# Analysis of Global Thermal Instability and Precipitates using TRANS_HEAT Code

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## Abstract

The TRANS_HEAT code has been used to analyze the transient cavity behaviors affected by the random distributed precipitates. The precipitates can be regarded as a kind of the residual resistance. For defect-free case, the steady state and quench can be reached in about 1 msec. $Q_0$ can reduce by two orders for Wasserbach Nb(RRR=300) and by three orders for Heraeus Nb(RRR=100) before quench; to keep $Q_0 \geq 2 \cdot 10^9$ field can reach 1300G for Wasserbach case and 900G for Heraeus case. The quench field is not very much affected by the precipitate concentration, while the time to reach quench $t_q$ reduces rapidly with the concentration. $t_q$ can reduce by one order when $Q_D = 1.4 \cdot 10^6$. From calculation we get $\Delta(lgt_q)/\Delta H = -3 \cdot 10^{-3} G^{-1}$. A comparison between our calculation and experimental data is discussed.

## 1 Introduction

Hydrogen absorbed in niobium during chemical polishing and precipitated as niobium hydrides on the surface of the superconducting cavity will reduce severely the Q value of the cavity[1]. These precipitates are about micron sizes and random distributed[2]. They have higher resistivity and even become non-superconducting during operation. The distances between these particles are about several microns, so these particles can cause almost uniform heating of an area of superconducting surface(~1mm). The denser the particles the higher the temperature, then the temperature distribution is still non-uniform in a larger area. When temperature exceeds $T_c$, the thermal breakdown occurs. This kind of thermal instability, called global thermal instability, was first observed and discussed by Padamsee's group[3]. In this paper we will use TRANS_HEAT code to discuss the global thermal instability in different precipitate density cases, including the defect-free case.

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## 2 Method

A finite cylindrical disk of thickness Z and radius R bounded by liquid helium on the lower surface and by a uniform of field on the top surface is assumed. A precipitate is on the center of the top surface, so the transient heat conduction equation can be written in the cylindrical coordinates as

$$rCv\frac{\partial T}{\partial t} = \frac{\partial}{\partial r} rK \frac{\partial T}{\partial r} + r\frac{\partial}{\partial z} K \frac{\partial T}{\partial z} + rP \qquad (2.1)$$

where $Cv$ is heat capacity, $K$ is thermal conductivity, $P$ heat production from rf field and absorption by helium. The heat flow on the rim of the disk is assumed to be zero:

$$\frac{\partial}{\partial r} T(R, z, t) = 0; \quad z \leq Z \qquad (2.2)$$

This is similar to the case of uniform precipitates in which the distance between precipitates is equal to 2R.

In defect-free case

$$Q_{BCS} = \frac{G}{R_{BCS}} \qquad (2.3)$$

where $R_{BCS}$ is the surface resistance predicted by BCS theory (including the residual resistance); G is the geometry constant of the cavity. For spherical cavity with beam holes $G = 280\Omega$.

The average surface resistance of defect

$$R_D^{av} = R_D (\frac{a}{R})^2 \qquad (2.4)$$

with $R_D$ the surface resistance of the defect, a the radius of the defect. Thus, the quality factor $Q_0$ of a cavity can be determined by

$$\frac{1}{Q_0} = \frac{1}{Q_D} + \frac{1}{Q_{BCS}} \qquad (2.5)$$

where

$$Q_D = \frac{G}{R_D^{av}} \qquad (2.5)$$

The $R_{BCS}$ reduces by two orders when temperature increases from 1.5K to 4.5K. The thermal conductivity and heat capacity also change rapidly during this temperature range. Therefore, the partial differential equation (2.1) is highly nonlinear and the mathematical instability is easy to happen during the solving process. In TRANS_HEAT the deviation of the temperature distribution between two iteration EX is used to determine automatically the appropriate the length of the time step. However, the value of EX must be input. If EX is too large, instability will happen and the execution of the program will be stopped. In this case a prompt to reduce EX will be printed out.

In the defect-free case, the heat source is plane and the heat conduction is one dimensional. By using Green function for diffusion[4], the temperature distribution for the step plane heat source can be deduced as

$$T(r,\tau) = F(\omega)\frac{2P\sqrt{D\tau}}{\sqrt{\pi}K} \qquad (2.6)$$

where the thermal diffusivity $D = K/Cv$; $\omega = r/(2\sqrt{D\tau})$ and

$$F(\omega) = \exp(-\omega^2) - \sqrt{\pi}\omega \cdot erfc(\omega) \qquad (2.7)$$

The temperature gradient can be written as

$$\nabla T(r,\tau) = \frac{P}{K} erfc(\omega) \qquad (2.8)$$

where $erfc(\omega)$ is the error function. Comparing with the temperature evolution in the point heat source[6]:

$$T(r,\tau) = erfc(\omega)/(4\pi D r). \qquad (2.9)$$

the temperature evolution in the plane heat source is much slower.

These predictions are in agreement with the calculations of the code.

## 3 Results and Discussion

The heat production, the Kapitza conductivity functions, thermal conductivity and BCS surface resistance of Nb are totally copied from program HEAT[5].

The calculations are carried out on 3GHz Niobium cavities with Wasserbach Nb ($RRR \geq 300$), $R_{res} = 30n\Omega$, annealed Niobium for Kapitza resistance, wall-thickness $Z = 1.5mm$, $T_B = 1.5K$, and number of thickness step $N = 30$. In calculation we assume R=0.01mm, the number of radius step M=4 and 10, and $R_D = 2 \cdot 10^{-5}$ and $2 \cdot 10^{-4}\Omega$. They are respectively equivalent to the cases: $Q_D = \infty$, $1.4 \cdot 10^9$, $2.24 \cdot 10^8$, $1.4 \cdot 10^8$.

The $Q_0$ and $T_{max}$ vs. H for different $Q_D$ at steady state are shown in Fig. 1 and 2, respectively. For defect-free case, $Q_0$ decreases and surface temperature $T_s$ increases mainly above 1300G. $Q_0$ increases by two orders when field increases to 1400G. For the cases of different $Q_D$, $Q_0$ decreases rapidly with $Q_D$ at the small field, but they approach each other when field is near to 1400G. $\Delta T_s/\Delta H$ increases with decreasing $Q_D$, while $T_s$ will approach to 4.6K when field is near to 1400G.

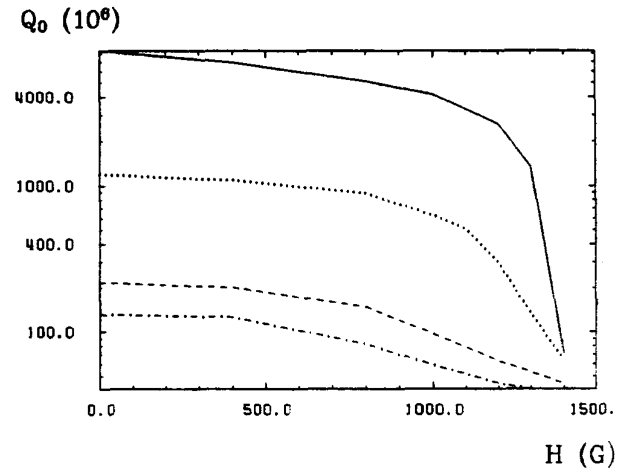


Figure 1: *$Q_0$ vs H for different $Q_D$*
full line: $Q_D = \infty$; dotted line: $Q_D = 1.4 \cdot 10^9$; dash line: $Q_D = 2.24 \cdot 10^8$; dash dotted line: $Q_D = 1.4 \cdot 10^8$

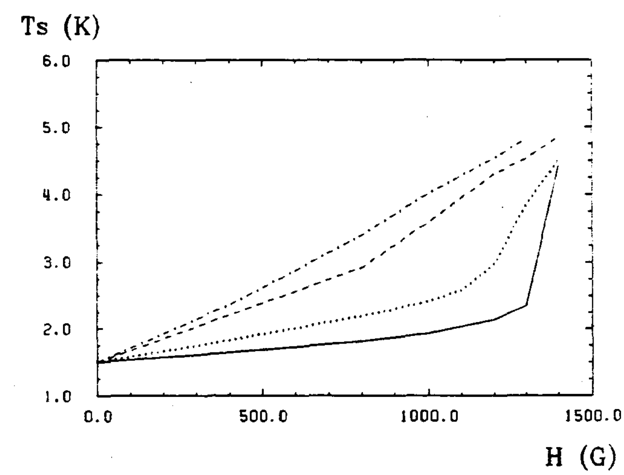


Figure 2: *surface temp.$T_s$ vs H for different $Q_D$*
full line: $Q_D = \infty$; dotted line: $Q_D = 1.4 \cdot 10^9$; dash line: $Q_D = 2.24 \cdot 10^8$; dash dotted line: $Q_D = 1.4 \cdot 10^8$

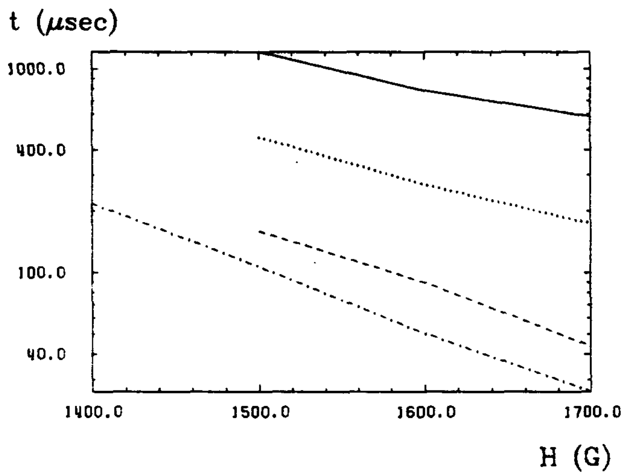


Figure 3: *quench time $t_q$ vs H for different $Q_D$* full line: $Q_D = \infty$; dotted line: $Q_D = 1.4 \cdot 10^9$; dash line: $Q_D = 2.24 \cdot 10^6$; dash dotted line: $Q_D = 1.4 \cdot 10^6$

For the case of $Q_D = 1.4 \cdot 10^6$ quench occurs at 1400G, and for other cases quenches occur at 1500G.

Temperature distributions for all cases are plane. even for the case of $Q_D = 1.4 \cdot 10^6$, the temperature increasing at defect on very beginning($1\mu$sec) is less than 0.01K. Thus we can use the average method to discuss this kind of defects and the precipitates can be regarded as a kind of residual resistance.

The precipitate distribution is still not uniform in a larger area, so many kinds of precipitates ($Q_{D,i}$; i=1,2,...,N)can exist in the cavity, then

$$\frac{1}{Q_0} = \frac{1}{Q_{BCS}} + \sum_{i=1}^{N} \frac{a_i}{Q_{D,i}} \quad (3.1)$$

where $a_i$ is the area fraction of ith kind of precipitates.

The time to reach quench $t_q$ vs. H for different $Q_D$ cases are shown in Figure 3.

The main characteristic of the defect-free case is that the temperature evolution is much slower. The time to reach quench $t_q$ for defect-free case at 1500G is 1.2msec. $t_q$ decreases rapidly with decreasing $Q_D$: $t_q$=0.1msec. when $Q_D = 1.4 \cdot 10^6$ and 1500G. For all cases, $t_q$ decreases with H and $\Delta(lgt_q)/\Delta H = -3 \cdot 10^{-3} G^{-1}$.

The global thermal heating depends mainly on the niobium thermal conductivity. If Heraeus Nb(RRR=100) is used instead of Wasserbach Nb(RRR=300) in the above-mentioned example, the $Q_0$ and $T_s$ vs H for the defect-free case are presented in Table 1.

In defect-free case(Heraeus Nb), the quench happens at 1200G and $Q_0$ reduces to $2 \cdot 10^6$ before quench.

If we require that $Q_0$ be large than $2 \cdot 10^9$, then the surface temperature $T_S$ must be lower than 2.1K. Thus the thermal conductivity in this temperature range (1.5-2.2K) is very important. Comparing with that of Wasserbach Nb the thermal conductivity of Heraeus Nb reduces by a factor of ~ 8, so the field can reach 1300G for Wasserbach case and only reach 900G for Heraeus case.

**Table 1: $Q_0$ and $T_S$ vs H for defect-free case**

| H(G) | 400 | 800 | 900 | 1000 | 1100 | 1200 |
|---|---|---|---|---|---|---|
| $Q_0(10^7)$ | 683 | 377 | 270 | 60 | 6 | 0.2* |
| $T_S$(K) | 1.66 | 1.94 | 2.07 | 4.64 | 5.17 | 5.81* |

* just before quench(Heraeus Nb)

For defect-free case the quench is not easy to happen because $Q_0$ can reduces by two orders(Wasserbach Nb) even three orders(Heraeus Nb). For a global thermal instability observed in Cornell University[3], the quench occurs at 1250G and $Q_0 \sim 0.8 \cdot 10^{10}$. So we can expect that the precipitates with $Q_D = 1.4 \cdot 10^6$ may exist in some part of equater region of the cavity, which will constraint the maximum field achievable during high pulsed-power processing. The outer wall temperature calculated by code is $\sim 3.4K$, which is much higher than the experiments[3]. The reason may be that it is difficult to satisfy the one dimensional conduction condition.

In our calculation we assume that the size of the precipitate particle is about $1\mu$. The size may be smaller than that in many cases. When the size of the particle is smaller than the mean path of conduction electron. the heat production may become proportional to the field instead of field square. This may be the reason why the $Q_0$ curves decrease rapidly below $E_{acc} = 1MV/m$ for the cavities in DESY[1].

## References

[1] G.Enderlein et al., "Investigation on hydrogen contamination of superconducting cavities", IEEE Particle Accelerator Conference, San Francisco, May 6-9 (1991)

[2] T.Schober and H.Wenzel, in "Hydrogen in Metalls II" Ed. by G.Alefeld. J.Völkl, Springer, Berlin 1978 pp.11-71

[3] J.Graber et al., "First Observation of a Global Thermal Instability in a Superconducting RF Cavity", CNLS 91-1061 (1991)

[4] P.M.Morse and H.Feshbach, Methods of Theoretical Physics, McGraw-Hill, New York (1953) P.857

[5] H.Padamsee, CERN/EF/RF 82-5 (1982)

[6] X.Cao and D.Proch, "Transient Heat Conduction Analysis in Superconducting Cavities", IEEE Particle Accelerator Conference, San Francisco, May 6-9 (1991)