# BEAM ENERGY LOSS VERSUS VELOCITY IN SC CAVITIES FOR SNS LINAC

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## Abstract

Particles in ion linacs are accelerated through the wide range of their velocities. The dependence of beam energy losses in cavities on the beam velocity has been studied recently [1]. We apply the method [1] to analyze the beam energy loss in 8-cell superconducting cavities, which have been proposed for the superconducting version of the Spallation Neutron Source linac.

#### **1 INTRODUCTION**

With the advance of superconducting (SC) RF technology its application is more and more attractive not only for electron machines, but also for high-intensity ion linacs. Projects like Accelerator Production of Tritium (APT) and the accelerator-driven transmutation of nuclear waste (ATW) consider using SC linacs to accelerate intense CW proton beams providing the final beam power up to 200 MW. With the final beam energies around 1-2 GeV, the ion beam during acceleration changes its velocity from a non-relativistic one to  $\beta = v/c = 0.85 - 0.95$ . Using SC accelerating cavities for the relatively high- $\beta$  part of the linac, say  $\beta > 0.5$ , reduces operational costs of such machines significantly. With a normal conducting pulsed H linac being the baseline for the Spallation Neutron Source (SNS) project [2], a SC option for the SNS linac has been under investigation recently [3].

The number of different types of SC cavities should be limited to a few due to cost and production reasons. The SC version [3] of the SNS linac proposes using only two types of the SC 8-cell cavities optimized for  $\beta$ =0.61 and  $\beta$ =0.76, respectively [4]. However, these cavities have to accelerate the H<sup>-</sup> beam in wide particle velocity ranges: the first type is to span the interval of  $\beta$  from 0.560 to 0.688 (beam energy 194-355 MeV), and the second one for  $\beta$ =0.688 to 0.875 (to 1 GeV). Obviously, for SC cavities it is important to know the amount of the beam energy deposited in them and how it depends on the beam velocity. Usually the beam energy loss factors in the cavities are computed using time-domain codes like MAFIA [5] or ABCI [6]. However, this approach works only for a relativistic beam. For the non-relativistic case the problem is more complicated because of difficulties with its numerical formulation in the time domain. A method to calculate the loss factors of a non-relativistic bunch has been developed recently [1], and in this note we apply it for the 8-cell SC cavities proposed in [4].

# **2** LOSS FACTORS FOR $\beta < 1$ .

The loss factor k is related to the energy  $\Delta U$  lost in the structure by a passing bunch as  $\Delta U = kq^2$ , where q is the bunch charge. The beam power deposited by the bunches following through the cavity with the bunch repetition rate  $f_{\rm rep}$  is  $P = kq^2 f_{\rm rep} = kI^2/f_{\rm rep}$ , where  $I = qf_{\rm rep}$  is the average beam current. Of course, this equation does not account for possible interactions of bunches through their wakes. The total loss factor can be written as a sum of contributions from all cavity modes  $k(\beta) = \sum k(\beta)$ , and in general the contributions from high-frequency modes can be significant. For SC cavities, however, we are mostly concerned about the lowest resonance modes, below the cutoff frequency, since they contribute to the heat load on the cavity itself. The beam energy transferred into the higher modes, which have frequencies above the cutoff and propagate out of the cavity into the beam pipes, will be deposited elsewhere outside the cavity. The loss factors of individual modes for a Gaussian bunch with the rms length  $\sigma$  can be written as

$$k_{s}(\boldsymbol{\beta},\boldsymbol{\sigma}) = \exp\left[-\left(\frac{\omega_{s}\boldsymbol{\sigma}}{\boldsymbol{\beta}c}\right)^{2}\right] \frac{|I_{s}(\boldsymbol{\beta},\omega_{s})|^{2}}{4W_{s}}, \quad (1)$$
where

where

 $I_s(\beta,\omega) = \int dz \exp(-i\omega z / \beta c) E_{sz}(z)$  (2) is the overlap integral. Here  $E_{sz}$  is the longitudinal component of the on-axis electric field of  $s^{\text{th}}$  mode, and  $\omega_s$ and  $W_s$  are the frequency and stored energy of the mode, see [1] for details.

In the next section, we apply Eqs. (1)-(2) to calculate the loss factors of the lowest modes in the SNS 8-cell cavities as functions of the beam velocity  $\beta c$ .

#### **3 SNS 8-CELL SC CAVITIES**

The two types of 8-cell SC cavities for the SNS linac have been designed [4], optimized for  $\beta$ =0.61 and  $\beta$ =0.76. They have axisymmetric shapes shown in Fig. 1. The SUPERFISH code [7] has been applied to calculate the eigenmodes of the cavities. The frequencies of 8 modes in the TM<sub>010</sub> band are listed in Table 1. The longitudinal on-axis fields of 8 TM<sub>010</sub> modes in the  $\beta$ =0.61 half-cavity are plotted in Figs. 2-3. Using the calculated fields, frequencies and stored energies of the modes, from Eqs. (1)-(2) we find the mode loss factors for any value of  $\beta$ . The results for the loss factors are plotted versus  $\beta$  in Figs. 2-3 along with the corresponding field profiles.

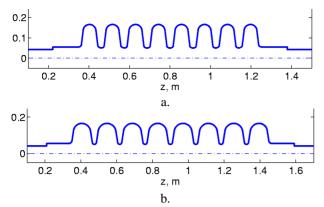


Figure 1: SNS 8-cell SC cavities: (a)  $\beta$ =0.61, (b)  $\beta$ =0.76.

Table 1: Frequencies of TM<sub>010</sub> modes

No	Cavity β=0.61	Cavity β=0.76
1 (π/8)	790.077	788.967
2 (2π/8)	791.804	790.756
3 (3π/8)	794.366	793.442
4 (4π/8)	797.357	796.623
5 (5π/8)	800.311	799.818
6 (6π/8)	802.787	802.536
7 (7π/8)	804.428	804.358
8 (π)	805.000	805.000

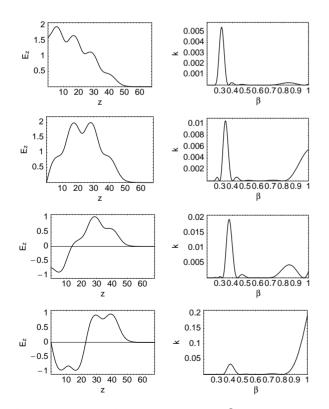


Figure 2:  $\text{TM}_{_{010}}$  modes 1-4 in 8-cell SNS  $\beta$ =0.61 cavity: longitudinal on-axis fields (arbitrary units) in the right half-cavity versus *z* (cm) (left), and loss factors (V/pC) versus  $\beta$  (right).

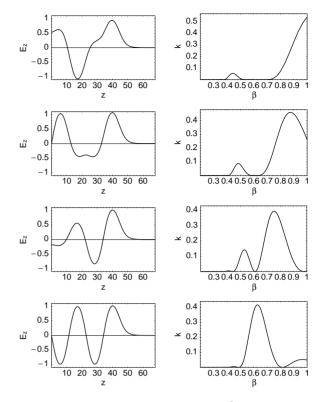


Figure 3:  $TM_{010}$  modes 5-8 in 8-cell SNS  $\beta$ =0.61 cavity.

One can see from Figs. 2-3 that the loss factors of individual modes exhibit non-monotonic behavior, with maxima at different values of  $\beta$ . As one should expect, the accelerating mode ( $\pi$ -mode) peaks near the design value of  $\beta$ =0.61. An interesting effect occurs when we plot the sum of the loss factors of all the modes in the TM<sub>010</sub> band, see in Fig. 4.

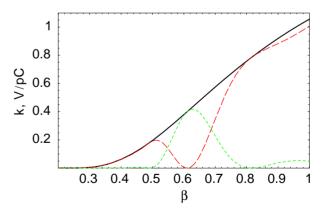


Figure 4: Loss factor of all  $TM_{_{010}}$  modes in 8-cell SNS  $\beta$ =0.61 cavity (black solid line): the accelerating mode contribution (green short-dashed) and that of all other modes (red long-dashed).

The contribution of all the  $TM_{_{010}}$  modes except the fundamental (accelerating) one in Fig. 4 has a minimum near the design  $\beta$ , but increases as we go apart from this value. Essentially, one should be only concerned about

this parasitic loss factor, especially near the ends of the  $\beta$ range for the cavity. The loss factor of the fundamental mode must be large near the design  $\beta$  to provide an effective acceleration of the beam; its consideration is related to the beam loading.

It is interesting to compare these results with those from time-domain calculations. Figure 5 shows the loss factor for the same 8-cell SNS  $\beta$ =0.61 cavity integrated up to a certain frequency versus that frequency, as computed for  $\beta$ =1 with the ABCI code [6]. The first sharp step in Fig. 5 near 800 MHz corresponds to the TM<sub>010</sub> band. The magnitude of this step is about 1.05 V/pC, which is exactly the value of the total loss factor for  $\beta$ =1 in Fig. 4. One can also see two more sharp steps, near 1.75 and 2.35 GHz, corresponding to two more resonance bands of the cavity below the beam pipe cutoff frequency 2732 MHz (the beam pipe radius is 4.2 cm). The loss factors of these higher bands are a few times smaller than that of the TM<sub>010</sub> one for  $\beta$ =1, and they decrease rapidly as  $\beta$  decreases to the design value, see in [1].

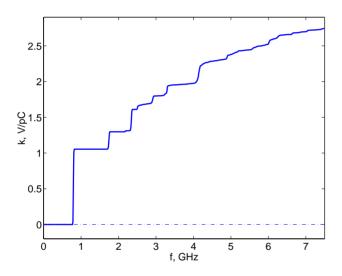


Figure 5: Loss factor for the 8-cell SNS  $\beta$ =0.61 cavity integrated up to a given frequency versus that frequency.

Similar calculations have been performed for the 8-cell SNS  $\beta$ =0.76 cavities. Figures 6-7 show the individual modes of the TM<sub>010</sub> band and the dependence of their loss factors on the beam velocity. The total loss factor for the cavity and contributions of the accelerating mode and all other ones are plotted in Fig. 8. And the results of the time-dome calculations with the ABCI code are presented in Fig. 9. One can see only two steps in the integrated loss factor below the cutoff frequency: the TM<sub>010</sub> contribution near 800 MHz is equal to 1.25 V/pC, in agreement with the result of Fig. 8 for  $\beta$ =1, and the next one, due to TM<sub>020</sub> band, near 1.8 GHz.

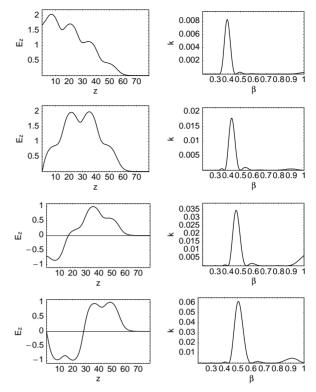


Figure 6: TM<sub>010</sub> modes 1-4 in 8-cell SNS  $\beta$ =0.76 cavity: on-axis fields (arbitrary units) in the right half-cavity versus *z* (cm), and loss factors (V/pC) versus  $\beta$ .

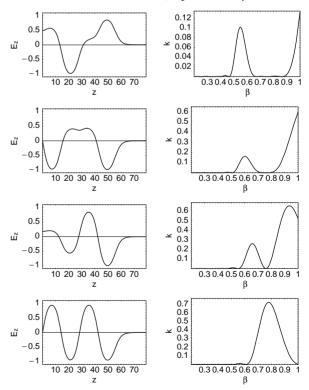


Figure 7: TM<sub>010</sub> modes 5-8 in 8-cell SNS  $\beta$ =0.76 cavity.

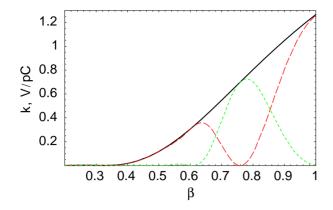


Figure 8: Loss factor of all  $TM_{_{010}}$  modes in 8-cell SNS  $\beta$ =0.76 cavity (black solid line): the accelerating mode contribution (green short-dashed) and that of all other modes (red long-dashed).

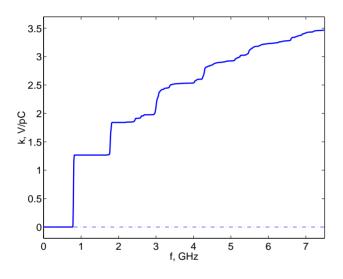


Figure 9: Loss factor for the 8-cell SNS  $\beta$ =0.76 cavity integrated up to a given frequency versus that frequency.

## **4 SUMMARY**

As it was mentioned in Sect. 3, the loss factor due to parasitic TM<sub>010</sub> modes (all those except the accelerating one) was very small at the design value of  $\beta$ , below 0.2% of the total loss factor. It becomes a significant fraction of the total loss factor for values of  $\beta$  near the ends of the cavity operational range. The beam power lost to those parasitic modes is deposited inside the cavity. To estimate this power, we use the SNS 2-MW beam parameters I =56 mA and  $f_{rep} = 402.5$  MHz. The SNS linac will operate in the pulsed regime with about 1-ms long macropulses at the repetition rate 60 Hz, and the macropulses have an internal chopped pattern with duty factor of 65%, that leads to the effective duty factor 0.04. For completeness, we present both values of the deposited power, for a hypothetical CW regime and for the SNS pulsed beam.

The results for the values of the loss factors and corresponding deposited beam power in the cavities near the ends of the cavity operational  $\beta$ -ranges are summarized in Table 2. Obviously, the power deposition for the SNS operational regime is rather modest.

Table 2: Parasitic loss factors and deposited beam power

	Cavity β=0.61	Cavity β=0.76
β-range:	0.560   0.688	0.688   0.875
$\beta_{\min} \mid \beta_{\max}$		
$k(\beta_{\min}) \mid k(\beta_{\max}),$	0.105   0.278	0.227   0.626
V/pC		
$k(\beta_{\min}) \mid k(\beta_{\max}),$	34.8   50.4	43.8   63.3
%% of total		
$P(\beta_{\min}) \mid P(\beta_{\max}),$	0.82   2.16	1.76   4.87
W, for CW		
$P(\beta_{\min}) \mid P(\beta_{\max}),$	0.03   0.09	0.07   0.19
W, for SNS		

One should notice that additional contributions from the higher modes with the frequencies below the beam-pipe cutoff can be calculated in a similar way, as it was done for the APT cavities in [1]. As one can see from Figs. 5 and 9, even for  $\beta$ =1 these contributions are small compared to those from the TM<sub>010</sub> modes. In addition, they decrease monotonically as  $\beta$  decreases [1], so that we expect only small additions from them to the figures in Table 2.

#### **5** ACKNOWLEDGEMENT

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