Longitudinal Dynamics of trains of bunches in Accelerating Structures

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Abstract

The case of interaction of not fully relativistic multi-bunch beams with the fundamental and higher order modes of a cavity is not yet covered by existing codes, nevertheless it is a fundamental problem in the design of RF guns or FEL and Collider injector cavities. A simple model that couples Newton and Maxwell equations, taking into account also space charge, beam loading and build-up effects of higher order modes under beam-tube cutoff frequency, is presented. This approach is intended to fill that gap, avoiding relativistic approximation ($\beta(t) \leq 1$). It uses a current density description of the beam and a slowly varying envelope approximation for the time evolution of the modes amplitude. A fast running code (HOMDYN), based on this model has been developed and the application to a 4-cell cavity, 500 MHz frequency, is illustrated.

Table 1 - Symbol definitions

A_n = mode amplitude	L = bunch length
$\hat{e}_n$ = mode spatial distribution on axis	q = bunch charge
ω_n = mode radian frequency	$z_{h,t,b}$ = head, tail and barycenter coordinates
ϕ_n = mode phase	R = bunch radius
α_n = mode complex amplitude	c = light velocity
	m_o = electron rest mas
Q_n = quality factor of the mode	ß = average velocity/light velocity
U_n = mode stored energy	J = beam current density
E_o = peak electric field	N_b = n. of bunches in train
C.C.= Complex Conjugate	ν_r = repetition rate

I. INTRODUCTION

When treating the evolution of high charge, not fully relativistic electron bunches in RF fields of an accelerating cavity, it is necessary to take into account also the field induced by the beam in the fundamental and higher order modes, and the variation of bunch sizes due to both the external fields and space charge.

For single bunches the problem has been already tackled using PIC codes, which describe the bunch as an ensemble of particles and track their motion, coupled to the E.M. field propagation. The case of long bunch trains would consume an unbearable amount of computer time if treated by a mere extension of the single bunch case.

We have therefore devised a simple model that uses a current density description of the beam and slowly varying envelope approximation (SVEA) for the evolution of the cavity modes. The present version deals only with TM monopole modes: an extended version comprehensive of dipole modes is under development.

Motion and field equations are coupled together through the driving current term.

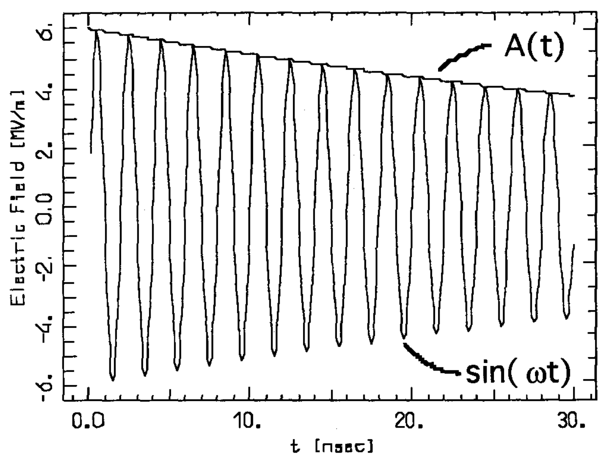

Fig. 1 - Only the slowly varying envelope effects are taken into account in the field equation.

The SVEA approximation supposes small field perturbations produced by any single bunch, that add up to give an envelope of any field mode slowly varying on the time scale of its period (Fig. 1). Because the characteristic cavity reaction time is of the order of $\tau = 2Q/\omega >> T$ we fulfill the SVEA hypothesis. This approximation allows to reduce the numerical computing time.

II. THE FIELD EQUATIONS

Expressing the electric field E as a sum of normal orthogonal modes [1] with complex variable:

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$$E(z,t) = \sum_n A_n(t)\,\hat{e}_n(z)\,\sin(\omega_n t + \phi_n(t))$$

$$= \sum_n \left(\alpha_n(t)\,\hat{\eta}_n(z)\,e^{i\omega_n t} + \alpha_n^*(t)\,\hat{\eta}_n^*(z)\,e^{-i\omega_n t} \right)$$

where $\alpha_n=(A_n/2)e^{i\phi_n}$, $\hat{\eta}_n(z)=\hat{e}_n(z)/i$, $\hat{e}_n(z)=\hat{\mathbf{e}}_n(r=0)$ and:

$$\nabla^2\hat{\eta}_n = -k_n^2\hat{\eta}_n \qquad \int_v \hat{\eta}_n \cdot \hat{\eta}_m^*\,dv = \delta_{nm}$$

the equation for the electric field complex amplitude α_n driven by a beam current distribution J(z,t) in the cavity is :

$$\frac{d^2\alpha_n}{dt^2} + \left(2i\omega_n + \frac{\omega_n}{Q_n}\right)\frac{d\alpha_n}{dt} + i\frac{\omega_n^2}{Q_n}\alpha_n =$$

$$= -\frac{e^{-i\omega_n t}}{\varepsilon}\int_v \left(\frac{\partial J}{\partial t}\cdot\hat{\eta}_n^*\right)dv$$

with the normalization relations:

$$\left|\alpha_n(t)\right| = \sqrt{\frac{U_n(t)}{2\varepsilon}} \qquad \hat{e}_n(z) = \sqrt{\frac{\varepsilon}{2U_{n,o}}}\,\hat{E}_{n,o}(z)$$

Applying the SVEA approximation hypotheses:

$$\frac{d\alpha_n}{dt} << \omega_n\alpha_n \qquad \frac{d^2\alpha_n}{dt^2} << \omega_n^2\alpha_n$$

we can neglect the second order derivative and we obtain the first order equation for each mode:

$$\frac{d\alpha_n}{dt} + \left(1+\frac{i}{2Q_n}\right)\frac{\omega_n}{2Q_n}\alpha_n =$$

$$= -\frac{e^{-i\omega_n t}}{2\omega_n\varepsilon}\left(1+\frac{i}{2Q_n}\right)\int_v\left(\frac{\partial J}{\partial t}\cdot\hat{e}_n\right)dv \;+$$

$$-\frac{1}{2\omega_n\varepsilon}\left(1+\frac{i}{2Q_n}\right)K\,e^{i(\Omega_{1,n}t + \psi_k)}$$

The last term accounts for a feeding sinusoidal current of amplitude K, phase ψ_k and detuning shift $\Omega_{1,n}= (\omega_1 - \omega_n)$, representing a power supply.

The beam current density term can be written as follows:

$$\int_v \left(\frac{\partial J}{\partial t}\cdot\hat{e}_n\right)dv = \frac{q\beta_b c}{L}\left(\hat{e}_n(z_h)\frac{dz_h}{dt} - \hat{e}_n(z_t)\frac{dz_t}{dt}\right) +$$

$$+\frac{qc}{2}\left(\hat{e}_n(z_h) + \hat{e}_n(z_t)\right)\frac{d\beta_b}{dt}$$

The evolution of the field amplitude during the bunch to bunch interval is given by the analytical solution of the equation driven by the generator only, which connects successive numerical integrations applied during any bunch transit.

III. THE BEAM EQUATIONS

The basic approximation in the description of beam dynamics lays in the assumption that each bunch is represented as a uniform charged cylinder, whose length and radius can vary under a self-similar time evolution, i.e. keeping anyway uniform the charge distribution inside the bunch. The present choice of a uniform distribution is dictated just by sake of simplicity in the calculation of space charge and HOM contributions to the beam dynamics. A further improvement of the model to include gaussian distributed bunches is under way. According to this assumption, and to the general hypothesis that the space charge and HOM effects on beam dynamics are perturbative, we can write, under a paraxial approximation, the equations for the longitudinal motion of the bunch barycenter:

$$\frac{dz_b}{dt} = \beta_b c$$

$$\frac{d\beta_b}{dt} = \frac{e}{m_o c\gamma_b^3}\sum_n\left(\alpha_n\hat{\eta}_n e^{i\omega_n t} + \alpha_n^*\hat{\eta}_n^* e^{-i\omega_n t}\right)$$

The evolution of the bunch radius R is described according to a recently proposed envelope equation [2], including RF-focusing and space charge effects, transformed into the time-domain:

$$\frac{d^2R}{dt^2} - \frac{dR}{dt}\frac{d\beta_b}{dt} g(\beta_b) + \frac{R}{4\gamma_b^2}\left(\left(\frac{d\beta_b}{dt}\right)^2 f(\beta_b) + K_r\right) +$$

$$- \frac{Ic^2}{I_A R\beta_b\gamma_b^3} - \frac{\varepsilon_n^2 c^2}{R^3\gamma_b^2} = 0$$

$$g(\beta) = \frac{1 - \beta\gamma^2 - 1/\gamma}{\beta} \qquad f(\beta) = \frac{1}{\beta^2}\left(3\beta^2\gamma^2 - 2\gamma^3(1+\beta) + \gamma^4 - 1 - \frac{2}{\beta}\left(1 - \beta^2\gamma^4\right)\right)$$

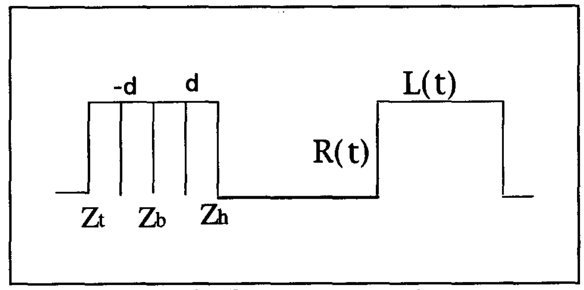

Fig. 2 - Bunch representation

where I_A is the Alfven current (17 kA), ε_n the rms normalized beam emittance and the RF average focusing gradient K_r is given by:

$$K_r = \frac{1}{8}\left(\frac{e E_{acc}}{\gamma m_o c^2}\right)^2 \sum_{s=2}^{\infty} \left\{ a_{s+1,1}^2 + a_{s-1,1}^2 + \right.$$

$$\left. + 2\, a_{s+1,1}\, a_{s-1,1} \cos(2\phi_1(t)) \right\}$$

where the sum run over the spatial harmonic coefficients $a_{s,1}$ of the mode form factor $\hat{e}_1(z)=\Sigma_s a_{s,1}\cos(s\, k_1 z)$, (n=1 for the fundamental π-mode).

The bunch lengthening is simply given by the head-tail velocity difference:

$$\frac{dL}{dt} = c\left(\beta_h - \beta_t\right)$$

The energy distribution inside the bunch is derived by specifying the energy associated to each slice (Fig. 2) located at a distance d from the bunch barycenter (z_b) and adding to the space charge effects as given by [3], the first order component coming from fundamental and HOM modes:

$$\frac{d\gamma_d}{dt} = \frac{e}{m_o c}\beta_d \left\{ \Sigma_n \left(\alpha_n(t)\, \hat{\eta}_n(z_b + d)\, e^{i\omega_n t} + C.C. \right) + \right.$$

$$\left. + \frac{q\, d}{2\pi\varepsilon_o \gamma_b R^2 L^2}\left(\sqrt{(\gamma_b L)^2 + R^2} - (\gamma_b L + R)\right) \right\}$$

IV. EXAMPLE: 4-CELL CAVITY

To test the validity of the simulation we have applied the computation to the case of a 500 MHz 4-cell resonator, computing the mode frequencies and field distributions by SUPERFISH and OSCAR2D codes. We take into account the fundamental mode family only (Fig.3), that is a drastic reduction in the field representation especially for transient behaviors but enough to have an idea of the beam-loading effects at least. Moreover space charge effects have not yet been checked with respect to other codes, and we neglect this effects in the following results.

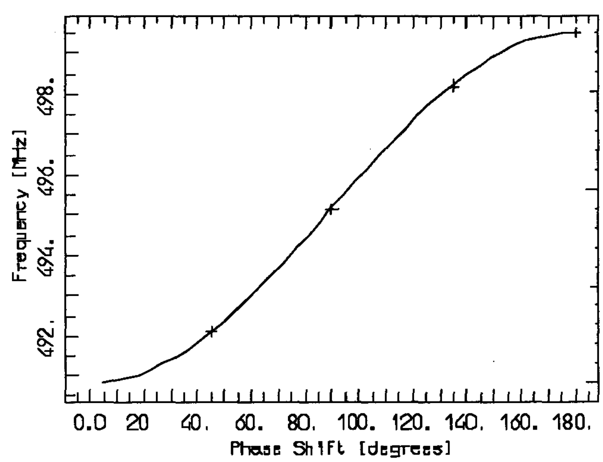

Fig. 3 - Dispersion curve of the fundamental mode family: ω_π=499.51, $\omega_{3\pi/4}$=498.20 , $\omega_{\pi/2}$=495.12 , $\omega_{\pi/4}$=492.09

The beam parameter are the same as for LISA beam at the entrance of the first Superconducting 4-cell cavity and are listed in Table 2 together with other data:

We have considered three cases with different starting conditions.

In the first case (Fig. 4-8) the beam has been injected in the cavity 20 msec after the RF pulse has been turned-on, that corresponds to 7 filling time constants (τ=2.9 msec). The dramatic beam loading effect induces a 15 % energy spread in the train.

In the second case (Fig. 9-12) we show a possible beam loading compensation by injecting the beam on the rising part of the accelerating voltage, 3 msec after the RF pulse has been turned-on.

In the third case (Fig. 12-17) the beam has been injected in an empty cavity and the voltage is induced by the beam current itself. This case can be interesting for the experimental study of beam-cavity interaction.

Notice that in some figures the cavity shape is also indicated and the 11 superimposed curves are plotted every 5000 bunches as functions of the bunch barycenter position z_b.

Table 2

Initial Energy	T= 1 MeV
Relativistic Factor	$\gamma = 3$
Number of bunches	$N_b = 50000$
Bunch Charge	q = 40 pC
Bunch Length	L = 8 psec
Train Length	1 msec
Repetition Rate	$\nu_r = 50$ MHz
Average Current	<I> = 2 mA
Accelerating Voltage	$V_{acc} = 6$ MV
External Q	$Q = 4.5 \times 10^6$
Cavity Length	1.7 m

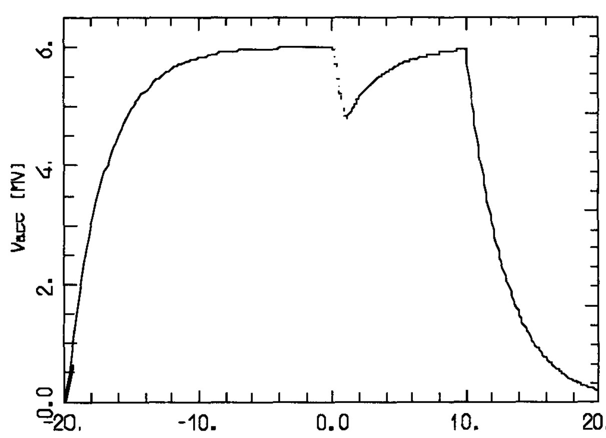

Fig. 4 - Accelerating Voltage versus time.

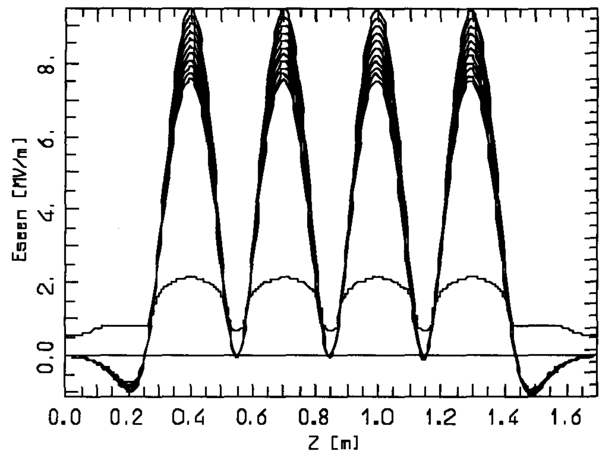

Fig. 5 - Total electric field seen by different bunches (superposition of the 4 resonant modes).

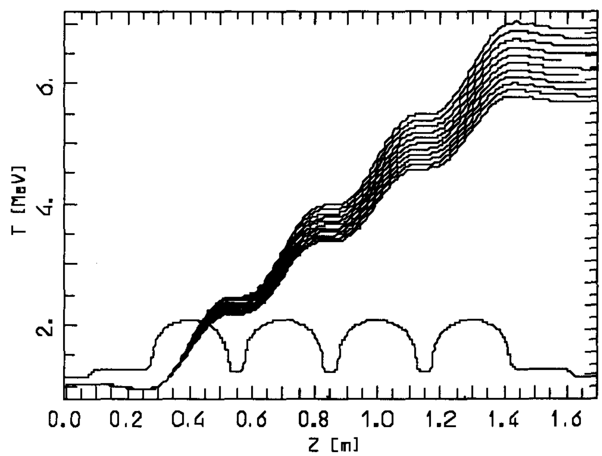

Fig. 6 - Energy of different bunches in the train. Notice the large train energy spread (15%) at the exit of the cavity.

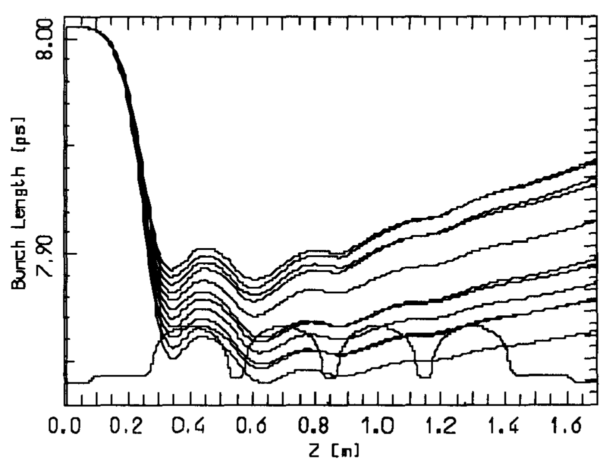

Fig. 7 - Bunch length L: the initial bunch compression is due to the decelerating fringing field seen by bunches in the tube.

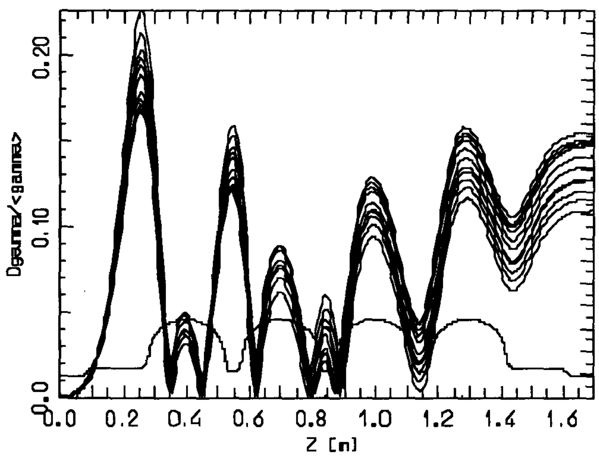

Fig. 8 - Bunch energy spread [%].

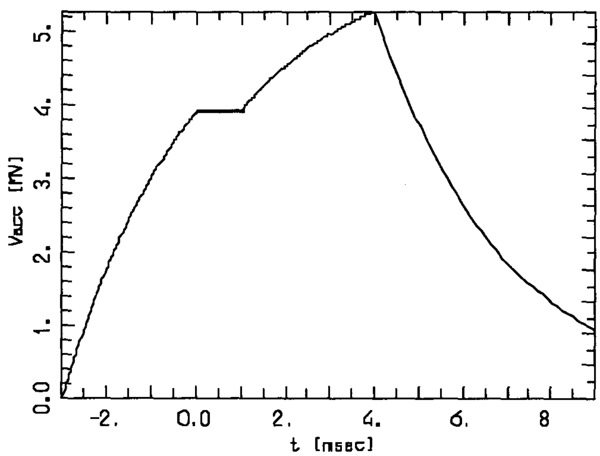

Fig. 9 - Accelerating Voltage versus time in case of beam loading compensation.

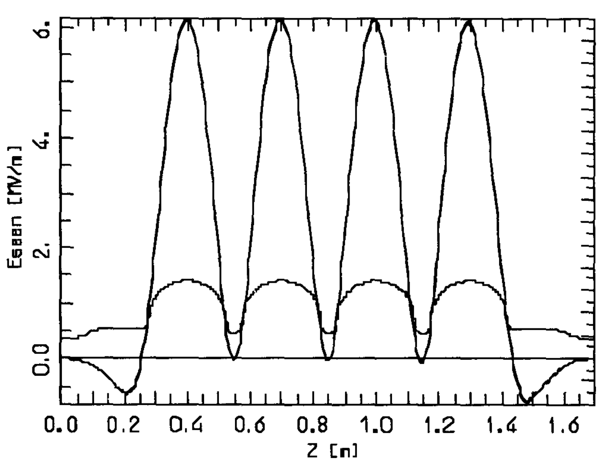

Fig. 10 - Total electric field seen by different bunches. In this case all the bunches see the same field.

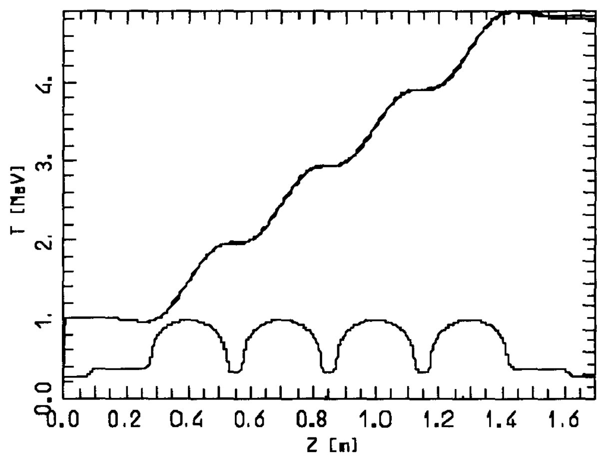

Fig. 11 - Energy of different bunches in the train. Notice the strong reduction of train energy spread at the exit of the cavity.

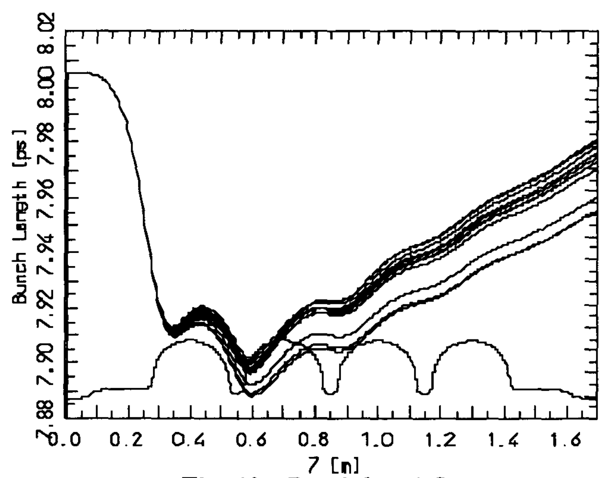

Fig. 12 - Bunch length L.

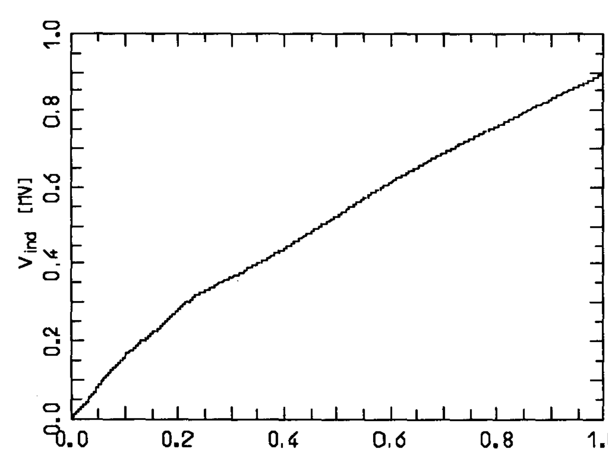

Fig. 13 - Induced Voltage versus time. Notice the initial oscillations due to the transient phase variations.

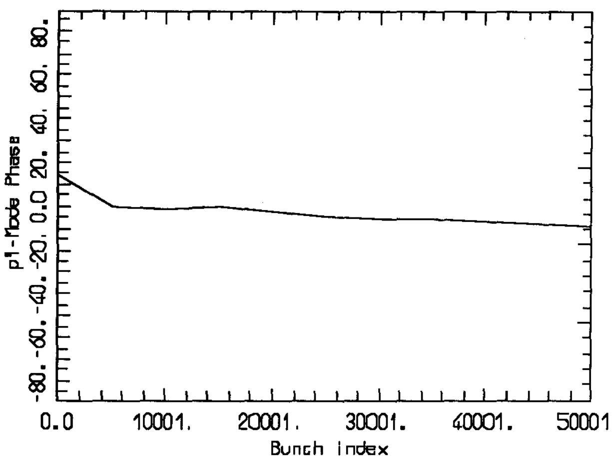

Fig. 14 - Phase evolution of the π-mode.

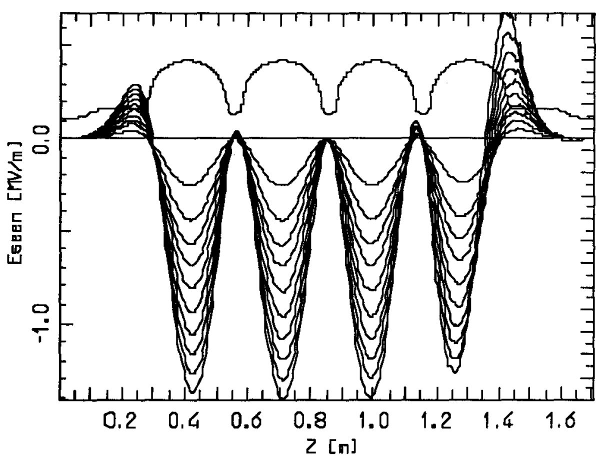

Fig. 15 - Total induced electric field seen by different bunches. The voltage is always decelerating for the incoming bunches. The field unflatness is due to transient interaction.

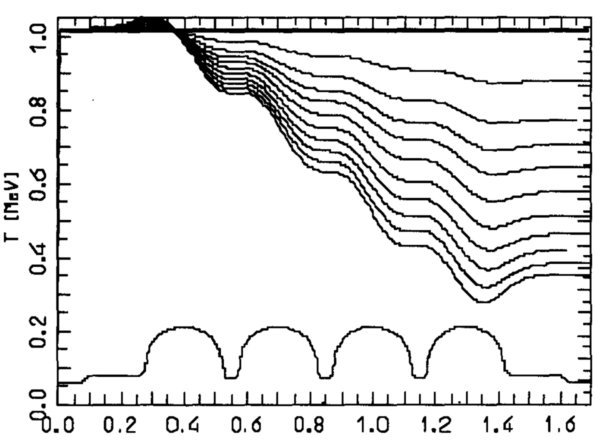

Fig. 16 - Bunch energy

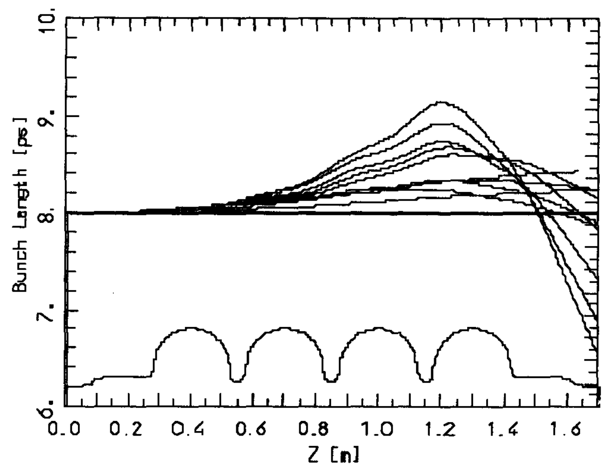

Fig. 17 - Bunch length L.

V. REFERENCES

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[3] G. Mavrogenes, "Space charge effects in high current linac transport systems," *ANL Report.*