Superconducting Billiard Cavities - a Non - Accelerator Application of RF - Superconductivity

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Abstract

The new method for experimental studies of quantum chaos with superconducting microwave cavities reported at the last Workshop on RF Superconductivity has been further developed. A description of the measurements mainly based on determination of the cavity resonances is given followed by examples of special cases. It is shown, that with the possibility of getting large and complete sequences of eigenmodes detailed comparisons with theoretical predictions can be made. Furthermore, the analysis of the amplitudes and the widths of the cavity resonances yields also important results.

1. *Experiment*

The main purpose of this experiment is the study of quantum chaos in special geometries by the use of microwaves in the GHz range. The basic idea here is to recognize, that the stationary Schrödinger equation to which the quantum particles obey has the same form as the Helmholtz equation,which is valid for microwave resonators. If the problem can be considered as two-dimensional, the eigenvalues are identical for a given geometry. The respective wavenumbers are defined as $k_{QM} = \sqrt{2mE}/\hbar$ in the quantum mechanical case and $k_{EM} = 2\pi\nu/c$ in the electromagnetic case. One is therefore able to study the properties of the spectrum of eigenmodes no longer only numerically but also experimentally, in particular for the geometries, which have been proven to be classically chaotic and where precice

analytic calculations iare very involved and partly not possible.
The experimental study of the spectral properties is therefore in principle possible, but for the analysis based mainly on statistics, a large and complete sequence of resonances is required and the limits are given by the resolution of modes at high frequencies, which is directly related to the Q-values of the cavity. For resonant cavities superconductivity is the most appropriate mean to improve the resolution by orders of magnitude [1]. In our first experiment, on which we reported two years ago in this conference [2], we built a flat microwave cavity shaped as a quarter of a stadium ($a = 36cm, r = 20cm, h = 0.8cm$) out of niobium and put it into one of the cryomodules of the superconducting electron accelerator S-DALINAC at Darmstadt [3]. By cooling the cavity down to 2K, Q-values from 10^5 up to 10^7 have been reached. Therefore it became possible for the first time to get a sequence of 1060 eigenmodes experimentally, which could be prooven to be complete. This was done in the range of 0.5 - 17.5 GHz with three different transmission spectra by the means of rf-antennas coupled very weakly to the electrical field in the cavity. The average width of the resonances is about 12 kHz, while the smallest observed spacing is 300 kHz. A detailed analysis of the shape of the resonances can be found in [4]. A comparison of transmission spectra taken at 300K and 2K is given in [1].

2. *Analysis*

The statistical analysis is mainly based on the examination of the short-range correlations, which shows up in the distribution of the spacings between adjacent modes, the nearest neighbor distribution, and the long-range correlations by the Dyson-Mehta or Δ_3 statistic [5], which exhibits correlations over a large frequency (or energy in the analogy) interval. Theory has provided distinct predictions for the cases of a purely regular or purely chaotic system. But the stadium billiard turned out to be a mixed system lying in between of the predictions. In contrast, the analysis of a measured spectrum while the cavity was in normal conducting state was in fact misleading, here the statistics showed good agreement with theory for purely chaotic systems; this can be explained very well by the fact, that one misses especially those modes, where the spacing gets of the order of their width. The result of the nearest neighbor distribution (NND) for the stadium billiard is shown in the upper part of fig. 1. The experimental distribution is tested against three models trying to describe the mixing with different ansatzes. This topic is explicitly discussed in [6], the mixing parameters (varying between 0 and 1 for purely regular and chaotic behavior, respectively) get values between 0.57 and 0.87 dependent on the model. The reason for obtaining a regular part in the non-integrable problem is one special class of eigenfunctions contributing to the *bouncing ball orbit* in the corresponding classical system. This can be explained theoretically with a formula developed afterwards by Littlejohn et al. [7], and the very good agreement with data can be seen in fig. 2. It shows the fluctuating part of the integrated level density as a function of frequency and one can easily recognize a slow oscillation with a frequency of 750 MHz. This oscillation can be described by the formula just inserting the geometry parameters and is given in fig. 2 by the full line. The agreement is impressive, there is no free parameter left. It is also possible now to eliminate the regular part which results in a modulation of the level density and to calculate the statistics again, best visible in the Δ_3 statistics in fig. 3. The upper data points correspond to the original spectrum, a clear deviation from the expected GOE graph is found tending towards regular behavior. After eliminating the regular contribution the lower points are calculated and result in a good agreement with theory, because also a saturation above a critical L-value given by the shortest periodic orbit (in our case $L_{max} = 15$) is expected [8].

The reason for the mixing is now clearly identified, also studied further in detail by [7]. The next billiard was then built with the intention to avoid such special classes of periodic orbits and resulted in the hyperbola geometry [9] . A desymmetrisized version sketched in the lower part of fig. 1 was fabricated and measured. It is 1350mm long and 190mm wide, the height here is choosen to be 7mm to remain in the two dimensional regime up to at least 20 GHz. Because the hyperbola extends to infinity we capped the end of the cavity with a quarter circle. A total number of 1052 modes have been counted and the result of the NND is given in the lower part of fig. 1. The fits of the models used give mixing parameters consistent with unity, so this geometry is therefore purely chaotic.

Furthermore, the amplitudes and the widths of the resonances were also measured. With a superconducting cavity one can neglect the losses of the walls, if the antennas are strongly overcoupled. The cavity therefore reacts as a chaotic scatterer without losses except through the *explicit channels* (antennas). In the case of few channels the S-matrix has very interesting features, e.g. the autocorrelation function of the matrix elements is no longer Lorentzian [10]. First results will be published soon.

Concerning the transition from regular to chaotic behavior a special cavity has been built consisting of two coaxial circles. The inner one can be moved, so that with different configurations the transition can be observed experimentally. Analysis of measurements done at the CERN cryolabs is in progress.

Future studies will concern three dimensional billiards and chains of superconducting two dimensional billiards to observe the exiting phenomenon of the Anderson localisation [11].

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FIGURES

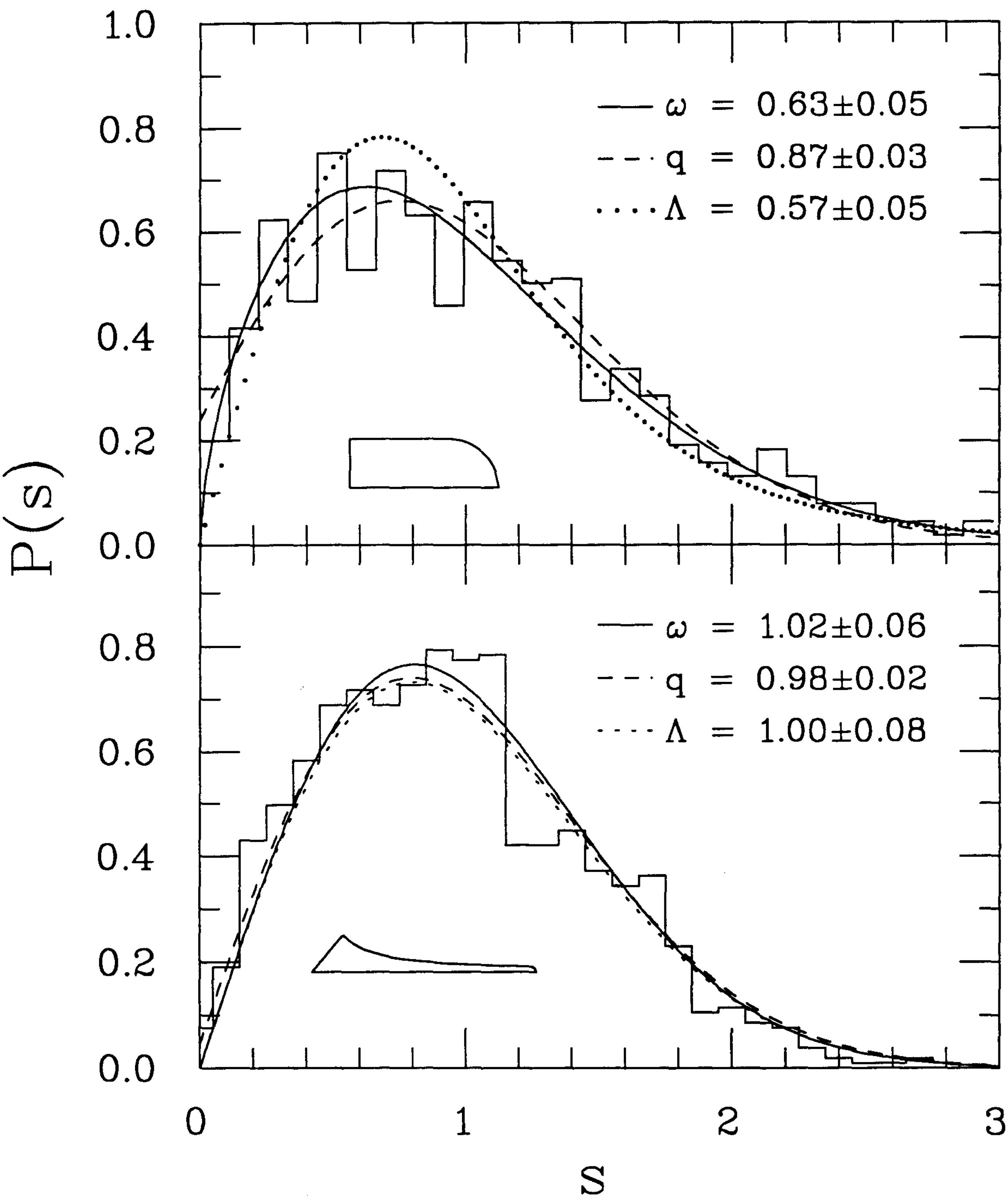

FIG. 1. The histogram in the upper part shows the experimental nearest neighbor distribution (NND) for the stadium billiard. The full line corresponds to a fit with the Brody distribution [12], the dashed and dotted lines to the Berry-Robnik [13] and the Lenz-Haake [14] models respectively. The lower part gives the same curves for the hyperbola billiard.

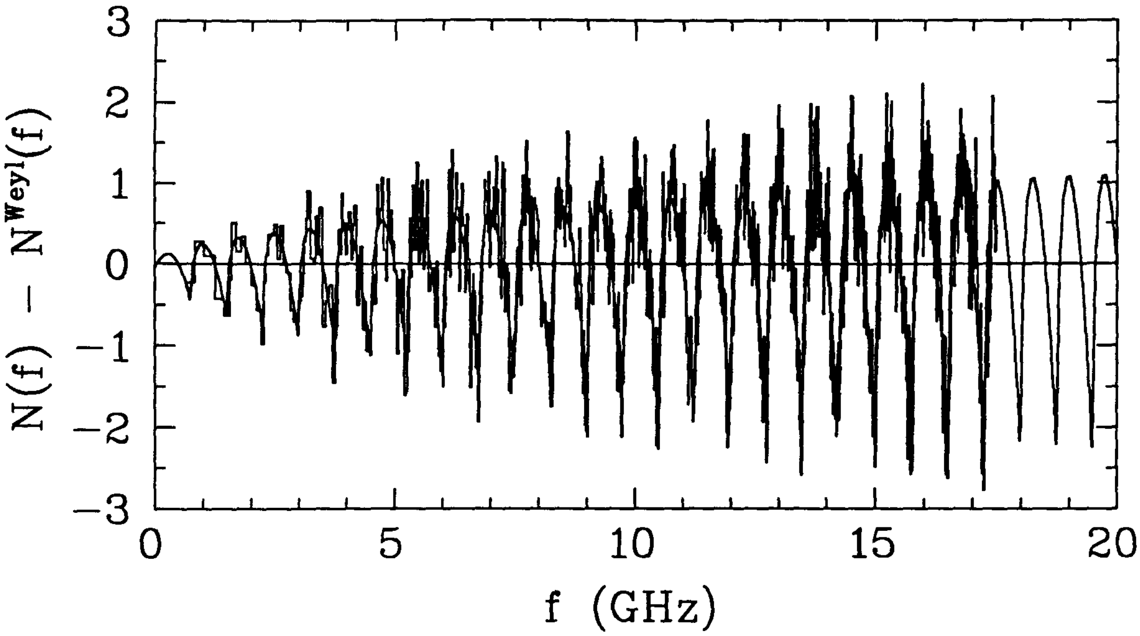

FIG. 2. The fluctuating part of the integrated level density is given by the histogram as a function of frequency, the full line shows the theoretical prediction by the formula of Littlejohn [7].

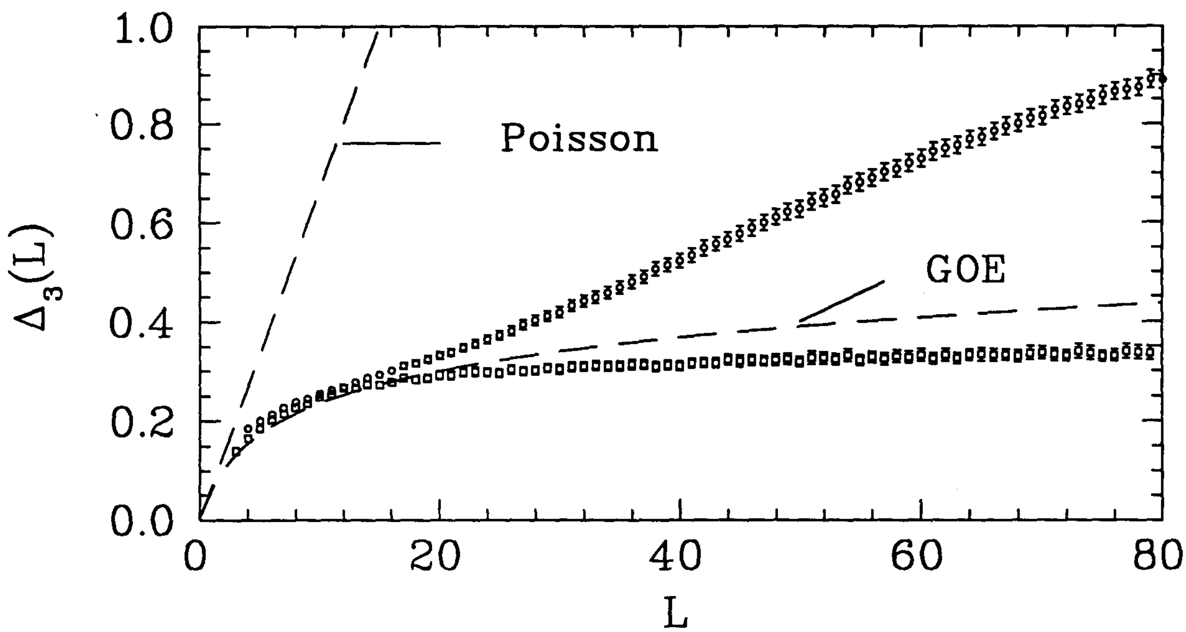

FIG. 3. $\Delta_3(L)$-statistics of the experimental data set compared with theoretical predictions (dashed lines). The open circles (squares) derive from the unfolded spectrum with (without) the bouncing ball orbit contribution.