# MATHEMATICAL MODEL FOR FAST AND SLOW DUMPING OF K-500 SUPERCONDUCTING CYLOTRON MAGNET

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# Abstract

A superconducting cyclotron (K-500) is under construction at Variable Energy Cyclotron Centre. Two superconducting coils – named 'Alpha' and 'Beta' – generates the main magnetic field of the cyclotron. Subsequent to the in-house fabrication of the two coils, it was assembled into the cryostat and the magnet assembly completed. Immediately after the cool down, the superconducting coils were energized. Two dump resistors – slow and fast – serve to dissipate the energy stored in the coils externally depending upon the situation.

Various sets of data were collected during dumping and based on these an approximate mathematical model was constructed to simulate the dumping characteristics. This model took into account the lead drops, the cable and the joint resistances along with the dump resistor values and the self- and mutual- inductances (that varied with current) of the coil.

## **INTRODUCTION**

The two coils are powered by M/S Danfysik make two power supplies rated 20V, 1000A, 10ppm stability class. Along with this, each of the magnet coils are always connected with two dump resistors, viz., fast dump and slow dump as shown in Figure 1.



Figure 1: Schematic of Power Supplies with Superconducting Coils and Dump Resistors

The two dump resistors – slow and fast – serve to dissipate the energy stored in the coils externally depending upon the situation. For example, during power failure, the energy stored in the coils is slowly dissipated through the slow dump resistors. Slow dumping is also automatically triggered for the following conditions viz, He level in upper part of cryostat falling below 95%, He pressure rising to more than 306mbar, horizontal support link forces not within permissible range (1000lbs – 10000lbs), panic button operated manually, power supply tripped due to internal interlock failure. For a potentially catastrophic situation (like quench, lead voltage drop

increasing above 160mV, He pressure in coil exceeding 680mbar, cryostat vacuum exceeding  $10^{-1}$ mbar, He level sensor in upper half of cryostat reading less than 80%) the energy is quickly dumped through the fast dump resistors.

## **MATHEMATICAL MODEL**

#### Theoretical Model

During dumping, the  $3\phi$  supply is withdrawn and the series contact gets automatically opened and the system shown in Figure 1 reduces to the circuit as shown in Figure 2.



Figure 2: The dumping scheme

The governing differential equations for the model:

$$L_{\beta}\frac{di_{\beta}}{dt} + M\frac{di_{\alpha}}{dt} + R_{\beta}i_{\beta} + R_{com}(i_{\beta} - i_{\alpha}) = 0$$
(1a)

$$L_{\alpha}\frac{di_{\alpha}}{dt} + M\frac{di_{\beta}}{dt} + R_{\alpha}i_{\alpha} + R_{com}(i_{\alpha}-i_{\beta}) = 0$$
(1b)

where,

$$\begin{split} M &= k \sqrt{(L_{\beta}L_{\alpha})} \\ R_{\beta} &= R_{dump\_\beta} + R_{diode\_\beta} + R_{cable\_\beta} + R_{lead\_\beta} + R_{joint\_\beta} + R_{coil\_\beta} \\ R_{com} &= R_{cable\_com} + R_{lead\_com} + R_{joint\_com} \\ R_{\alpha} &= R_{dump\_\alpha} + R_{diode\_\alpha} + R_{cable\_\alpha} + R_{lead\_\alpha} + R_{joint\_\alpha} + R_{coil\_\alpha} \end{split}$$

It is to be noted that during fast dump condition, the diode will be absent.

Neglecting the diode drop, cable resistances, joint resistances, lead resistances and considering the inductance values of the coils to be independent of current, the system reduces to

$$i_{\beta} = (0.62i_{\beta0} + 0.41i_{\alpha0})e^{-t/3065.92} + (0.38i_{\beta0} - 0.41i_{\alpha0})e^{-t/901.42}$$
(2a)  
$$i_{\alpha} = (0.38i_{\alpha0} + 0.58i_{\beta0})e^{-t/5065.92}$$

$$\begin{aligned} & I_{\alpha} = (0.58I_{\alpha0} + 0.58I_{\beta0})e \\ & + (0.62i_{\alpha0} - 0.58i_{\beta0})e^{-t'^{901.42}} \end{aligned} \tag{2b} \\ & \text{during slow dumping} \end{aligned}$$

and

$$i_{\beta} = (0.60i_{\beta 0} + 0.40i_{\alpha 0})e^{-t/155.88} + (0.40i_{\theta 0} - 0.40i_{\alpha 0})e^{-t/25.97}$$

$$+ (0.40i_{\beta 0} - 0.40i_{\alpha 0})e^{-t/25.97}$$
(3a)  

$$i_{\alpha} = (0.40i_{\alpha 0} + 0.60i_{\beta 0})e^{-t/155.88}$$

$$+ (0.60i_{\alpha 0} - 0.60i_{\beta 0})e^{-t/25.97}$$
(3b)

+ 
$$(0.60i_{\alpha 0} - 0.60i_{\beta 0})e^{-0.23.97}$$
 (3b  
uring fast dumping

during fast dumpir with

 $L_{\beta} = 26H, L_{\alpha} = 13H,$ 

$$\begin{split} R_{slowdump\_\beta} &= 7m\Omega, \ R_{slowdump\_\alpha} = 4.9m\Omega \\ R_{fastdump\_\beta} &= 250m\Omega, \ R_{fastdump\_\alpha} = 167m\Omega \\ k &= 1/\sqrt{2} \end{split}$$

 $i_{\beta 0}$  = initial current in Beta coil

 $i_{\alpha 0}$  = initial current in Alpha coil

It can be observed that both the equations for slow dumping (2a &2b) and fast dumping (3a & 3b) have a fast decaying component and a slow decaying component.

The time constants for them are as follows:

For slow dump

Time constant for the slowly decaying component =  $5065.92 \text{sec} (\sim 84.5 \text{min})$  Time constant for the fast decaying component = 901.42sec (~ 15min)

For fast dump

Time constant for the slowly decaying component =  $155.88 \sec (\sim 2.5 \min)$ 

Time constant for the fast decaying component =  $25.97 \text{sec} (\sim 0.5 \text{min})$ 

From the equation sets (3a) & (3b) and (4a) & (4b), it can further be observed that when the initial currents in both the coils are same (i.e.,  $i_{\beta 0} = i_{\alpha 0}$ ), the fast decaying component almost vanishes and the currents in the coils, in that case, will be monotonically decaying with a time period of 84.5min in the case of slow dump and 2.5min in the case of fast dump (Actually starting from equation 1a & 1b, if we make the self inductances of  $\beta$  exactly twice of that of  $\alpha$ , the coupling coefficient =  $1/\sqrt{2}$  and the ratio of the dumping resistance of  $\beta$  to that of  $\alpha$  to be exactly 1.5, then it can be shown that for the same initial currents in both the coils, the fast decaying component vanishes altogether).

# Results

During actual dumping, we have to consider all the sources of energy dissipations viz, the diode drops, the lead drops, the cable drops, the joint resistance and the coil resistance. These were measured by monitoring the voltages across the load, leads, power supply output leads at different currents and obtaining an average resistance from the results.

The results hence obtained are shown in Figures 3 & 4.



Figure 3: Comparison of the experimental and theoretical results during a Slow Dump



Figure 4: Comparison of the experimental and theoretical results during a Fast Dump

## Discussion

From the above two plots it is fairly apparent that the model is useful in the higher current region and shows a marked error in the lower current region (current < 50A). The time of an overshoot occurring due to the difference in the coil currents can also be predicted from the model.

The main source of error in the lower current region is believed to be due to the fact that the inductances of the magnet coil are functions of current in the coil and the non-linearity is more pronounced near the low current zone. The inductance value is fairly constant when the iron part of the magnet gets saturated. The other deviations of the theoretical model from the experimental results arise from the non-linear nature of the diode and the resistance build up at the joints. Also since the dumping phenomenon is inherently slow, the resistances in the dumping path get ample time to get heated and hence may change their values.

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## REFERENCES

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