OPTICAL BRAGG ACCELERATION STRUCTURES

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Abstract

It is demonstrated that a Bragg waveguide consisting of a series of dielectric layers may form an excellent optical acceleration structure. Confinement of the accelerating fields is achieved for both planar and cylindrical configurations by adjusting the first dielectric layer width. A typical structure made of Silica and Zirconia may support gradients of the order of 1GV/m with an interaction impedance of a few hundreds of Ohms and with an energy velocity of less than 0.5c. An interaction impedance of about one thousand Ohms may be obtained by replacing the Zirconia with a (fictitious) material of $\varepsilon = 25$. Special attention is paid to the wake-field developing in such a structure. In case of a relatively small number of layers, a qualitative approach shows that the emitted power is inversely proportional to the number of micro-bunches. Quantitative results are given for a higher number of dielectric layers, showing that in comparison to a structure bounded by metallic walls, the emitted power is significantly smaller due to propagation bands allowing electromagnetic energy to escape.

INTRODUCTION

Indications that solid-state lasers will reach wall-plug to light efficiencies of 30% or more make a laser-driven vacuum-accelerator increasingly appealing. Since at the wavelength of relevant lasers, dielectrics may sustain a significantly higher electric field and transmit power with reduced loss comparing to metals, the basic assumption is that laser accelerator structures will be made of dielectrics. Closed optical structures and near-field accelerators with dimensions comparable to the wavelength are both being considered. Examples of these two are: a) the LEAP [1] crossed laser beam accelerator where the interaction between the crossed laser beams and the particles is limited by slits to satisfy the Lawson-Woodward theorem [2, 3], and b) the *photonic band-gap* (PBG) concept where a laser pulse is guided in a dielectric structure with a vacuum tunnel bored in its center [4]. Lithography, which would result in planar structures, and optical fiber drawing are manufacturing techniques that seem well suited for laser driven structures that have typical dimensions of a few microns.

Motivated by the low-loss Bragg dielectric *planar* mirrors used in high-power lasers, it is suggested to harness this concept in order to confine the laser-field in an optical acceleration structure [5, 6]. Its essence is to form a hollow dielectric waveguide consisting of an almost perfect reflector made of a planar array of quarter-wavelength dielectric layers. In the transverse direction the geometry is similar to that of a dielectric mirror, however, its characteristics are slightly different since the wave number has a significant component ($k = \omega/c$) *parallel* to the dielectric surfaces, whereas in the case of a high-power laser mirror, the wave impinges *perpendicularly*.

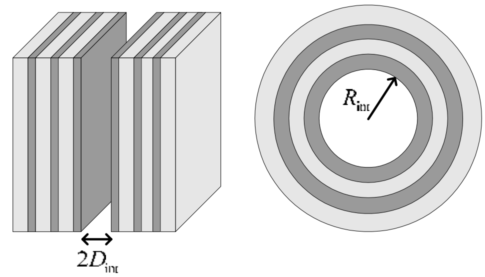

Figure 1: Optical Bragg acceleration structures.

FIELD CONFINEMENT

Two types of devices are reported in this communication, one is a Planar Bragg Accelerator (PBA) and the other is a Cylindrical Bragg Accelerator (CBA), as illustrated in Fig. 1. Both devices consist of alternating dielectric layers of two materials ($\varepsilon^{\rm I}, \varepsilon^{\rm II}$), surrounding a vacuum region. The half-width of the PBA vacuum region is $2D_{\rm int}$, and the radius of the CBA vacuum tunnel is $R_{\rm int}$. The PBA is symmetric relative to the central plane, and the CBA is azimuthally symmetric. The theory of Bragg reflection waveguides was developed by Yeh *et al.* [7, 8].

Subject to a design procedure [5, 6], it is possible for the Bragg waveguides to support the TM mode with phase velocity c required for acceleration. Imposing the continuity of the transverse electric and magnetic fields at the boundaries between the layers, a matrix formulation is obtained, with which the location of the interfaces between the dielectric layers is determined. Aside from the first layer, which is adjacent to the vacuum tunnel, all the layers are of transverse quarter-wavelength width $\lambda/(4\sqrt{\varepsilon - 1})$. The first layer may be conceived as a matching layer between the vacuum region to the subsequent periodic structure, as it rotates the amplitude vector dictated by the vacuum mode, to overlap the eigen-vector of the periodic structure. Confinement entails vanishing *real* part of the transverse component of the complex Poynting vector, meaning that in each dielectric layer there is a standing wave. It is therefore evident that for the structure to truly support the mode, there must be an infinite number of layers, otherwise energy would "escape" and there would be no confinement. In a practical structure, the number of layers should be sufficient so that the outward power flow is negligible.

ACCELERATOR PARAMETERS

Interaction Impedance. The interaction impedance is a measure of the accelerating gradient experienced by the electrons for a given amount of power injected into the system. Denoting by P the flowing power, the interaction impedance is defined by $Z_{\rm int} \triangleq |\lambda E_0|^2/P$. As a typical structure we shall consider dielectric layers made of Silica and Zirconia. Assuming that the materials' characteristics are known ($\varepsilon^{\rm I} = 2.1, \varepsilon^{\rm II} = 4$) and so is the laser wavelength (λ_0), the only free parameter left is the width of the internal vacuum layer. Simulations show that for the PBA, $Z_{\rm int}/\lambda_0$ decreases monotonically from about 150Ω to about 25Ω, when the internal half width is changed between $D_{\rm int} = 0.3\lambda_0$ to $D_{\rm int} = 0.8\lambda_0$. For the CBA, ($R_{\rm int} = 0.3\lambda_0$ to $0.8\lambda_0$), the interaction impedance decreases from about 270Ω to 20Ω. Increasing the dielectric coefficient of one of the materials can improve the interaction impedance significantly. For example, taking Silica as one material and a material with $\varepsilon = 25$, leads to an interaction impedance of about 1000Ω in the CBA.

Energy Velocity. Denoting the energy per unit length of the structure by W, the energy velocity is defined by $v_{\rm EN}/c \triangleq P/(cW)$. According to simulations, for $0.3 \le D_{\rm int}/\lambda_0 \le 0.8$, the energy velocity increases monotonically from about $0.42c$ to about $0.53c$ in the Silica-Zirconia PBA. For the CBA, the increase is between $0.38c$ to $0.48c$.

Maximum Electric Field. The last parameter of interest is the maximum electric field sustained by the structure before the probability of breakdown becomes significant. Since for most practical purposes the maximum field may be assumed to occur at the vacuum-dielectric interface, this quantity may be evaluated analytically. Moreover, if we assume a gradient of 1GV/m and that the fluence imposes a maximum electric field of 2GV/m, then the maximum internal radius allowed is $0.55\lambda_0$ for the CBA, and $0.28\lambda_0$ for the PBA.

WAKE-FIELD ANALYSIS

The last topic that needs to be considered in the present study is an estimate of the wake-field generated by a train of micro-bunches in such a dielectric structure. In a recent study [9] a general formulation of the wake-field for an arbitrary dielectric structure with a cylindrical vacuum tunnel was given. Based on this formulation, we determine the (symmetric) wake-field of a line charge moving in the PBA. Similar results are obtained for the CBA.

Wake-Field in the PBA

The solution of the wake-field is obtained via the wave equation of the magnetic vector potential, and imposing the boundary conditions of the relevant components of the electromagnetic field. The field generated by the line-charge in free space is called the primary field, and the remainder, which is due to the effect of the surrounding structure, is called the secondary field. Assuming that the material adjacent to the vacuum region has a dielectric coefficient $\varepsilon^{\rm I}$, it is convenient to define the relation between the outgoing and incoming waves just outside the vacuum tunnel as a function of frequency $\mathbb{R}(\omega)$. This reflection coefficient is directly dependent on the surrounding layers and their dielectric constants.

For the case $\mathbb{R} \equiv 0$, i.e., a vacuum tunnel within a homogeneous material with $\varepsilon^{\rm I}$, the secondary field is evaluated to obtain the decelerating field on the moving line-charge with charge per unit length q'

$$\mathcal{E}_{\parallel} = \frac{-q'}{2\pi\varepsilon_0 D_{\rm int}} {\rm Re}\left\{ j \ln\left(1 + j\frac{\gamma\sqrt{\beta^2\varepsilon^{\rm I} - 1}}{\varepsilon^{\rm I}}\right)\right\}. \quad (1)$$

It is evident that below the Cerenkov velocity ($v = c/\sqrt{\varepsilon^{\rm I}}$), the decelerating force is zero. It increases monotonically with the velocity, and for the ultra-relativistic regime ($\gamma \to \infty$) the decelerating field

$$\mathcal{E}_{\parallel} = \frac{q'}{2\pi\varepsilon_0 D_{\rm int}} \times \frac{\pi}{2}, \quad (2)$$

is $\pi/2$ times the radial field of a static line-charge at distance $D_{\rm int}$. Although this expression was calculated for a simple structure consisting of a vacuum core in an otherwise uniform dielectric, in case of an ultra-relativistic line-charge it is valid also if the structure has surrounding layers [9]. Causality arguments may be shown to lead to the conclusions that reflections caused by such layers reach the axis only after the original source has moved away. In other words, the reacting (decelerating) field on an ultra-relativistic particle itself is *independent* of the details of the structure layers, and it depends only on the charge and the size of the vacuum core.

From here on we shall focus on the ultra-relativistic regime, where it can be shown that the expression for the longitudinal electric field within the vacuum tunnel is uniform across the vacuum core, and is given by the inverse Fourier transform integral

$$E_z^{(s)}(\bar\tau) = \frac{q'}{2\pi\varepsilon_0 D_{\rm int}} \times \frac{1}{2}\int_{-\infty}^{\infty} d\bar\omega\, e^{j\bar\omega\bar\tau} \frac{1+\mathbb{R}}{(1+j\bar\omega)-(1-j\bar\omega)\mathbb{R}}, \quad (3)$$

where $\bar\omega \triangleq \omega\sqrt{\varepsilon^{\rm I}-1}D_{\rm int}/(\varepsilon^{\rm I}c)$ is the normalized frequency and $\bar\tau$ is the normalized space-time such that $\bar\omega\bar\tau = \omega(t - z/v)$.

Emitted Power in the PBA

It was discussed above that the decelerating force on a single ultra-relativistic charge does not depend on the surrounding structure. In practice, the *macro-bunch* of a future optical acceleration structure may consist of a train of M (~ 1000) micro-bunches separated by a wavelength (λ_0) of the driving laser field. A qualitative expression for $M \ge 1$ may be evaluated for the case of a "weak mirror" ($\mathbb{R} \sim 0$), by neglecting the effects of the micro-bunches on each other. In this configuration (or for a small number of layers) the emitted power was found to be proportional to the square of the number of electrons in the macro-bunch, and inversely proportional to the number of micro-bunches. This power is also inversely proportional to the square of the internal radius of the structure for the CBA , and to the width of the vacuum core in the PBA.

The electromagnetic power generated by a macro-bunch of total charge per unit length q' distributed to M micro-bunches, each one being $\lambda_0 \frac{\alpha}{2\pi}$ long, is given by

$$P = \frac{v\,(q')^2}{2\pi\varepsilon_0 D_{\rm int}} \frac{1}{2} \int_{-\infty}^{\infty} d\bar{\omega}\, \frac{1+\mathbb{R}}{(1+j\bar{\omega})-(1-j\bar{\omega})\mathbb{R}} \times \left\{ \mathrm{sinc}\left[\frac{\alpha}{2}\frac{\bar{\omega}}{\bar{\omega}_0}\right] \frac{\mathrm{sinc}\left[\pi\frac{\bar{\omega}}{\bar{\omega}_0}M\right]}{\mathrm{sinc}\left[\pi\frac{\bar{\omega}}{\bar{\omega}_0}\right]} \right\}^2 , \quad (4)$$

where $\mathrm{sinc}(x) \equiv \sin(x)/x$ and $\bar{\omega}$ is as defined above. Simulations indicate that if $0 \le \alpha < \pi/2$ the power is virtually independent of α and within a good approximation it is inversely proportional to M. Fig. 2 illustrates the normalized power $\bar{P} \triangleq P\left(v\,(q')^2/(2\pi\varepsilon_0 D_{\rm int})\right)^{-1}$ for a PBA with different values of the number of layer N, for the no-reflections case, and for closed structures with metallic and magnetic walls. For the PBA, it shows that for a stronger confinement (large N) the power emitted is higher, but is more moderately dependent on the number of bunches. For a small number of bunches, it is seen that the general behavior is $\sim 1/M$. The curves of $N = 80$ and $N = 120$ almost coincide, indicating that this is the limiting curve when the number of layers is increased. A significant difference between the PBA and *closed* structures is seen in Fig. 2. Unlike the two top curves of the magnetic and metallic cases, the PBA allows for energy to escape out of the structure through the propagation bands, the trailing bunches are less affected by the wake-field, and hence the interaction power is smaller. This clearly reveals the significant advantage of a Bragg structure over any other closed structure.

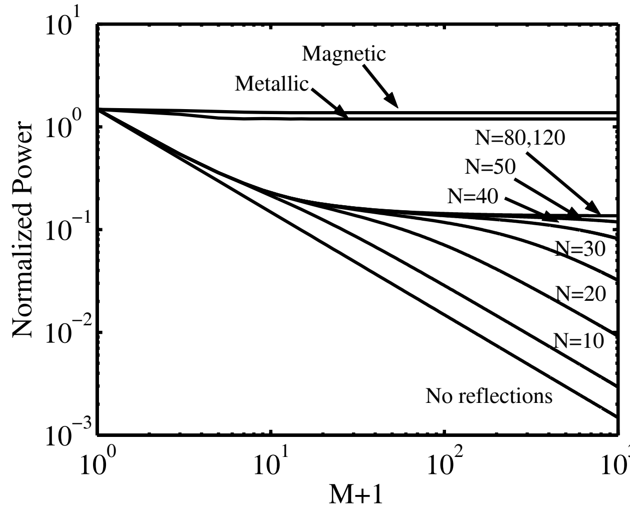

Figure 2: Emitted power as a function of the number of micro-bunches.

CONCLUSION

In the present study we have designed and analyzed an acceleration structure based on a Bragg reflection waveguide. The first layer is designed to match between the required mode in the vacuum region and the eigen-mode of the periodic structure, so that the field is confined within the structure. An interaction impedance of over 250Ω is feasible with existing materials. Materials of high dielectric coefficient can significantly improve the interaction impedance to about 1000Ω.

The total emitted power due to a train of micro-bunches was shown to be limited by two extreme cases. The *lower limit* is set by the power emitted by a train of bunches moving along a vacuum tunnel bored in a dielectric medium that extends to infinity. The *upper limit* corresponds to a closed structure, namely, a dielectric loaded waveguide with either metallic or perfect magnetic walls.

ACKNOWLEDGEMENT

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