

# ESTIMATION OF BEAM POLARIZATION FROM THE MEASURED BEAM LIFETIME IN AN ELECTRON STORAGE RING

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## Abstract

The Touschek lifetime is determined by Möller scattering. Since the Möller scattering depends on the beam polarization, the Touschek lifetime (and thus the total beam lifetime) also depends on the beam polarization. The polarization dependence of the Möller scattering is calculated and the result is applied to Pohang Light Source (PLS) beam lifetime.

## INTRODUCTION

Stored electron beam gets polarized transversely. This radiative polarization proceeds according to the formula [1]

$$P = P_0 \left( 1 - e^{-t/T} \right), \quad (1)$$

where  $P$  is the degree of beam polarization and the limiting polarization  $P_0$  can have the maximum value 0.924 when there is no depolarizing effect. Without depolarizing effects, the characteristic time constant  $T$  is given by

$$T = T_0 \equiv 98 \text{sec} \times \frac{R_{bnd}^2 R_{avg}}{E^5 (\text{GeV})}, \quad (2)$$

where  $R_{bnd}$  is the bending radius in meters and  $R_{avg}$  is the mean radius of the ring. With the PLS parameters,  $E = 2.0$  GeV,  $R_{bnd} = 6.30$  m, and  $R_{avg} = 44.65$  m, we find  $T_0 \simeq 1.5$  h.

The Touschek lifetime depends on the beam polarization through the polarization dependence of the Möller scattering. The loss rate gets smaller as the degree of beam polarization gets bigger. Hence the Touschek lifetime also depends on the beam polarization. Since the PLS beam lifetime is Touschek dominant, the measured beam lifetime would contain the information of the radiative polarization. In this paper, we will analyze the measured PLS beam lifetime and deduce the saturated beam polarization.

## POLARIZATION DEPENDENCE OF TOUSCHEK EFFECT

The Möller scattering cross section that depends on transverse polarization is given by [2, 3]

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{4p^{*4}E^{*2}} \frac{1}{\sin^4 \theta} \left[ 4 - 3 \sin^2 \theta + 4 \times (4 - 3 \sin^2 \theta)p^{*2} + (4 - \sin^2 \theta)^2 p^{*4} - P_1 P_2 \sin^2 \theta \left\{ 1 + 2(1 + \sin^2 \phi + \right. \right. \right. \quad (3)$$

$$\left. \left. \left. \sin^2 \cos^2 \phi \right) p^{*2} + \sin^2 \theta \cos(2\phi) p^{*4} \right\} \right],$$

where  $r_0$  is the classical electron radius,  $P_1$  and  $P_2$  are the transverse polarizations of the two electron beams, and  $p^*$ ,  $E^*$  are mass normalized momentum and energy such as  $p^* = p/mc$ ,  $E^* = E/mc^2$ . The curly bracket that is proportional to  $P_1 P_2$  gives the polarization dependence of Touschek lifetime. We will consider the case that  $P_1$  and  $P_2$  have the same value  $P$ , the radiative polarization. Consider a bunch with a volume  $V$  that contains  $N$  particles. After complicated algebra that is omitted here [4], Touschek lifetime  $\tau_t$  is given by

$$\begin{aligned} \frac{1}{\tau_t(P)} &= -\frac{1}{N} \frac{dN}{dt} \\ &= \alpha \left[ C(\epsilon) + DP^2 \right] N. \end{aligned} \quad (4)$$

The  $N$ -independent parameter  $\alpha$  is defined as

$$\alpha = \frac{\sqrt{\pi} N c r_e^2}{\gamma^3 V \sigma_{x'} (\Delta p_m / p)^2}, \quad (5)$$

where  $r_e$  is the classical electron radius,  $\Delta p_m / p$  is the momentum acceptance,  $\gamma$  is the Lorentz factor,  $V = (4\pi)^{3/2} \sigma_x \sigma_y \sigma_l$  is the bunch volume, and  $C(\epsilon)$  gives the polarization-independent part of Touschek lifetime and is defined by [5]

$$\begin{aligned} C(\epsilon) &= \epsilon \int_{\epsilon}^{\infty} \frac{1}{u^2} \left\{ \left( \frac{u}{\epsilon} \right) - \ln \left( \frac{u}{\epsilon} \right)^{1/2} - 1 \right\} e^{-u} du, \\ \epsilon &= \left( \frac{\Delta p_m}{\gamma^2 \sigma_{x'} mc} \right)^2. \end{aligned} \quad (6)$$

When  $\epsilon < 1$ ,  $C(\epsilon)$  is approximately 1 [6].

The parameter  $D$ , which gives the polarization dependent contribution to Touschek lifetime, is also proportional to an integral that does not allow analytic integration. But in the limit  $\Delta p_m / p \ll 1$ , which is usually the case,  $D$  is approximated to a simple number, [4]

$$D = -\frac{1}{4}, \quad \frac{\Delta p_m}{p} \ll 1. \quad (7)$$

Then it is easy to show that

$$\frac{\tau_t(P) - \tau_t(0)}{\tau_t(P)} = \frac{P^2}{4C(\epsilon)}. \quad (8)$$

Hence if we measure the relative increase of Touschek lifetime, we can determine the polarization  $P$  from the above

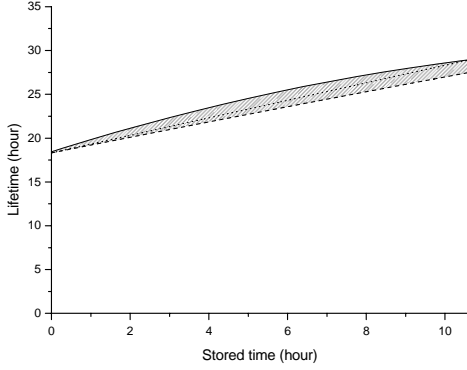


Figure 1: Measured PLS beam lifetime. The solid line represents the beam lifetime with polarization and the dashed line represents the beam lifetime with the polarization suppressed.

Eq. (8). However, what we measure is not Touschek lifetime but the total beam lifetime  $\tau$ ,

$$\frac{1}{\tau} = \frac{1}{\tau_0} + \frac{1}{\tau_t}, \quad (9)$$

where  $\tau_0$  denotes other partial beam lifetimes. But in many (especially small and intermediate) third generation light sources, Touschek lifetime is the determining factor for the total beam lifetime ( $\tau_t < \tau_0$ ). In these cases, we still have the simple relation

$$\frac{\tau(P) - \tau(0)}{\tau(P)} = \frac{P^2}{4C(\epsilon)}, \quad \tau_t < \tau_0. \quad (10)$$

## APPLICATION TO PLS

In real storage rings, there always exist depolarizing effects. The main source of the depolarizing effects is the horizontal magnetic field caused by magnet misalignment and closed orbit distortion. The skew quadrupoles are, if any, strong depolarizing sources. Because of these depolarizing effects, the limiting polarization  $P_0$  is almost always less than the maximum value 0.924 and the characteristic time  $T$  is also different from  $T_0$  given in the above Eq. (2). Figure 1 displays PLS beam lifetimes measured with a stored 2.0 GeV beam. In order to block the radiative polarization, skew quadrupoles were turned on. The strength of skew quadrupoles were chosen appropriately so as not to decrease the dynamic aperture. The dashed line of Fig. 1 denotes the measured beam lifetime with skew quadrupoles turned on. The initial current was 157 mA and the initial lifetime was 18.3 h. The final current was 98 mA. The monitored pressure showed slight but non-negligible dependence on the stored current.

Before we proceed, note that [7]

$$\frac{d\tau_t}{dt} = \frac{d\tau_t}{dN} \frac{dN}{dt} = \left( -\frac{1}{\alpha C(\epsilon) N^2} \right) \left( -\alpha C(\epsilon) N^2 \right) = 1. \quad (11)$$

Hence the unpolarized Touschek lifetime has a time derivative that is equal to 1. To be sure that the radiative polarization is suppressed, it is compared with the dotted line which is a simple straight line with a slope 1 ( $d\tau/dt = 1$ ) representing a pure Touschek lifetime  $\tau_t$  and the solid line which is a measured beam lifetime with skew quadrupoles turned off. The dashed line and the solid line have almost the same initial current and initial lifetime. Although  $\tau_t$  has a slope 1 ( $d\tau_t/dt = 1$ ), since the other part  $\tau_0$  is almost time independent ( $d\tau_0/dt \approx 0$ ) the combined value  $\tau$  can never have  $d\tau/dt$  greater than 1. But in the process of radiative polarization,  $d\tau/dt$  has an additional contribution from  $dP/dt$  and can be greater than 1 as in

$$\begin{aligned} \frac{d\tau_t(P)}{dt} &= 1 + \frac{d\tau_t(P)}{dP} \frac{dP}{dt} \\ &= 1 + \frac{P}{2\alpha(C - P^2/4)^2 N} \frac{dP}{dt}, \end{aligned} \quad (12)$$

which is always greater than 1. Hence  $d\tau/dt$  can be greater than 1 even after including  $\tau_0$ . The solid line clearly has a slope greater than 1 at the early stage of beam storing. This is an evidence of radiative polarization procedure going on. But the beam lifetime with skew quadrupoles (dashed line) has  $d\tau/dt$  less than 1 in Fig. 1, which is a clear evidence that its polarization has been suppressed completely or to a small value. The skew quadrupoles may give a slight change to  $\tau_t$  by modifying the bunch volume, but  $\tau_0$  is not affected. Hence both the dashed line and the solid line have the same  $\tau_0$ .

From Fig. 1, we see that  $[\tau(P) - \tau(0)]/\tau(P) \simeq 0.1$ . Hence we determined the left side of Eq. (10). In the right side, we have to determine  $\epsilon$ . Note that the momentum acceptance  $\Delta p_m/p$  is determined by the RF momentum acceptance, the dynamic aperture, and the physical aperture of the chamber. But here we will assume that it is entirely determined by the RF acceptance. In PLS it is 0.15 approximately. On the other hand, the radial momentum spread  $\sigma_{x'}$  is computed by

$$\sigma_{x'} = \sqrt{\epsilon_x \gamma_x + \eta'^2 (\Delta E/E)^2}. \quad (13)$$

It is straightforward to see that  $\epsilon$  is actually much less than 1 and thus  $C(\epsilon) \simeq 1$  to a good accuracy. Therefore, Eq. (10) becomes now

$$P^2 \approx 0.4. \quad (14)$$

Hence the radiative polarization of PLS is determined to be

$$P \approx 0.6. \quad (15)$$

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