Transient Heat Conduction Analysis in Superconducting Cavities*

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Abstract

A vector algorithm has been developed for solving two dimensional nonlinear transient diffusion equations. An analytical solution of transient heat conduction for the step point heat source is obtained, too. The calculations show that the steady state or thermal breakdown for sub-millimeter size defects can be reached in about one millisecond. The quench time decreases rapidly with increasing RF-field. The smaller defect grows more rapidly than the larger one. The quench may even occur within several micro-seconds if the cavity field is nearly doubled as in high power processing technique. The sub-micron size defect can reach very high temperature in a nanosecond, then the dynamic effect of the atomic diffusion may also be considered. We discuss the results of our calculation and give a comparison with experimental data.

1 Introduction

Defect induced thermal instability and electron field emission from point sources are the main reasons to limit the performance of superconducting cavities

The first calculations of thermal stability with a computer program by Padamsee [1] are useful for the understanding of the mechanisms and the dependents of the quench field on different parameters. Latest, the experiments in Cornell[2] show that by using high power processing, the field emission can be significantly reduced. In order to understand the high power processing and quench process, we developed a program for transient heat conduction analysis in superconducting cavities.

The behaviors of sub-millimeter size defects during quench process or high power processing can be describes quantitatively by the computer program.

A probable mechanism for field emissiom in cavities is proposed and the effects of high power processing on electron field emission are discussed qualitatively.

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2 Method

A finite cylindrical disk of thickness Z and radius R bounded by liquid helium on the lower surface and by a uniform of field on the top surface is assumed. A defect is on the center of the top surface, so the transient heat conduction equation can be written in the cylindrical coordinates as

$$rCv\frac{\partial T}{\partial t} = \frac{\partial}{\partial r}rK\frac{\partial T}{\partial r} + r\frac{\partial}{\partial z}K\frac{\partial T}{\partial z} + rP \qquad (2.1)$$

where Cv is heat capacity, K is thermal conductivity, P heat production from rf field and absorption by helium. The heat flow on the rim of the disk is assumed to be zero:

$$\frac{\partial}{\partial r}T(R,z,t) = 0: \quad z \leq Z \qquad (2.2)$$

The disk is divided into a series of $M \times N$ ring-like elements of average radius $(m - \frac{1}{2})\Delta r$, thickness $\Delta z = Z/N$, and width $\Delta r = R/M$. By using finite difference method, the temperature distribution $T(r, z, t)$ can be represented as:

$$T^l_{mn} \equiv T(m,n,l); \quad m = 1,\dots M; \quad n = 1,\dots N$$

and the partial differential equation (2.1) can be reduced to

$$[Cv](\{T\}^{l+1} - \{T\}^l) = ([CR] + [BZ])\{T\}^l + \{P\}^l \quad (2.3)$$

where { } denotes vector of length MN and [ ] denotes MN by MN matrix, [CR] contains all terms involving Δr^2 and [BZ] contains all terms involving Δz^2. The matrix [Cv] can be transformed into the unit matrix under transformation [U]:

$$[U][Cv][U]^T = [I]$$

Thus, eq.(2.3) can be reduced to

$$\{T'\}^{l+1} = [I + C + B]\{T'\}^l + \{P'\}^l \qquad (2.4)$$

where $[C + B] = [U][CR + BZ][U]^T$; $\{T'\}^l = [U]^{-1}\{T\}^l$ In order to avoid instability the implicit method must be used, i.e.

$$[I - C - B]\{T'\}^{l+1} = \{T'\}^l + \{P'\}^l \qquad (2.5)$$

For the small change of temperature distributions

$$[I-B-C] \sim [I-B][I-C]$$

so the eq.(2.5) can be rewritten as

$$[I-C][I-B]\{T'\}^{l+1} = [I+CB]\{T'\}^{l} + \{P'\}^{l} \quad (2.6)$$

Comparing with eq.(2.5) the calculations of eqs.(2.6) is greatly reduced. Besides, a vector algorithm for solving eq.(2.6) has been developed and proved to be efficient.

The magnetic field for heat production is

$$H(t) = H_0 + (\Delta H/\Delta t)t \quad (2.7)$$

where the initial value H_0 and the velocity of magnetic field increasing $\Delta H/\Delta t$, including $\Delta H/\Delta t = 0$, can be input in the program.

When the steady state approaches, we double the time step length then the steady state can be reached easily; while thermal breakdown occurs, the time step length is reduced by ten times, so the quench process can be calculated smoothly until the heat production is larger than 10 watts.

By using Green function for diffusion[3], the temperature distribution for the step point heat source can be deduced as

$$T(r,\tau) = erfc(\omega)/(4\pi D r) \quad (2.8)$$

where the thermal diffusivity $D = K/Cv$; $\omega = r/(2\sqrt{D\tau})$ and $erfc(\omega)$ is the error function.

When $\tau \to \infty$, $\omega \to 0$, thus $erfc(0) = 1$. Therefore, the equilibrium state can be written as

$$T_{eq}(r) = 1/(4\pi D r) \quad (2.9)$$

From eq.(2.10) the temperature gradient $\nabla T(r,\tau)$ can be written as

$$\nabla T(r,\tau) = -tgf(\omega)/(4\pi D r^2) \quad (2.10)$$

where the coefficient function of temperature gradient

$$tgf(\omega) = (2/\sqrt{\pi})\omega \exp(-\omega^2) + erfc(\omega)$$

The diffusion depth d is defined as

$$d \equiv \sqrt{D t_d} \quad (2.11)$$

where t_d is the correspondingly required diffusion time. Thus, for the step point source at diffusion depth d the heat current $q = K\nabla T$ reaches 92% of its steady state value and temperature reaches near half of the steady state value.

The heat production of defects sensible to the magnetic field is

$$P = \frac{1}{2}H^2 R_D \pi a^2 \quad (2.12)$$

where R_D is the defect resistance; a is the radius of defects.

For the propagating defect(the defect can create a n.c. ring and assume its radius is r_n), the equilibrium temperature distribution at the n.c. ring ($r \le r_n$) can be written as

$$T_{eq}(r) = \frac{(R_D - R_N)a^2 + R_N r^2}{4Kr} H^2 + T_{DB} \quad (2.13)$$

where R_N is the resistance of normal state, T_{DB} is the defect temperature of at the outer surface.

It can be easily proved that when $R_D > R_N$ and

$$r = r_c = a\sqrt{R_D/R_N - 1} \quad (2.14)$$

the temperature reaches its minimum:

$$T(r_c) = \frac{R_N H^2 r_c}{2K} + T_{DB} \quad (2.15)$$

Since the radius r_c must be larger than the defect radius a, to observe the ring we must have the conditions:

$$R_D \ge 2R_N;\ T(a) \ge T_c \ and\ r_n \le r_c \quad (2.16)$$

where T_c is the superconducting transition temperature. When r_n is larger than r_c, the temperature will increase with increasing the radius of n.c. ring, so the quench occurs when the temperature at the critical radius r_c reaches T_c.

3 Results and Discussion

3.1 Sub-millimeter size defects

These tests were carried out on 3GHz Niobium cavities with Wasserbach Nb ($RRR \ge 300$), wall-thickness $Z = 1.5mm$. In calculation we assume defect radius $a = 0.05mm$, n.c. resistance of Nb $R_N = 2 \cdot 10^{-3}\Omega$ $T_B = 1.5K$, $\Delta z = \Delta r = 0.05mm$, $M = 40$, and $N = 30$. The heat production, the Kapitza conductivity functions, thermal conductivity and BCS surface resistance of Nb are totally copied from Padamsee's program[1]

The transient temperature distribution of point defects can be divided into two stages:

1. The heat current produced from the defect gradually reaches the outer surface of the cavity. In this term the temperature gradients are gradually established and isotherms remain about spheres. this term lasts about 1 μsec.

2. the heat current will get through the cavity wall gradually until the heat current into cooling bath is nearly equal to the production of the heat current by the defect. The defect temperature at the outer surface increases gradually. In this term isotherms are deformed to adapt the boundary conditions. This term can last about 1 msec.

When the temperature at the critical radius r_c(2.14) reaches T_c, the thermal breakdown will occur. The smaller defect grows more rapidly than the larger one because r_c is smaller for the smaller defect.

At the high field (900G), the thermal breakdown occurs at the stage 1. The small defect can cause the thermal breakdown. For the 0.25w defect , 2.34 micro-joules can cause the thermal breakdown in $9.2\mu sec$; while for the 2.5w defect , 10 micro-joules can cause the thermal breakdown in $4\mu sec$, so the impact of pulsed high energy electron beams or laser beams is not easy to cause thermal breakdown.

At the low field (400G), the thermal breakdown occurs at the stage 2. In this case, only large defect($r_c > 0.4mm$) can cause the thermal breakdown.

In high power processing experiments[2], the cavity was limited by available power being dissipated into field emission ($E_{pk} = 18.9MV/m$, $P_{diss} = 12.1watts$). After 300 pulses of $100\mu sec$ duration. $E_{pk} \sim 33.5MV/m$. the field emission was significantly reduced and the cavity reached $Epk = 19.5MV/m$ limited by a thermal breakdown.

During high power processing. the field increasing velocity $\Delta H/\Delta t$ is about $5G/\mu sec$ and E_{pk} can be nearly double. However, according to our calculation, Epk can only increases about 10%. If Epk is nearly double, then $\Delta H/\Delta t$ must be larger than $100G/\mu sec$. Therefore, we can estimate that there is no sub-millimeter size defect in the cavity, otherwise high power processing can not be carried out. These defects may have sub-micron sizes and should have more complicated structures.

3.2 Sub-micron size defects

The experiments[2,4] pointed out that the field emission in cavity usually occurs at a non-metallic sub-micron size defect. The the graphite and MoS_2 can be strong field emitters.

Field emission is usually considered as a quantum tunnelling effect by Fowler-Nordheim(FN)[5] and the emission is temperature-independent.

From the point of transient heat analysis, if the emission occurs at low temperature by quantum tunnelling effect, then it will occur immediately and high power processing will not affect the emission. However,the time constant for defect growth from high power processing experiments is about $0.1msec$, so that the field emission in cavities must occur least at high temperature. caused by electron and phonon interactions, even some atomic diffusion may be included to enhance the field emission.

The temperatures and temperature gradients around the particle are determined by (2.8) and (2.10). They are respectively proportional to r^{-1} and r^{-2}, and depend on the average thermal conductivity K. Therefore, The submicron size defect can reach very high temperature in a nanosecond. This means that the temperature of submicron size defects depends on the RF phase, thus we can not distinguish the thermionic emission from Fowler-Nordheim's by experiments[6]

If the size of the particle a is nearly to the mean free path of conduction electrons ℓ, then the heat production of the particle can be approximately written as

$$P = nE\ell\pi a^2 \tag{3.1}$$

where n is the electron density in the conduction band. The field enhancement factor β in FN emisson can be determined by the temperature and its gradient of the particle. Thus, this emission in cavities can still be described by FN emission formulas.

The thermal diffusivity of Nb decreases greatly with temperature ($D = 10^{-1}m^2/sec$ at 1.5 K; $D = 10^{-5}m^2/sec$ at 20 C). It is possible for the time constant of the defect growth to reach 0.1msec if the emission occurs at high temperature.

The drive force for diffusion is a chemical potential gradient. The chemical potential for the impurity can be written as

$$G = kTln(\gamma C) \tag{3.2}$$

where C is the impurity concentration and γ is the activity coefficient which represents the interaction between the impurity and the matrix. If C is the electron concentration, then γ and G are the electron activity coefficient and electron chemical potential, respectively. The electron emission is similar to the impurity segregation, but the temperature gradient may play an important role in the electron emission.

In high power processing the field strength E can increase, so the temperature and temperature gradient can also increase. Increase of temperature may eliminate the emission just like high temperature treatment. In the cavity the smaller defect grows more rapidly than the larger one, so the small defect is easy to be eliminated by the high power processing, while the large one still remains. This may be the reason why thermal breakdown often occurs after high power processing.

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