R-MATRIX ANALYSIS OF THE CSR EFFECT IN A FUTURE ERL LIGHT SOURCE

R. Hajima*, JAERI, Tokai, Ibaraki 319-1195, Japan

Abstract

The R-matrix method, which has been widely used in beam-optics analyses, is extended to study the emittance growth caused by coherent synchrotron radiation in an energy-recovery linac (ERL). The R-matrix method enable us to make quick estimation of the emittance growth through an ERL arc and to obtain an optimized arc for minimizing the emittance at each straight section of the arc. In this paper, emittance growth in a 3 GeV ERL is analyzed by R-matrix method, and the results are compared with particle tracking simulations.

INTRODUCTION

Next generation light sources based on energy-recovery linacs (ERL) have been proposed [1]. These devices will provide sub-picosecond X-ray pulses with very high brilliance, which cannot be obtained in existing X-ray light sources using electron storage rings. In the development of ERL light sources, study on coherent synchrotron radiation (CSR) is one of the critical issues, because the emittance of an electron bunch is diluted by CSR when the electron bunch travels a circular path such as a beam transport loop of an ERL.

The beam transport loop of an ERL is a series of achromatic cells, and designed to keep the electron bunch length constant or to compress it gradually. We recently reported that the CSR-induced emittance dilution in such a beam transport system can be estimated by an analytical method using R-matrices instead of time-consuming computations of particle tracking [2]. In this paper, the R-Matrix analysis of the CSR effect is briefly described and the emittance growth in a 3 GeV ERL is discussed.

CSR ANALYSIS BY R-MATRICES

A first-order equation of electron motion in a uniform field of a dipole magnet is

$$x'' = -\frac{x}{\rho^2} + \frac{1}{\rho}\left(\delta_0 + \delta_{csr} + \kappa[s - s_0]\right) \;, \qquad (1)$$

where ρ is the bending radius, δ_0 is the initial momentum deviation normalized by the reference momentum. The last two terms on the right-hand side are related with the CSR effect: δ_{csr} the normalized momentum deviation caused by CSR in the upstream path ($0 < s < s_0$), $\kappa = W/E_0$ the normalized CSR wake potential in the bending path defined by CSR wake potential W and the reference energy E_0.

* hajima@popsvr.tokai.jaeri.go.jp

In the constant CSR wake regime, eq.(1) can be solved analytically, and electron dynamics in the bending plane can be calculated using 5x5 R-matrices

$$\vec{x} = (x \;\; x' \;\; \delta_0 \;\; \delta_{CSR} \;\; \kappa)^T \;, \qquad (2)$$

$$\vec{x}(s_1) = R_{0\to 1}\, \vec{x}(s_0) \;, \qquad (3)$$

and the displacement of bunch slices in the (x, x') phase space is expressed by CSR-wake dispersion function (ζ_x, ζ_x') [2]. A transform R-matrix for a sector dipole magnet is obtained by Green's function method:

$$R_{bend} = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) & \rho(1-\cos\theta) & \rho^2(\theta-\sin\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & \sin\theta & \sin\theta & \rho(1-\cos\theta) \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \rho\theta \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} . \qquad (4)$$

The beam emittance along the beam path is calculated by

$$\varepsilon^2 = (\varepsilon_0\beta_x + D^2)(\varepsilon_0\gamma_x + D'^2) - (\varepsilon_0\alpha_x - DD')^2 \;, \qquad (5)$$

where $(\alpha_x, \beta_x, \gamma_x)$ are Courant-Snyder parameters, ε_0 is the initial emittance and (D, D') is rms spread of the CSR wake dispersion defined as

$$(D, D') = \Delta\kappa_{rms}(\zeta_x, \zeta_x') \;, \qquad (6)$$

$$\Delta\kappa_{rms} = \Delta E_{rms}/E_0/L_b \;, \qquad (7)$$

where L_b is the bending path length.

EMITTANCE GROWTH IN A 3-GeV ERL

Emittance compensation by envelope matching

In this section, we discuss two kind of techniques for the emittance compensation in an ERL loop, and see calculation results by R-matrix analysis and particle tracking. As an example of a future ERL light source, we consider a beam transport for a 3GeV ERL consisting of triple-bend achromat (TBA) cells as shown in Fig.1. The bending radius is 25m, and the bending angle is $(3+6+3) = 12$ deg. The quadrupole magnets inside the cell are tuned so that the TBA cell becomes isochronous, $R_{56} = 0$.

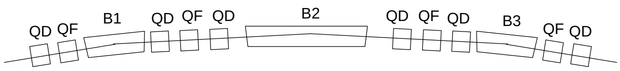

Figure 1: A triple bend achromatic cell: $\rho = 25$m, $\theta = 3 + 6 + 3 = 12$degree.

We assume the electron bunch parameters as follows: bunch charge $Q = 770$ pC, bunch length $\sigma_s = 30$ μm, normalized rms emittance $\varepsilon_{n,rms} = 0.1$ π mm-mrad, uncorrelated energy spread $\sigma_E/E_0 = 0.02\%$, and Gaussian distribution of electrons in the 6D phase space of motion. The energy spread caused by CSR is estimated from the equation:

$$\Delta E_{rms} \simeq 0.22 \frac{eQL_b}{4\pi\epsilon_0 \rho^{2/3} \sigma_s^{4/3}} \; . \tag{8}$$

and found to be $\Delta E_{rms} \simeq 990$ keV at the cell exit.

If the bunch is decomposed into numerous slices along the longitudinal direction, each bunch slice has an independent error in bending angle due to the longitudinal CSR wake. This CSR kick on a bunch slice dilutes the projection emittance after the bending path. The amount of emittance growth depends on the orientation of the beam ellipse and the CSR kick in the (x, x') phase space as shown in Fig.2. Consequently, the emittance growth can be minimized by choosing an appropriate beam envelope along an achromatic cell. This is the emittance compensation by beam envelope matching.

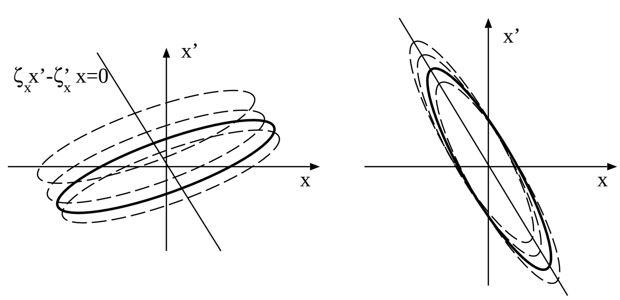

Figure 2: Two-dimensional phase ellipse in the (x, x') plane, and displacement of bunch slices due to the CSR kick. Two cases for large emittance growth (left) and small emittance growth (right).

The CSR-induced emittance growth in the 3-GeV TBA cell is calculated with varying the beam envelope. The beam envelope is changed by scanning Courant-Snyder parameters at the cell entrance: $-5.0 < \alpha_x < 5.0$, $\gamma_x = 0.29m^{-1}$. Figure 3 shows the projection emittance at the cell exit as a function of α_x. The results from the R-matrix analysis are compared with a particle tracking code `elegant` [3]. It can be seen that the emittance growth can be compensated by choosing an apropriate beam envelope.

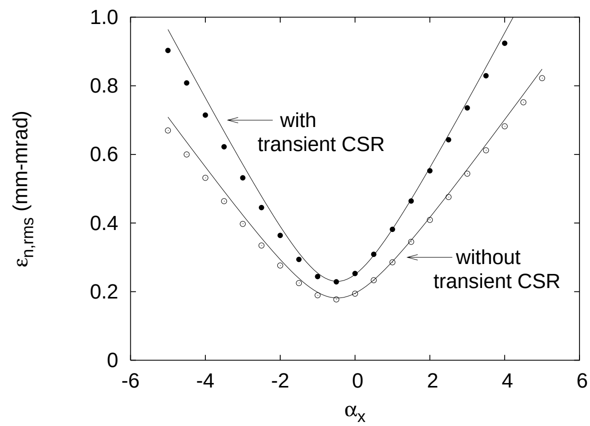

Figure 3: CSR-induced projection emittance growth with scanning the beam envelope calculated by R-matrix (solid lines) and particle tracking (dots).

Effect of transient CSR

In the R-matrix analysis, we assume the constant CSR regime, where each electron feels constant CSR wake potential along a bending path and the normalized CSR wake potential κ can be attributed to each electron. This assumption is valid under the following conditions: (1) all the dipole magnets have the same bending radius, (2) the electron bunch does not change its longitudinal profile in a single achromatic cell, and (3) the transient CSR effect at the entrance and exit of the magnet is not large. The first two conditions are apparently fulfilled in our example of a 3GeV ERL. The last criterion should be checked carefully.

The transient CSR can be neglected for the case of a bending magnet: $\rho/\gamma^3 \ll l_b \ll \rho\phi_m^3/24$, where l_b is the bunch length and ϕ_m is the bending angle [4]. For our example, three terms are, 1×10^{-10}, 3×10^{-5}, 1.5×10^{-4}, respectively, and the transient CSR has only a small effect.

This minor effect of the transient CSR can be included in the R-matrix analysis by correcting the normalized CSR wake potential. From the simulations of `elegant` without the transient CSR, the rms energy spread induced by CSR is found to be 960 keV at the 3rd bend exit. The energy spread becomes 1190 keV when the transient CSR is included by Stupakov's model [5]. Note that the transient CSR after the 3rd bend does not contribute to the emittance growth. The emittance growth estimated by R-matrix analysis after the correction $\Delta\kappa_{rms}^{transient} = (1190/960)\Delta\kappa_{rms}$ is plotted in Fig.3 together with the result of `elegant` including the transient CSR. We can see that the R-matrix analysis with the correction of normalized CSR wake potential can predict the emittance growth including the transient CSR.

Emittance compensation by cell-to-cell phase matching

The CSR-induced emittance growth along a series of achromatic cells can be compensated by setting cell-to-cell betatron phase advance at an appropriate value [6].

This kind of emittance compensation technique can be also adopted to ERL light sources.

We choose the quadrupole parameters of the TBA cell in the 3GeV ERL so that the betatron phase advance at each cell becomes $\Delta\psi_x = (8/3)\pi$ with keeping isochronous condition. With this design of TBA cells, the CSR kick applied to the bunch slices are expected to be canceled every three-cells. Figure 4 shows the beam envelopes in three TBA-cells designed as $\Delta\psi_x = (8/3)\pi$. We can see both the momentum dispersion and the CSR dispersion disappear after the third cell: $\eta_x = \eta'_x = 0$, $\zeta_x = \zeta'_x = 0$. The emittance compensation by cell-to-cell phase matching is confirmed by R-matrix analysis and `elegant`. Figure 5 shows the calculated emittance along the designed TBA cells. It is clearly seen that the emittance compensation is achieved at the third cell exit. In the cell-to-cell phase matching, the emittance after the compensation is independent of the beam envelope during the cells.

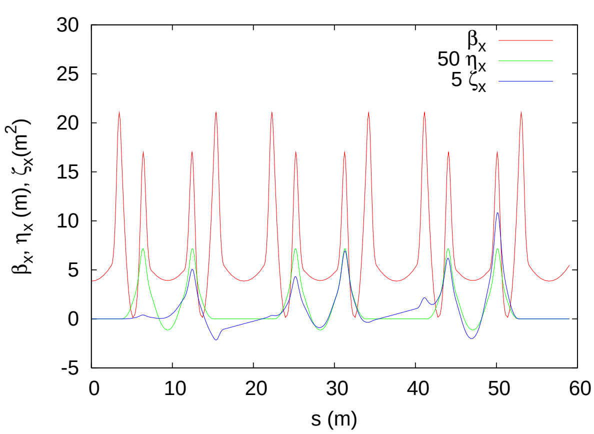

Figure 4: Beam envelopes for a series of TBA cells, where the betatron phase advance for a single cell is $\Delta\psi_x = (8/3)\pi$.

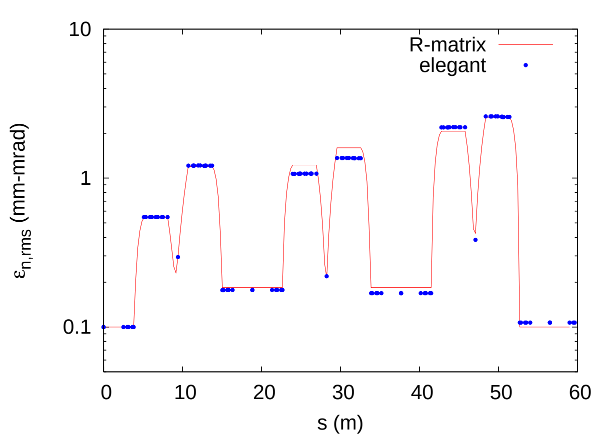

Figure 5: Beam emittance calculated by R-matrix and `elegant`.

SUMMARY

In this paper, CSR-induced emittance growth and its compensation in a future ERL light source have been discussed. We have seen that the R-matrix analysis can be applied to the beam optics design for the emittance compensation in ERL light sources. Two kind of emittance compensation techniques, beam envelope matching and cell-to-cell phase matching, have been presented. We compared the results from the R-matrix analysis and particle simulations, and confirmed that the CSR-induced emittance growth and its compensation can be well described by R-matrix analysis.

The effect of transient CSR is not large in a high-energy ERL for an X-ray light source such as our example of 3 GeV device. For these cases, the R-matrix method can be applicable to the emittance analysis including the transient CSR by correcting the CSR wake potential.

Because of the page limitation, we have limited our analysis to the unshielded CSR in isochronous cells. The matrix approach, however, can be easily extended to the shielded CSR with an arbitrary bunch shape as far as the CSR wake potential is given. For the case of bunch compression in an ERL loop, particle tracking calculation is necessary to refine the cell design including higher-order beam dynamics. However, the matrix approach is still helpful even in this case, because we can prescreen the design parameters by the matrix method before applying time-consuming particle tracking.

This work was supported in part by Japan Society for the Promotion of Science, Grant-in-Aid for Scientific Research (B) 15360507.

REFERENCES

[1] S. M. Gruner et al., Rev. Sci. Instrum. **73** (2002) 1402.

[2] R. Hajima, Jpn. J. Appl. Phys. **42** (2003) L974-L976.

[3] **ELEGANT** version 14.8, Nov. 26, 2002; M. Borland: Argonne National Laboratory Advanced Photon Source Report No. LS-287 (2000).

[4] E.L. Saldin et al., Nucl. Instrum. Meth. **A398** (1997) 373.

[5] G. Stupakov, P. Emma, LCLS-TN-02-12 (2001).

[6] D. Douglas, Thomas Jefferson Natl. Acc. Lab. Tech. Note, JLAB-TN-98-012 (1998); J. H. Wu et al., *Proc. 2001 Particle Accelerator Conference, Chicago, USA* (IEEE, New Jersey, 2001) p. 2866.