An Analytical Approach for Calculating the Quench Field in Superconducting Cavities

H. Safa

C.E. Saclay , DSM/DAPNIA/SEA, 91191 Gif-sur-Yvette, France

Abstract :

A semi-analytical approach is described here giving the quench field level in a superconducting RF cavity. This thermal resolution using the Hankel transform function might be interesting as it immediately points out the influence of various parameters involved. In particular, quench field is plotted as a function of RRR and thickness of the material and, in the case of a local defect, the importance of its size and resistivity is shown.

Introduction

One of the basic limitation of superconducting cavities is their thermal instability called "quench" leading to a transition towards the normal state. This is certainly an important issue for future high energy colliders requiring high accelerating gradients. In principle, for a perfect superconductor, there is a theoretical limit given by the critical magnetic field. But in real cavities, it appears that the quench field is somewhat lower than this theoretical limit. As in most elliptical (β=1) cavities, the ratio between the accelerating field E_{acc} and the magnetic field on the equator is almost the same,

$$B_{[mT]} = 4.\ Eacc_{[MV/m]}$$

all the results will be given for obvious practical reasons as a function of the accelerating gradient E_{acc}. For niobium, the ideal critical field is $B_{c0} = 190\ mT$ @T=0K, and using the temperature variation of the critical field $B_c = B_{c0}\left[1 - (T/T_c)^2\right]$, one can deduce the maximum accelerating field achievable for niobium cavities to be $Eacc = 45\ MV/m$ at T=1.8K (this assuming no heat generation and an ideal cooling). Heat losses deposited at the inner surface of the cavity and thermal resistances in the niobium sheet and at the interface with the helium bath will reduce this ideal quench limit value.

Generally, the quench field value is calculated by using finite elements computer codes [1,2,3]. A different simplified approach is described here aiming at trying to give a better feeling of what might be the importance of the different parameters involved in the system stability. After describing why thermal instability occur in the uniform case (without local defects), the quench field limit is shown to be in that case quite close to the ideal one given above. Then, one can assume that there is locally a "defect" either a normal inclusion or a degraded niobium zone having bad superconducting properties. The variation of the quench field will be given as a function of the defect size (assumed for simplicity to be a flat disk of diameter Φ=2a), the RRR of the bulk niobium and the thickness e of the sheet.

Thermal analysis

Let us consider an infinite plane sheet of thickness e having one face in vacuum and the other in contact with a helium bath. On the vacuum side, a given amount of heat flux q [in W/m^2] is deposited which flows through the bulk towards the helium bath.

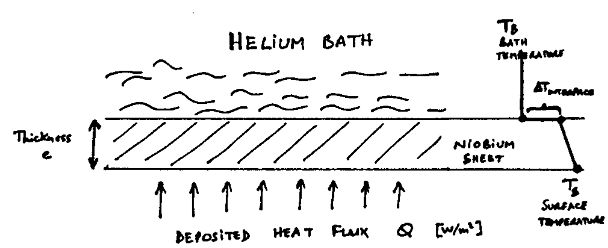

Figure 1– Thermal scheme of a superconducting niobium cavity.

Thus, the vacuum face will have the highest temperature T_2, while the temperature T_1 of the helium face will be higher than the bath temperature T_b due to the interface cooling mechanism (which is Kapitza phenomenon in the case of a superfluid bath T<2.17K @P=50mb) :

$$q_z(z = e) = h_k\ (T_1 - T_b)$$

If κ is the thermal conductivity of the solid, the general equations can be written as follows (assuming no heat generation inside the bulk) :

$$\begin{cases} q = & -\kappa \ gradT \\ divq = & 0 \end{cases}$$

The problem will be to determine the "hot" face temperature T_2 from which will strongly depend the amount of deposited heat flux $q_z(z=0)$.

Heating

RF Losses

Of course, the main source of heating is due to the surface resistance R_s, the dissipated heat flux for a given surface magnetic flux B beeing

$$q = \frac{1}{2\mu^2} \ R_s B^2$$

The surface resistance can be splitted in two parts : One is the standard BCS resistance which temperature, RRR and frequency dependence are well understood. But there is no general agreement concerning the remaining part called "residual" resistance which is not predictible. The BCS part is predominant at high temperatures (above 2K at 1.5GHz) whereas at low temperatures, only the residual resistance remains (BCS resistance should be zero at 0K). Causes of non-zero residual resistance are identified as static magnetic flux pinning [4], grain boundaries [5], residual 100K effect [6] or impurities [7]. Inhomogeneities and surface defects may also contribute to part of these residual losses. Nevertheless, the residual resistance can be made quite small (a few nanoOhms) and should not affect the quench limit value in an important way.

The BCS part is of major importance. In a previous paper [7], it has been shown that the real order parameter of superconductors is roughly varying as $\Omega = 1 - \left(\frac{T}{T_c}\right)^3$. The BCS surface resistance can be calculated using

$$R_s \ = \ \mathcal{R}e \left(\frac{i \, \omega \mu \lambda}{\sqrt{\Omega \ + \ \mathrm{i} \, \omega \tau \, (1 - \Omega)}} \right)$$

where ω is the pulsation of the electromagnetic field, λ the London penetration depth and τ the normal collision time of electrons.

Electron Emission

Field emitted electrons from the cavity surface can be accelerated in the RF field and impinge on another point with high energies (order of MeV). This can be another source of heating locally the inner surface. Usually, when electron emission occur, the field in the cavity will be limited by the amount of available input power. But if the corresponding power is dissipated in a thin meridian, an "electronic" quench might sometimes be observed. As this obviously is not an intrinsic phenomenon, it will be assumed in the following that the cavity do not show any field emission.

Cooling

The heating previously described lead to a certain amount of deposited heat flux q on a so thin layer of the inner face that it can be considered as deposited on the surface z=0. The heat will propagate in the sheet and be evacuated by the helium bath assumed to be at a uniform temperature T_b. Two thermal gradients appears : the first is due to the finite thermal conductivity of the sheet, and the other to the cooling mechanism at the solid/liquid interface.

Conduction resistance

The ability of heat transport in a solid is given by the value of its thermal conductivity κ. For superconductors, heat transport is essentially ensured by normal conducting electrons so κ is strongly increasing with temperature (A good superconductor is a bad thermal conductor). It can be easily understood that the electronic part will be proportionnal to the electrical conductivity and hence the RRR [8,9] :

$$\kappa_{elec} \propto \ (1 - \Omega) \, .RRR$$

There is also a phonon contribution to the thermal conductivity which only prevails at low temperatures (T<2.5K) thus resulting in a bump in the κ(T) curve more or less important depending on the grain size of the material. In practice, it won't have a great influence on the quench field level at which the corresponding temperature of interest is in all cases higher than 3K.

Interface resistance

Heat flowing through a solid/liquid interface results in a temperature discontinuity (ΔT) roughly proportionnal to the flux for low fluxes [10]

$$\Delta T = \frac{q}{h_k}$$

In the case of the superfluid helium, Kapitza conduction writes $q = \frac{h_0}{4}\left(T^4 - T_b^4\right)$ hence $h_k \simeq h_0 T_b^3$. Above a critical flux q_{fb}, the liquid boils just near the surface decoupling thermally the solid from the bath : this is the film boiling limit. Experimentally, q_{fb} ranges between 5000 and 20000 W/m^2 [11]. An average value of 10000 W/m^2 will be taken.

The uniform case

This is a one-dimension resolution along the z axis. If q_s is the uniformly deposited heat flux, then the "cold" face temperature is $T_1 = T_b + q_s\left(\frac{1}{h_k}\right)$. In principle, the "hot" face temperature T should be given by integrating $\kappa \frac{\partial T}{\partial z} = -q_s$ as κ depends on z via the temperature profile T(z). But, one can assume a mean value for κ (for example taking an average temperature at z=e/2) which greatly simplifies to $(T - T_1) = q_s\left(\frac{e}{\kappa}\right)$ (this approximation will be justified later on as it will be shown that the temperature difference between the hot and cold face is quite small even at the quench limit). Thus, the temperature should be solution of the equation :

$$\mathbf{T - T_b = q_{s(T)} \left\{ \frac{e}{\kappa_{(T)}} + \frac{1}{h_k} \right\}}$$

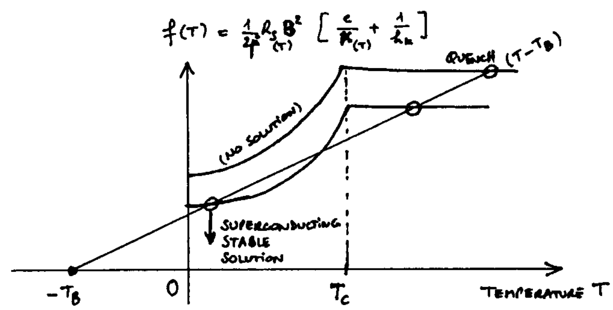

Figure 2– Graphical resolution in the uniform case

There, instability can be easily understood. If one plots the right hand side exponentially increasing function of temperature $f(T) = q_{s(T)} \left\{ \frac{e}{\kappa_{(T)}} + \frac{1}{h_k} \right\}$, it can be graphically seen that, depending on the values, there can either be three intersecting points with the line ($T-T_b$) or only one. In the first case, there is one stable superconducting solution, an unstable one, and one stable normal solution. In the second case, only the stable normal solution remains. A quench will occur when the curve of f(T) will be higher than the line ($T-T_b$) for $T<T_c$. As a consequence, in order to avoid the quench, one should try to lower f. That means :

— Increase the thermal conductivity κ (i.e. increase RRR)
— Decrease the thickness sheet e.
— Increase the Kapitza conductance h_k.
— Decrease the surface resistance R_s.

The thermal instability described here above does not take in account the magnetic field limit. It can take place even if there were no critical magnetic field. But if the function f is small enough, one can hit the critical magnetic field before the thermal instability crossing the (B,T) phase line (see figure 3).

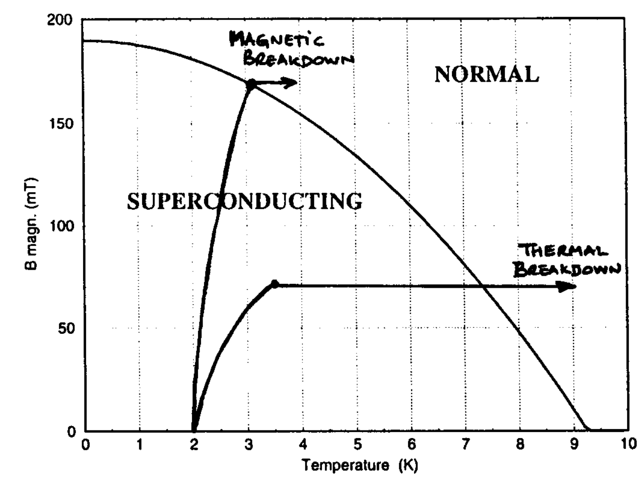

Figure 3– Themo-magnetic transition (Quench) in the (B,T) phase diagram of a superconductor

This case is calculated for a standard niobium sheet where the quality factor Q_0 has been plotted as a function of E_{acc} (F=1.3GHz, Rresidual=7.5nΩ, RRR=300) (fig. 4). At T=1.8K, the quench field is found to be E_{acc}= 42MV/m. This is very close to the ideal value (45MV/m) indicating that, in the defect-free case, very high gradients should be achieved.

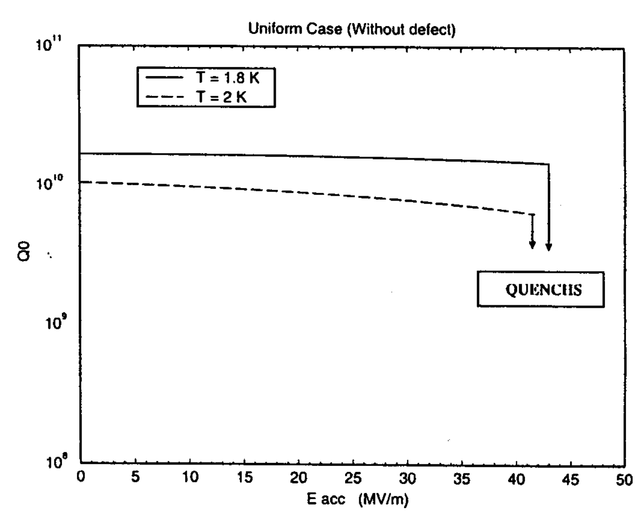

Figure 4– Quality factor of a cavity as a function of accelerating field in the free-defect case

The defect case

It is assumed here a local flat zone of diameter Φ=2a where the heat deposited flux q differs from the uniform surface. The problem is a 2–dimensional one using cylindrical coordinates (r,z).

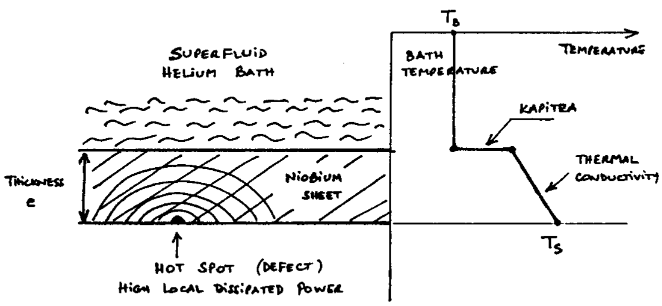

Figure 5– The 2–dimensionnal defect case. Heat is assumed to be deposited on the inner surface.

The hottest point is therefore located at (r=0,z=0). Here again, taking an average value for κ results in writing the general equation inside the sheet as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial r^2} = 0$$

with the following boundary conditions

$$\begin{cases} q_z(z=0) = q(r) & Warm\ side \\ q_z(z=e) = h_k\ [T_{(r,e)} - T_b] & Cold\ side \end{cases}$$

The use of the Hankel transform (see Appendix) appear to be very helpful giving directly the analytical solution as an integral form. Writing q(k) as the Hankel transform of the function [q(r)-q_s] (the uniform heat flux q_s have been separated)

$$q(k) = \int_0^\infty r\ J_0(kr)\ [q(r) - q_s]\, dr$$

Then, the temperature in the sheet is given by

$$T(r,z) = \int_0^\infty k\ J_0(kr)\ \frac{q(k)}{ch(ke)}\left[\frac{sh[k(e-z)]}{\kappa\ k} + \frac{1}{h_k}\right] dk + q_s\left[\frac{e-z}{\kappa} + \frac{1}{h_k}\right] + T_b$$

and, consequently, the heat fluxes are

$$q_z(r,z) = \int_0^\infty k\ J_0(kr)\ \frac{q(k)}{ch(ke)}\ ch[k(e-z)]\, dk\ +\ q_s$$

$$q_r(r,z) = \int_0^\infty k^2\ J_1(kr)\ \frac{q(k)}{ch(ke)}\left[\frac{sh[k(e-z)]}{k} - \frac{\kappa}{h_k}\right] dk$$

One immediately recognizes the uniform solution given above when q(k)=0 with $T(r,z) = q_s\left[\frac{e-z}{\kappa} + \frac{1}{h_k}\right] + T_b$.

In the general case, the temperature of the hottest point can be determined as

$$T_{max} = \int_0^\infty k\ \frac{q(k)}{ch(ke)}\left[\frac{sh[ke]}{\kappa\ k} + \frac{1}{h_k}\right] dk + q_s\left[\frac{e}{\kappa} + \frac{1}{h_k}\right] + T_b$$

Quench can occur for three reasons :

a) A thermal breakdown, similar to the one described in the uniform case, if the hot spot temperature T_{max} exceeds the critical temperature T_c.

b) A magnetic breakdown if the magnetic field exceeds the critical magnetic field $B_c(T)$. Note that due to the temperature rise, this local field can be substantially lower than the uniform case one.

c) A cooling breakdown when the heat flux on the cold side exceeds the film boiling limit q_{fb} $(q_z(r=0, z=e) > q_{fb})$.

The resistive defect solution

Let us consider here a resistive defect having a uniform surface resistance. Then the generated heat flux is constant q(r)=q_n for (r<a) and q(r)=q_s for (r>a). Of course, one must bare in mind that q_n is orders of magnitude higher than q_s.

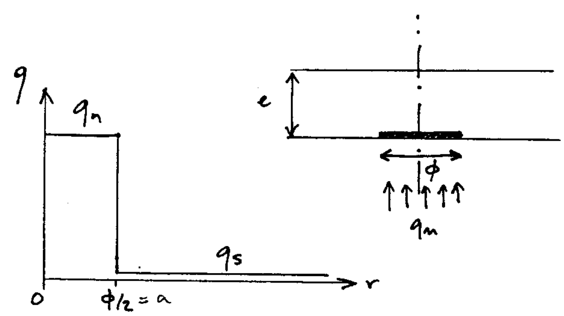

Figure 6– Flux profile in the case of a pure resistive cylindrical defect..

As shown in the appendix, the Hankel transform in this case writes as

$$q(k) = a^2(q_n - q_s)\left[\frac{J_1(ka)}{ka}\right]$$

giving in a straightforward way

$$\begin{cases} T - T_b = (q_n - q_s)\left(\frac{a}{e}\right)^2\left(\int_0^\infty x\frac{J_1\left(x\frac{a}{e}\right)}{x\left(\frac{a}{e}\right)}\left[\frac{e}{\kappa}\frac{shx}{x} + \frac{1}{h_k}\right]\frac{dx}{chx}\right) \\ q_{z(r=0,z=e)} = q_s + (q_n - q_s)\left(\frac{a}{e}\right)^2\left(\int_0^\infty x\frac{J_1\left(x\frac{a}{e}\right)}{x\left(\frac{a}{e}\right)}\frac{dx}{chx}\right) \end{cases}$$

At that point, one have to distinguish between two cases.

— If the defect is bigger than the sheet thickness (a>>e), then nearly all the flux q_n will flow to the cold side. Therefore, one will rapidly hit the film boiling limit (Quench c) at low fields.

— If the defect is small compared to the thickness of the sheet (a<<e) —which should be the general case in a real cavity —, then the temperature rise will scale roughly as

$$\left(\frac{a}{e}\right)^2 \left[\frac{e}{\kappa} + \frac{1}{h_k}\right] q_n$$

Again, as in the uniform case, the conclusions concerning κ and h_k remain valid : One must try to increase the thermal conductivity as well as the Kapitza conductance. Figure 7 show as an example the quench field as a function of RRR (κ is approximately proportionnal to the RRR).

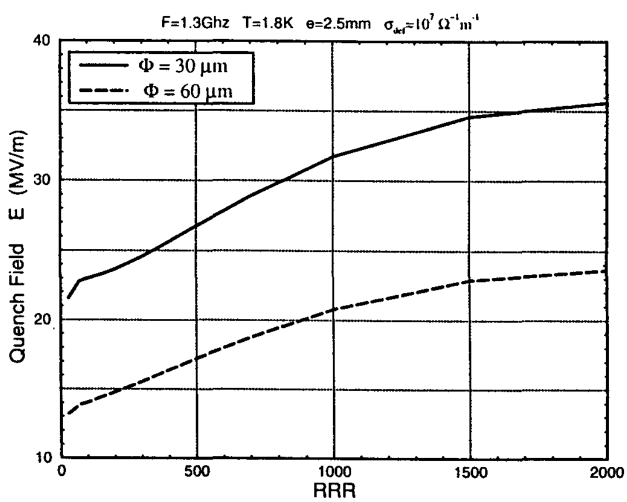

Figure 7– Quench field as a function of RRR.

But as opposed to the uniform case, here appears the $(a/e)^2$ term showing the square dependence upon the defect size. Indeed, in figure 8, the quench field is plotted versus the diameter of the defect Φ=2a for a given normal resistance ($R_n = \sqrt{\frac{\omega\mu}{2\sigma}}$). Big defects can drastically reduce the quench field (in that example, a 100μm defect induces a quench at a field as low as 10MV/m). Therefore, if defects in a sheet were to be unavoidable, it should be highly recommended at least to try minimizing their size.

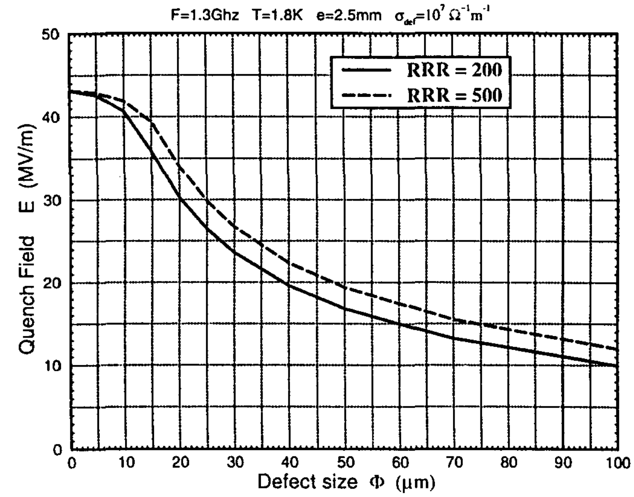

Figure 8– Quench field as a function of defect size.

Concerning the sheet thickness e, a trade off between two tendencies appears : The bracket term $\left[\frac{e}{\kappa} + \frac{1}{h_k}\right]$ (the same that in the uniform case) should be minimized inclining to decrease e, but the strongest term $(a/e)^2$ clearly favour the use of thick sheets. This can be understood by the fact that a thin sheet makes a smaller temperature drop between the two sides, but conversely, a thick sheet helps diffusing the heat in the radial direction thus minimizing the flux on the cold side. The conclusion is that, for a given defect, *there is an optimum value for the sheet thickness* (fig 9). In practice, that optimum is very broad (between 3 and 7 millimeters in our example), and the choice of thickness will be determined by other considerations (mechanical properties, stiffness, material weight and cost, ...). But the thin sheets (less than 1mm) have to be avoided, in contradiction with the result obtained in the uniform case.

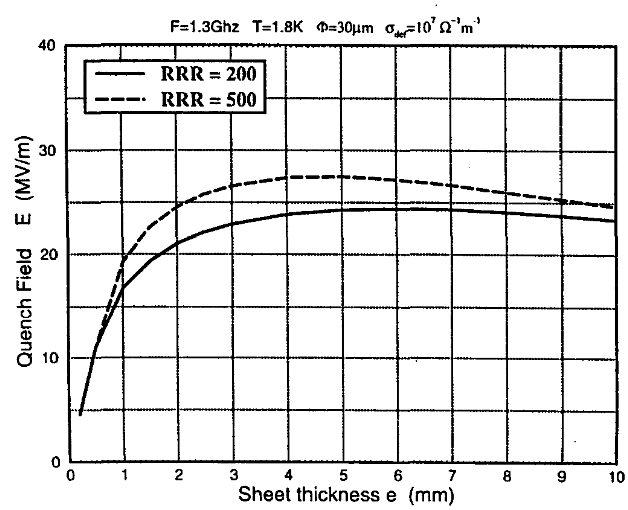

Figure 9– Quench field as a function of sheet thickness.

Conclusion

As a result, main conclusions are :

1) First, as expected, the higher the RRR, the higher the quench field level.
2) In our niobium cavities, quenches (at 20–30 MV/m) are presumably due to a local defect having poorer superconducting properties than pure niobium because in the uniform defect-free case, quench field is calculated to be substantially higher (over 40 MV/m). The size of defects appears to be of major importance.
3) In the uniform defect-free hypothesis, the cooling of the superconducting surface is better for thinner walls; But in the realistic case of a defect, it turns out that there is an optimum value for the thickness of the niobium wall.

References

[1] G. Müller, "Superconducting niobium in high RF magnetic fields", Proceedings of the 3rd Workshop on RF Superconductivity, p.331, Argonne, Illinois, USA, [Sept 1987]
[2] H. Padamsee, "Calculations for breakdown induced by large defects in superconducting niobium cavities", CERN/EF/RF, 82–5
[3] R. Röth, EPAC 92, Berlin [1992]
[4] C. Vallet, Ph. D., University C. Bernard, Lyon, France [1994]
[5] B. Bonin & H. Safa, "Power dissipation at high fields in granular RF superconductors", Superconductor Science and Technology, 4, 257–261, [1991]
[6] B. Aune & al, "Degradation of niobium superconducting RF cavities during cooling time", LINAC 90, Albuquerque, USA [1990]
[7] H. Safa, "Surface resistance of superconductors", Proceedings of the 5th Workshop on RF Superconductivity, Hamburg, Germany [1991]
[8] E.A. Lynton, Superconductivity, p.101, London [1969]
[9] F. Kœchlin & B. Bonin, "Parametrisation of thermal conductivity of niobium", this conference
[10] P.L. Kapitza, J. Phys. USSR, 4, 181 [1941]
[11] K. Kraft, "Thermal transport and thermal magnetic breakdown in superconducting cavities made of high thermal conductivity niobium", Ph. D., Cornell University [1983]

Appendix : The Hankel transform

For any function f(x), one can define its Hankel transform as

$$F(k) = \int_0^\infty x \; J_0(kx) \; f(x) \, dx$$

where J_0 is the standard Bessel function of zero order. The inverse transform will be

$$f(x) = \int_0^\infty k \; J_0(kx) \, F(k) \; dk$$

Some examples of Hankel transform functions are :

Function f(x)	Transform F(k)
$\begin{cases} f(x) = y_0 & (x < x_0) \\ f(x) = 0 & (x > x_0) \end{cases}$	$F(k) = x_0^2 y_0 \dfrac{J_1(kx_0)}{kx_0}$
$f(x) = y_0 \, e^{-\frac{x^2}{2\sigma^2}}$	$F(k) = y_0 \sigma^2 \, e^{-\frac{k^2\sigma^2}{2}}$
$f(x) = y_0 \dfrac{sin(lx)}{lx}$	$\begin{cases} F(k) = \dfrac{y_0}{l\sqrt{l^2 - k^2}} & (k < l) \\ F(k) = 0 & (k > l) \end{cases}$
$f(x) = y_0 \dfrac{J_m(lx)}{x^m}$	$\begin{cases} F(k) = \dfrac{y_0 (l^2 - k^2)^{m-1}}{2^{m-1} m! \, l^m} & (k < l) \\ F(k) = 0 & (k > l) \end{cases}$