t=T/T_c)$ does not fit the experience but we also notice that a law

$$\Omega = 1 - t^3 \tag{4}$$

would be much more likely to approach the experimental data.

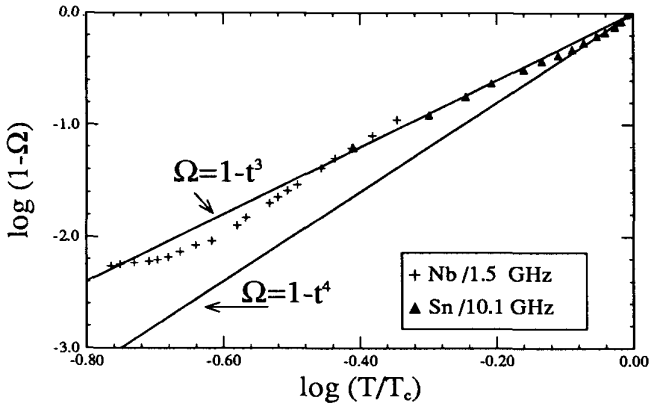

Figure 1 — Order parameter Ω for Nb [2] and Sn [8] as a function of temperature.

This feature is not particular to niobium. We have also done the same analysis using experimental results from literature on Indium [Ref. 5], Lead [Ref. 5,6,7,8] and Tin [Ref. 9,10]. We notice on figure 2 that the variation of the order parameter with temperature seems to follow a universal law since it is valid for different superconductors and at various frequencies.

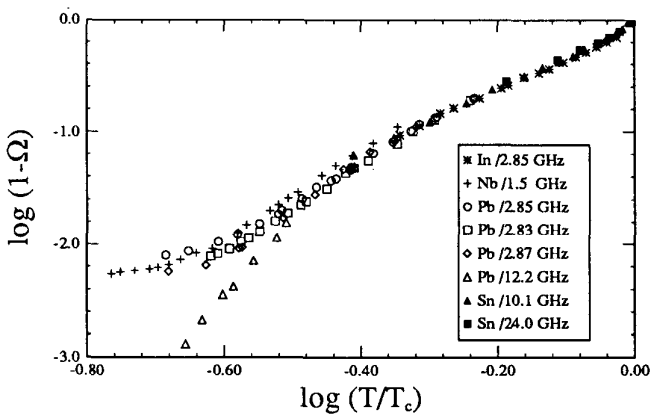

Figure 2 — Order parameter for different superconductors at different frequencies

Frequency Shift

The penetration of the electromagnetic field inside the conductor will shift the resonance frequency of the cavity by an amount given by [Ref. 11]

$$\frac{\delta F}{F_0} \simeq \frac{W_m - W_e}{W_{stored}}$$

F_0 is the frequency of the cavity assuming perfect conducting boundaries with no field penetration,

W_{stored} the total stored energy in the cavity,

$W_m = \frac{1}{2}\int \mu H^2$ and $W_e = \frac{1}{2}\int \epsilon E^2$ the magnetic and electric energy inside the conductor.

Using the geometrical factor G of the cavity, $W_{stored} = \frac{G}{2\omega R_s}\left(\int_{surface} H^2\right)$

and integrating E and H in the cavity wall using (xx), we get

$$W_m - W_e = \frac{\lambda H_0^2}{4\ \mathcal{R}e(\sqrt{\chi})}\left(\mu - \epsilon |Z|^2\right)\left(\int_{surface} H^2\right)$$

Thus, we deduce

$$\frac{\delta\omega}{\omega_0} = \frac{\omega\mu\lambda}{2G\,\mathcal{R}e(\sqrt{\chi})}\left[1 - \left(\frac{Z}{\eta}\right)^2\right] \qquad (5)$$

where $\eta = \sqrt{\frac{\mu}{\epsilon}} \sim 377\Omega$ is the free space impedance. The factor $\left(\frac{Z}{\eta}\right)^2$ is generally negligible in conductors.

Measuring the frequency as the temperature changes allows us to calculate Ω and is another way to confirm the above variation. This has been done on the TE011 mode of a pillbox niobium cavity at 4GHz (fig. 3). The counterpart of this measurement is that it is only sensitive in the neighbourhood of T_c as the frequency does not vary more for lower temperatures but it is very convenient to use to obtain accurate values for λ and T_c.

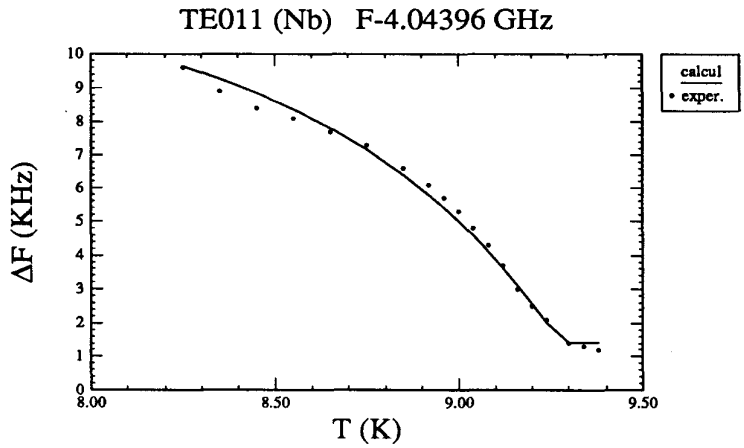

Figure 3 — Frequency shift as a function of temperature

Effect of impurities

The above analysis rests for an extremely pure material. In practical, we know that we have impurities and different kinds of defects that may disturb the superconducting state. As the RRR —which we take to be the ratio $[\sigma(T = T_c)/\sigma(T = 300K)]$ — is influenced by both impurities and lattice defects, we can assume it to be a suitable variable for the analysis of a non pure material.

We use, in connection with the two fluid model presented above, the BCS theory in order to extract the order parameter from the gap parameter Δ [see for example Ref. 12] :

$$\Omega_{BCS} = \frac{\Delta}{\Delta_0} \; th\left(\frac{\Delta}{2kT}\right) \tag{6}$$

where Δ is the solution of the self consistent equation

$$\frac{1}{NV} = \int_0^{\epsilon_c} \frac{th\left(\frac{1}{2kT}\sqrt{\epsilon^2 + \Delta^2}\right)}{\sqrt{\epsilon^2 + \Delta^2}} \, d\epsilon$$

Δ_0= gap parameter at T=0K

ϵ_c= cut-off energy of electron band

V = BCS interaction potential for superconducting pairs

We further assume that impurities, creating electron energy levels inside the superconducting gap, act as if a certain amount of electrons N_{n0} could not be

paired and remain normal conducting. Thus, the order parameter can be written as :

$$\Omega = \Omega_0 \, . \Omega_{BCS} \tag{7}$$

where $\Omega_0 = 1 - \left(\frac{N_{n0}}{N}\right)$ is the value of the order parameter at T=0K which is not equal to unity because of impurities.

Using this description, we have analysed all experimental results obtained at GECS (Saclay) on Nb superconducting cavities at a frequency of F=1.5GHz.

The RRR is deduced from RF measurements above the critical temperature. Almost all the cavities at Saclay are made of Nb sheet of bulk RRR ~200 and have a normal Q at T_c=9.35K around 100000 which gives a surface RRR of 100. For clarity, we have only kept two of these cavities (L1–05 and L1–04) which are typical. The other cavities reported have different RRR. The low RRR is generally due to surface contamination while the high RRR —namely the cavity L1–10 — is due to heat treatment (1500°C at Cornell) [Ref. 13]. The cavity L3–01 has been made with a Nb sheet having a better bulk RRR (~300).

We have fitted the experimental data of R_s(T) using the two parameters Ω_0 and (Δ_0/kT_c) and substracting a fixed external residual resistance of 3.5nΩ due to the static magnetic field in the cryostats(*). For all the cavities λ is taken to be 580Å and the critical temperature T_c=9.35K. One of the plots is drawn in figure 4 showing a close matching of our theory with experience.

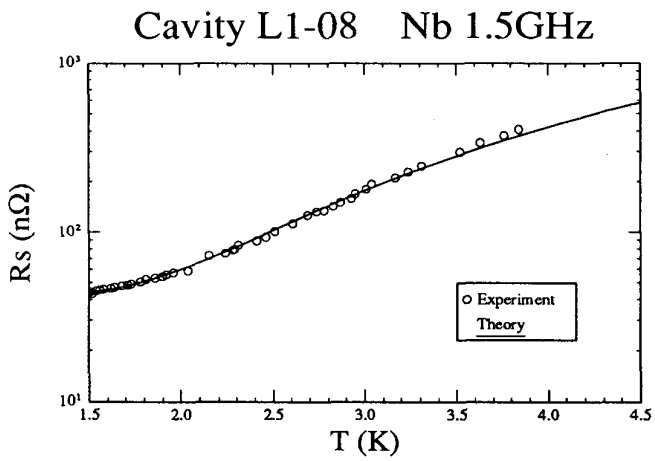

Figure 4 — Example of one fit obtained on a cavity

(*) The residual magnetic field measured is about 10mG and the residual resistance due to flux pinning has been measured to be roughly 0.35nΩ/mG.

All the results are summarized in the following table, the mean error being the departure from the experimental points.

CAVITY	Q(Tc)	RRR	(Δ_0/kT_c)	$(1-\Omega_0)$ x10^6	Rs(T=0K) (nΩ)	Mean Error (%)
L1-08	52800	31.15	1.228	12484	40.92	3.2
L1-10*	59000	38.9	1.401	9181	37.69	5.2
L5-08	88000	86.5	1.548	2193	21.48	5.1
L1-05	94420	99.6	1.577	1199	14.80	6.5
L1-04	96680	104.5	1.537	1086	14.23	3.6
L5-04	121600	165.	1.704	460	10.68	2.6
L3-01	135000	204.	1.733	93	5.29	10.0
L1-10	170000	323.	1.786	124	7.28	8.2

We clearly see that increasing RRR makes the gap parameter closer to the ideal value and simultaneously decreases the intrinsic residual resistance. If we plot the variation of the gap parameter (Δ_0/kT_c) (fig. 5) and the intrinsic residual resistance (fig. 6) as a function of (1/RRR), we can deduce the following behaviour

$$(\Delta_0/kT_c) = 1.835 - \frac{17.03}{RRR}$$

$$and \qquad R_s(T=0K) = \frac{1400}{RRR} \qquad (in \quad n\Omega)$$

which only holds for niobium at 1.5GHz but which, of course, could also be calculated at any other frequency.

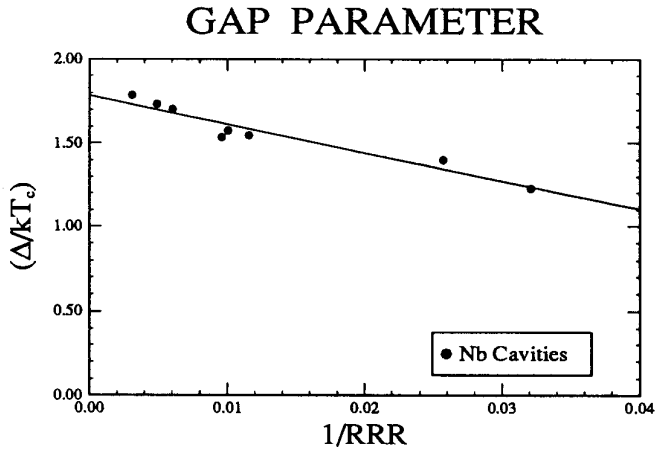

Figure 5 — Gap parameter as a function of RRR.

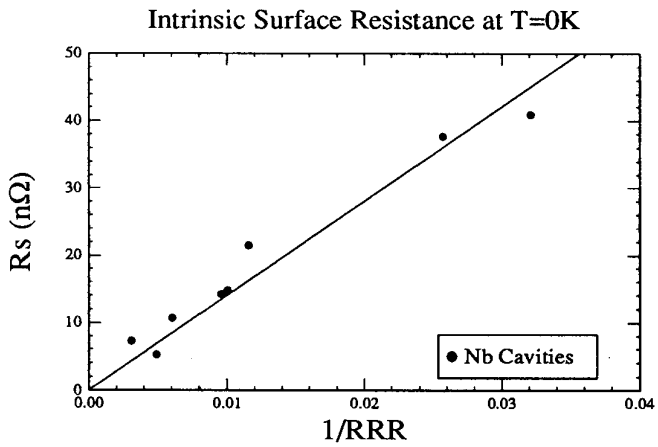

Figure 6 — Residual resistance as a function of RRR.

Conclusion

We have described a way to calculate the surface resistance of a superconductor using no other parameters than the penetration depth λ (at T=0K) and the mean collision time τ (related to the normal conductivity σ). This approach gives an excellent matching with experience no matter what temperature, frequency or even

material. From this, it turns out that there is a clear correlation between the residual losses and the normal conductivity (or RRR) of the material. That result is of great importance as, until now, the general belief was that the residual resistance was due to external and somewhat uncontrolled causes (chemical preparation, adsorbed gases, dirt, etc..). We have demonstrated that it is an *intrinsic* property for a given purity of the material. Consequently, the only way to approach the BCS value would be to further purify the material itself.

Finally, we note a quadratic variation of the residual normal electrons with the impurity concentration [$(1-\Omega_0) \propto 1/RRR^2 \propto c^2$, or, which is equivalent, $R_{res} \propto 1/RRR \propto c$]. This kind of variation is a feature which remains to be understood theoretically.

References :

[1] LONDON, Superfluids, Vol.1, New York, Wiley — (1950)

[2] B. AUNE & al., CE Saclay, France, Proceedings of the Linear Accelerator Conference, Albuquerque, USA (1990)

[3] GECS Laboratory, DAPNIA/SEA, CE Saclay, France

[4] GORTER & CASIMIR, Physica 1, 306 — (1934)

[5] Y. BRUYNSERAEDE & al., CERN Geneva, Physica 54, 137–159 North Holland Publishimg Co.— (1971)

[6] P. FLECHER &al, Karlsruhe, Germany, IEEE Trans. Nuclear Sci., USA, Vol. S16, N°3, 1018–1022 — (1969)

[7] H. HAHN & al., Brookhaven National Laboratory New York, USA, Journal Applied Physics, Vol. 39, N°6, 2606 — (1968)

[8] J. PIERCE, University of Virginia, USA, Journal Applied Physics, Vol.44, N°3, 1342 — (1973)

[9] S. SHRIDAR, California Institute of Technology, USA, Journal Applied Physics, Vol.63, N°1, 159 — (1988)

[10] Y. ZOHTA, Elechtrotechnical Lab., Japan, Physics Letters, Vol.31A, N°1, 21 — (1970)

[11] J. SLATER, Microwave Electronics, Van Nostrand Company, Princeton, New Jersey, USA

[12] TINKHAM, Introduction to Superconductivity, New York, Mc Graw-Hill — (1975)

[13] P. KNEISEL, Cornell University, USA, J. Less Common Metals 139, 179–188 — (1988).