MESH ANALYSIS OF COHERENT SYNCHROTRON RADIATION IN A VACUUM CHAMBER

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Abstract

We developed a numerical method to calculate coherent synchrotron radiation (CSR). It is based on mesh calculation of electromagnetic field in the frequency domain. The approximated Maxwell equations are solved numerically with boundary condition. In this paper, we consider the resistive boundary conditions and apply it to KEKB LER.

INTRODUCTION

Electrons travelling along curved trajectories emit synchrotron radiaition. The component of radiation whose wavelength is longer than the electron bunch length is emitted coherently. This component is called coherent synchrotron radiation (CSR).

We presented a numerical method based on the mesh calculation of electromagnetic field in the frequency domain[1]. In general, in order to carry out the correct simulations, the mesh size should be much smaller than the wavelength of the field. In our formalizm, however, the mesh size is allowed to be larger than the radiaition wavelength.

We begin with Maxwell equations in vacuum and Fourier transform them into frequency domain. Then we approximate these equations on some assumptions, and solve them numerically by finite difference with boundary conditions. In our previous paper we assumed that the vacuum pipe is perfectly conducting. In this paper we consider the resistive boundary and calculate the energy change and impedance in the bending magnet.

THEORY

Equations

At first, we give an outline of our previous paper[1]. To derive the basic equations, we adopt some approximations as below. (a) The size of the chamber cross section a is much smaller than the bending radius ρ, namely, $\epsilon \equiv \sqrt{a/\rho} \ll 1$. (b) The bunch consists of ultrarelativistic electrons $\gamma = \infty$. (c) The radiation components propagating at large angles with respect to the beam are ignored (paraxial approximation). (d) The dynamic change of the bunch shape due to CSR is negligible.

We employ a coordinate system (x, y, s), s the length along the reference orbit, x and y perpendicular to s. We denote the electric and magnetic field by $\boldsymbol{E}$, $\boldsymbol{B}$, the charge density and current density by J_0, $\boldsymbol{J}$.

We mainly work in the frequency domain. The variables in the time domain have a symbol $^{(t)}$, and others are variables in the frequency domain. Let k be the wave number. We define Fourier transformation of a function f as follows.

$$f^{(t)}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \, f(k) \, e^{-ik\tau} \tag{1}$$

where $\tau \equiv t - s$. We reduce the Maxwell equation to a simpler form. Fourier transformation of Maxwell equations with higher order terms $O(\epsilon^2)$ ignored gives

$$B_x = -E_y, \quad B_y = E_x \tag{2}$$

$$E_s = \frac{i}{k} \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} - \mu_0 J_s \right) \tag{3}$$

$$B_s = \frac{i}{k} \left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right) \tag{4}$$

From Maxwell equations, we obtain

$$\frac{\partial}{\partial s} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \frac{i}{2k} \left[\left(\boldsymbol{\nabla}_\perp^2 + \frac{2k^2 x}{\rho} \right) \begin{pmatrix} E_x \\ E_y \end{pmatrix} - \mu_0 \boldsymbol{\nabla}_\perp J_0 \right] \tag{5}$$

where $\boldsymbol{\nabla}_\perp = (\partial_x, \partial_y)$ and $\boldsymbol{\nabla}_\perp^2 = \partial_x^2 + \partial_y^2$. In our previous paper[1] we suppose that the chamber is rectangular with constant size and perfectly conducting. We assumed that the transverse beam size is zero. We calculated the longitudinal electric field E_s. These results agree well with the analytic theories of (A) steady states without shielding, (B) steady states between parallel plates, (C) transient states without shielding.

Resisive Wall

In this paper we consider the resistive boundary as the vacuum chamber which has a rectangular cross section. We introduce finite difference to eqs.(5) and (3). $x_i = i\Delta x, (i = 0 \sim m)$, $y_j = j\Delta y, (j = 0 \sim n)$, $s_\ell = \ell\Delta s, (\ell = 0 \sim u)$, where i, j, ℓ are the indices of x, y, s respectively.

Let $\boldsymbol{e}_t$ be a unit tangent vector perpendicular to $s-$axis, $\boldsymbol{e}_n$ be a unit normal vector directing inside of the beam pipe. In the case of rectangular cross section,

$$\boldsymbol{e}_n = +\boldsymbol{e}_x, \; \boldsymbol{e}_t = +\boldsymbol{e}_y, \; (i = 0, \, 1 \le j \le n-1) \tag{6}$$

$$\boldsymbol{e}_n = -\boldsymbol{e}_x, \; \boldsymbol{e}_t = -\boldsymbol{e}_y, \; (i = m, \, 1 \le j \le n-1) \tag{7}$$

$$\boldsymbol{e}_n = +\boldsymbol{e}_y, \; \boldsymbol{e}_t = -\boldsymbol{e}_x, \; (j = 0, \, 1 \le i \le m-1) \tag{8}$$

$$\boldsymbol{e}_n = -\boldsymbol{e}_y, \; \boldsymbol{e}_t = +\boldsymbol{e}_x, \; (j = n, \, 1 \le i \le m-1) \tag{9}$$

The resistive boundary condition is given by

$$E_t = \alpha B_s \tag{10}$$

$$E_s = -\alpha B_t \tag{11}$$

where $\alpha = e^{-i\pi/4}\sqrt{k/(\mu_0 c\sigma_{cond})}$, σ_{cond} is the conductivity of the metal pipe. From eq.(4) and (10), the following equations approximately hold.

$$E_{x(i,j)} = 0, \quad (j = 0, n, \ 1 \le i \le m) \quad (12)$$
$$E_{y(i,j)} = 0, \quad (i = 0, m, \ 1 \le j \le n) \quad (13)$$

These conditions eq.(12), (13) are the same as the boundary condition of perfect conductivity.
On the other hand, from eq.(3) we obtain

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} - \mu_0 J_0 = ik\alpha E_n \quad (14)$$

Application of eq.(14) on the walls gives the effect of the resistive boundary.

Integration of the Field to Infinity

We integrate the electric field to obtain the imedance or energy change. CSR, emitted in a bending magnet, goes out of the magnet and propagates in the drift space. The particles which have gone out the magnet are still affected by CSR in a straight beam pipe. In order to estimate the whole influence by CSR, we integrate the field not only in the magnet but also in the drift space. We have to continue tracking eq.(5) with $\rho = \infty$ until the next bending magnet, but if the next magnet is far enough, we can ignore the interference and replace the finite beam pipe with and infinite straight pipe.

We can semi-analytically integrate the electric field from the exit of the magnet to infinity as follows. Let $E_x^{(\infty)}$ be the value of E_x at infinity $s = \infty$, which is steady state in the drift space. $E_x^{(\infty)}$ satisfies the following equation.

$$\nabla_\perp^2 E_x^{(\infty)} - \mu_0 \frac{\partial J_0}{\partial x} = 0 \quad (15)$$

We can compute $E_s^{(\infty)}$ from $E_x^{(\infty)}$ and $E_y^{(\infty)}$ using eq.(3). This gives the resistive wall impedance (per unit length) of the exit beam pipe.
We separate the field $E_x^{(\infty)}$ from E_x,

$$E_x = E_x^{(\infty)} + F_x \quad (16)$$

Then the rest part F_x satisfies the following equation.

$$\frac{\partial F_x}{\partial s} = \frac{i}{2k}\nabla_\perp^2 F_x \quad (17)$$

We here define a vector $\boldsymbol{f}_x$ in which the elements of $F_{x(i,j)}$ are arrayed in one dimension. Using a matrix M, we can write eq.(17) in the linear algebric form

$$\frac{\partial \boldsymbol{f}_x}{\partial s} = \frac{i}{2k}M\boldsymbol{f}_x \quad (18)$$

The matrix elements of M are given by

$$M_{\mu\nu} = \frac{1}{\Delta x^2}(\delta_{i+1,i'} - 2\delta_{i,i'} + \delta_{i-1,i'})\delta_{j,j'}$$
$$+ \frac{1}{\Delta y^2}(\delta_{j+1,j'} - 2\delta_{j,j'} + \delta_{j-1,j'})\delta_{i,i'} \quad (19)$$

where $\mu = i + (m+2)j + 1$, $\nu = i' + (m+2)j' + 1$. The solution of eq.(18) can be written as

$$\boldsymbol{f}_x(s) = \exp\left(\frac{i}{2k}M(s - s_1)\right)\boldsymbol{f}_x(s_1) \quad (20)$$

Here $\boldsymbol{f}_x(s_1)$ is the value at the exit and is obtaind from the tracking result $E_x(s_1)$ with $E_x^{(\infty)}$ subtracted. Eq.(20) can be integrated as

$$\int_{s_1}^{\infty} \boldsymbol{f}_x(s)ds = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon - (1/2k)M}\boldsymbol{f}_x(s_1)$$
$$= 2ikM^{-1}\boldsymbol{f}_x(s_1) \quad (21)$$

Similarly, we can obtain the integrated value of $\boldsymbol{f}_y$.
Separating the longitudinal electric field as $E_s = E_s^{(\infty)} + F_s$, the integrated value $\int \boldsymbol{f}_s ds$ can be obtained via eq.(3) from $\int \boldsymbol{f}_x ds$ and $\int \boldsymbol{f}_y ds$.

SIMULATION RESULTS

We apply this method to super-KEKB LER reference. The bunch length is $\sigma_s = 3$mm, bending radius $\rho = 16.3$m, number of particles $N = 3.3 \times 10^{10}$ in a bunch. Since we make the bunch length shorter for high luminosity, CSR is a serious concern. To suppress CSR, the size of the pipe cross section might be reduced, though there can be side effects. We choose the size of the cross section $h \times w = 40 \times 40$mm in this paper ($h$ = full height, w = full width). All of the results shown below are calculated at the center of the cross section. As is in our previous paper, we assume that the transverse beam size is zero. The pipe is made of copper: $\sigma_{cond} = 5 \times 10^7[\Omega^{-1}\text{m}^{-1}]$.

First, we show the result of the impedance in the drift space and compare it in Figure.1 with the well-known analytic solution. The pipe of square cross section gives the same longitudinal wakefield as the round cross section at the center of the cross section. The result agrees well with the analytic formula.

The energy change in the bending magnet is shown in Figure 2. This result includes the integration to infinity. The resistive wall effect of the beam pipe in the magnet is included but not that of the exit pipe. (The latter is separated as $E_s^{(\infty)}$) One finds that the effect of resistive wall can not be ignored in this machine.

The real part and imaginary part of the impedance is shown in Figure.3 and 4 respectively. Both the resistive wall and the perfectly conductive wall decrease exponentially in $k \to 0$. The resistive wall contributes in the low frequency region because the typical scale length is large.

CONCLUSION

Owing to the paraxial approximation, CSR can be calculated in the beam pipe by using mesh. The method enable us to consider beam pipes of finite conductivity. The resistive wall contributes to energy change and is not negligible
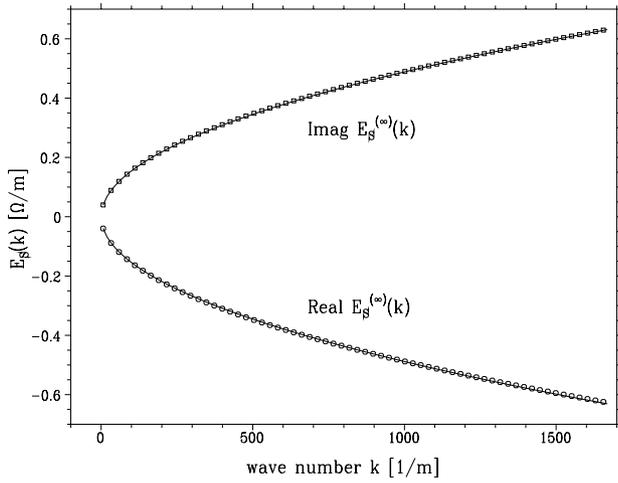
Figure 1: Impedance in the drift space in steady state. The real part (bottom) and imaginary part (top) of the analytic solution is plotted with the solid lines. The dots are the results by numerical calculation.
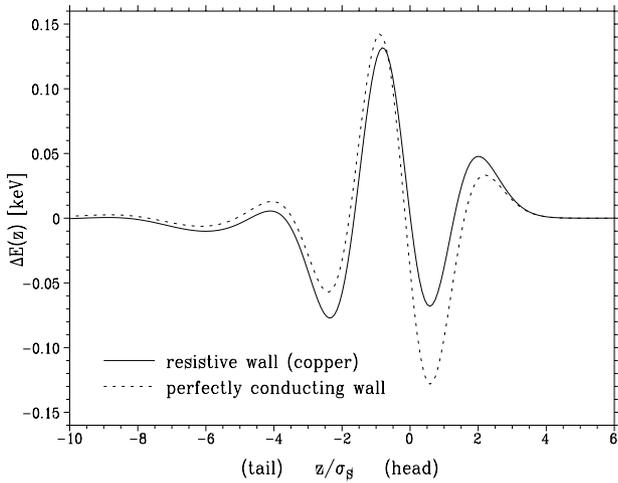

Figure 2: The energy change by CSR in super KEKB LER. The horizontal axis is the coordinate in the bunch in units of rms. The result in the copper pipe is plotted with the solid line, perfectly conductive pipe with the dotted line.

in the case of strong shielding such as KEKB LER. Considering the resistive boundary, this simulation includes the resistive wall wakefield automatically. In practice it is impossible to distinguish between CSR and wakefield.

Although we showed only simple cases, this method is very flexible and can be extended to more general cases, for example, chamber cross section other than rectangular, finite beam energy (but still large γ), changing beam profile (but not due to CSR itself).

Another important subject is the transverse force. We will discuss it in the next opportunity.
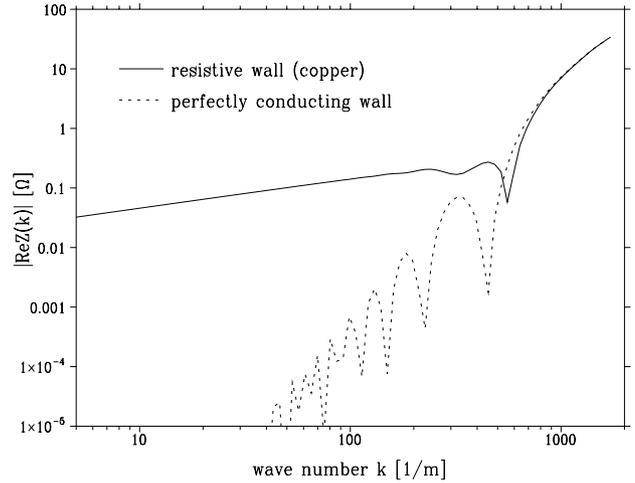

Figure 3: The real part of the impedance in logarithmic scale. The horizontal axis is the wave number.
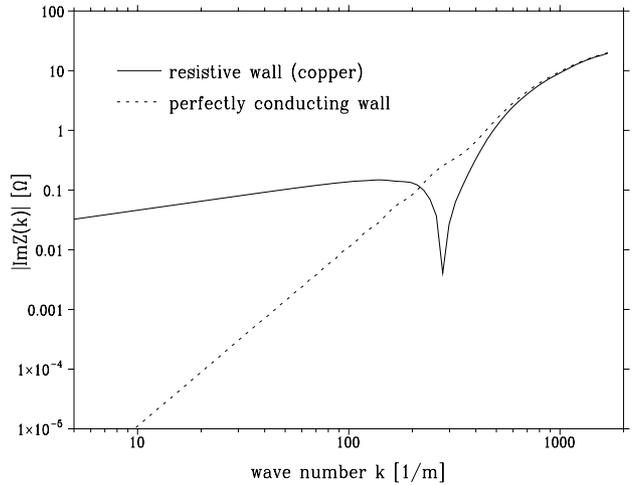

Figure 4: The imaginary part of the impedance in logarithmic scale.

ACKNOWLEDGEMENTS

REFERENCES

[1] T.Agoh and K.Yokoya, submitted to Phys.Rev.ST-AB