

## EXPERIMENTAL STUDY OF SEXTUPOLE COUPLING RESONANCES AT VEPP-4M\*

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### Abstract

In this paper the results of experimental investigation of coupling resonances excited by sextupole magnets at VEPP-4M collider are presented. Such aspects of nonlinear beam dynamics as the amplitude dependent tune shift, dynamic aperture and phase space portrait are considered. Measurement results are compared with those obtained by numerical simulation and analytic approach.

### INTRODUCTION

The work was carried out at VEPP-4M on experimental study of the nonlinear beam dynamics in the vicinity of 2D sextupole resonances ( $\nu_x + 2\nu_z = 24$  and  $\nu_x - 2\nu_z = -7$ ) excited by the chromatic sextupole magnets. We have considered such issues of the nonlinear dynamics as the amplitude-dependent tune shift, dynamic aperture and the phase trajectories of the nonlinear system. The results of experimental measurements are compared with the computer simulation and analytic estimations.

Earlier, the sextupole coupling resonances were studied experimentally at the ALADDIN storage ring [1] concentrated mainly on the beam trajectories. In our case, we added measurements of the nonlinear detuning and two-dimensional area of the beam stable motion.

### MEASUREMENT TECHNIQUE

Measurements were based on the fast pulse (duration less than one turn) excitation of the coherent beam oscillations [2]. The turn-by-turn measurements of the beam centroid enable determining the basic characteristics of the nonlinear motion as the dynamic aperture, tune dependence on amplitude, topology of phase trajectories, presence of the higher order resonances, etc.

Measurements were carried out at an energy  $E = 1.85$  GeV. The coherent betatron oscillations were excited horizontally by the separation electrodes, to which the pulse of 50 ns with the amplitude of 25 kV was applied. The positron inflector pulse (150 ns in duration and amplitude of 20 kV) made the vertical kick.

For the beam position measurement, the turn-by-turn BPM SRP3 is used. The signal from four pickup electrodes is digitised independently by the fast 10-bit 100 ns ADC and then processed by computer.

The typical beam current in measurements is  $1 \div 3$  mA. In this case, the rms coordinate resolution of BPM is around 70  $\mu\text{m}$ .

### RESULTS

The main goal of the measurements was obtaining of the quantitative information on the properties of the nonlinear motion of particles in VEPP-4M as the value of dynamic aperture depending on the betatron tune, the amplitudes of the main resonance driving terms, etc. The comparison of measurements with the results of the numerical simulation and analytic estimates provides the information on the VEPP-4M nonlinear lattice model.

#### Amplitude dependent tune shift

At the lowest order of the perturbation theory, the dependence of betatron frequency on the amplitude is written in the form

$$\Delta \nu_x = C_{xx} A_x^2 + C_{xz} A_z^2, \quad \Delta \nu_z = C_{zx} A_x^2 + C_{zz} A_z^2, \quad (1)$$

By varying the kicker voltage and measuring the signal from the turn-by-turn BPM one can determine the coefficients in (1).

The main sources of the nonlinear detuning are the sextupole and octupole magnets, fringe fields of the final focus quadrupoles and small nonlinear errors in the main magnetic elements. The contribution of sextupole components to (1) depends on the tunes in resonant way. Therefore, by measuring the coefficients in (1) as a function of the tune point one can separate different types of perturbations and find out their values. In the vicinity of coupling resonances, the sextupole fraction of coefficients (1) can approximately be described with the use of amplitudes only two azimuthal harmonics:

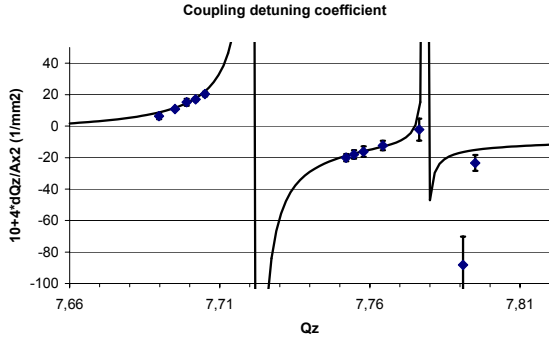
$$C_{zx} = C_{zx}^{(o)} + C_{zx}^{(s)}(\nu_x, \nu_z) \\ \approx C_{zx}^{(o)} + \frac{18}{\beta_{xp}} \left( \frac{B_{p,24}^2}{\nu_x + 2\nu_z - 24} - \frac{B_{m,-7}^2}{\nu_x - 2\nu_z + 7} \right), \quad (2)$$

and similarly for  $C_{xx}$  and  $C_{zz}$ . Fitting of experimental data by the expression (2) enables one to determine the contribution of the cubic nonlinearity and values of the resonance harmonics  $B_{p,24}$  и  $B_{m,-7}$ .

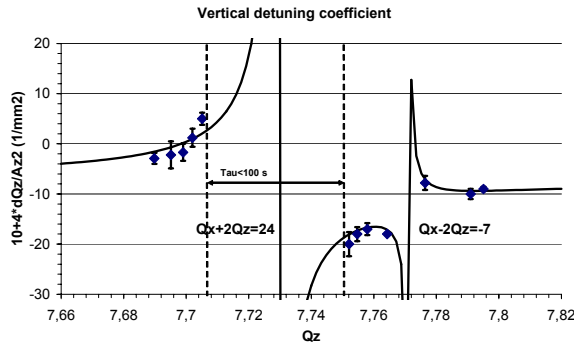
Figs. 1 and 2 show the coefficients  $C_{zx}$  and  $C_{zz}$  as functions of the vertical betatron frequency. The resonance behavior of coefficients is seen. In addition, Fig.2 shows the width of the resonance  $\nu_x + 2\nu_z = 24$ , where the beam lifetime does not exceed 100 s.

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 Figure 1: Experimental and theoretical values of  $C_{zx}(v_z)$ .

Estimates by (2) give the following values for the main perturbation harmonics:  $B_{p,24}=0.1 \text{ mm}^{-1/2}$  and  $B_{p,-7}=0.03 \text{ mm}^{-1/2}$ . In this case, the cubic nonlinearity contribution can be evaluated as follows  $C_{xz}^{(0)}=-6 \times 10^{-4} \text{ mm}^{-2}$ ,  $C_{zz}^{(0)}=-2 \times 10^{-4} \text{ mm}^{-2}$  and  $C_{zx}^{(0)}=-1 \times 10^{-4} \text{ mm}^{-2}$ .


 Figure 2: Experimental and theoretical values of  $C_{zz}(v_z)$ .

### Dynamic aperture

Main features of the nonlinear beam behaviour close to the sextupole coupling resonances can be obtained from the well-known Hamiltonian [3]

$$H_r = \delta \cdot I_1 + v_z I_2 \mp 6B_{\pm n} (2I_1)^{1/2} (2I_1 - I_2) \cos \phi, \quad (3)$$

where  $\delta = v_x \pm 2v_z - n_{\pm}$  is the resonance detuning,  $\phi = \phi_x \pm 2\phi_z - n_{\pm}$  is the phase variable,  $I_1 = J_x$  is the horizontal action,  $B_{\pm n}$  is the resonance harmonic amplitude and the constant parameter  $I_2$  determined by the initial conditions as:

$$I_2 = 2J_x \mp J_z, \quad (4)$$

where the upper sign corresponds to the sum resonance.

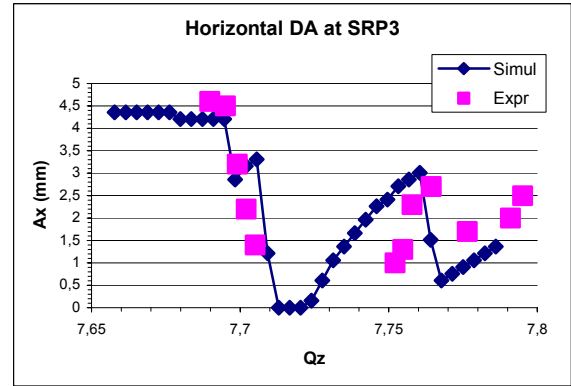
As is known, from (3) it follows that the motion near the sum resonance is unstable whereas the motion on the differential resonance is stable but the amplitudes of betatron oscillations are subjected to beatings. So despite the principal stability of motion near the differential resonance, the amplitudes beatings also can lead to a decrease in dynamic aperture. Since the beating values reach maximum at  $A_{z0} \approx 0$  it mainly limiting the horizontal aperture. From (3) and (4) one can obtain for the exact resonance condition ( $\delta=0$ )

$$A_x = 2A_{x0} - \sqrt{A_{x0}^2 + A_{z0}^2 \frac{\beta_{xp}}{2\beta_{zp}}},$$

$$A_z = 2A_{z0} - \sqrt{A_{z0}^2 + A_{x0}^2 \frac{2\beta_{zp}}{\beta_{xp}}},$$

where  $A_{z0}(A_{x0})$  is the initial dynamic aperture with no account for the differential resonance.

Figs.3 and 4 show the dynamic aperture measured near the sum and differential sextupole resonances. It is clear seen that the difference resonance decreases really the horizontal aperture and does not affect the vertical aperture.


 Figure 3: Horizontal aperture in vicinity of sextupole coupling resonances:  $v_x + 2v_z = 24$  occurs at  $v_y \approx 7.72$ , and  $v_x - 2v_z = -7$  at  $v_y \approx 7.77$ .

An analysis (3) gives the following value of the maximum dynamic aperture near the sum resonance:

$$A_{x0} = \sqrt{\beta_{xp} \frac{v_x + 2v_z - n_+}{12B_{+n}}} \text{ at } A_{z0} = 0$$

$$A_{z0} = \sqrt{\beta_{zp} \frac{v_x + 2v_z - n_+}{12B_{+n}}} \text{ at } A_{x0} = 0.$$

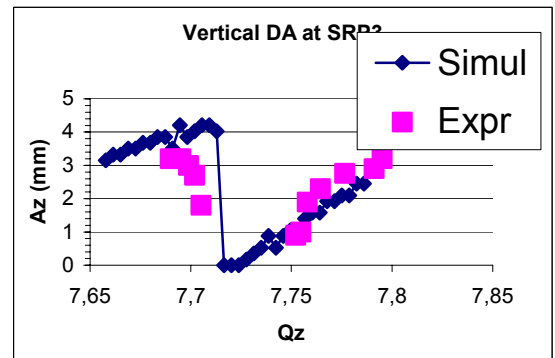


Figure 4: Vertical aperture in the vicinity of sextupole coupling resonances.

### Trajectories of beam motion

The phase surfaces of two-dimensional resonances in 3D phase space can hardly be depicted on a paper, however the oscillation amplitude beatings are well recognized at the Fig.5.

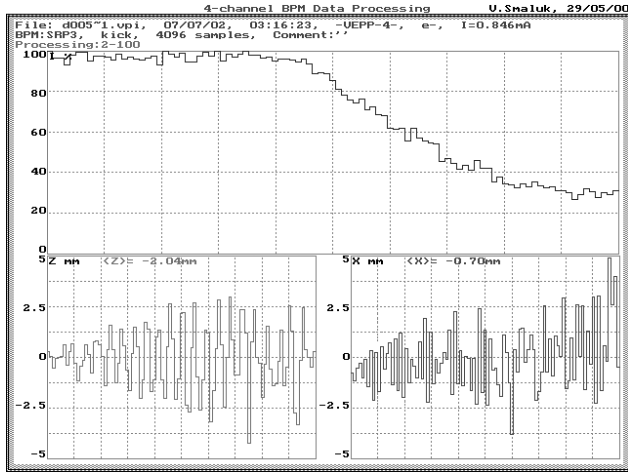


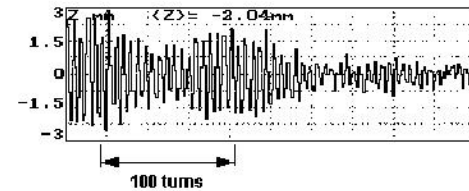
Figure 5: Beatings of amplitude of vertical oscillations near the differential resonance for various initial conditions.

Measurements near the sum resonance are complicated because of the beam instability. We can only fix the simultaneous growth in the amplitude of both transverse modes of oscillations (Fig.6) as it predicted by (4).

### CONCLUSIONS

An experimental study of the beam behaviour near the sum and differential resonances was performed at the VEPP-4M collider. The following aspects of the beam nonlinear motion as the dependence of the oscillation frequency on the amplitude, dynamic aperture, beam trajectories were measured and studied. The numerical characteristics of the model dynamic system describing VEPP-4M are obtained near the sextupole coupling resonances.

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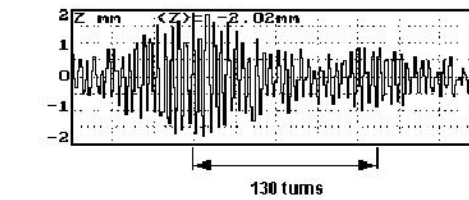


Figure 6: Beam instability near the sum resonance. Upper – the beam intensity, down – of the vertical and horizontal oscillations.

### REFERENCES

- [1] J.Liu, et al., Particle Accelerators, 1993, Vol.41, pp.1-12.
- [2] E. Levichev, V.A. Kiselev, A. Naumenkov. A Dynamic Aperture of VEPP-4M. Proc. of 2001 Particle Accelerator Conference (PAC2001), Chicago, Illinois, June 18-22, 2001, TPPH004, p.1659.
- [3] K.Symon. Applied Hamiltonian Dynamics, AIP249, 1992, Vol.1, p.277.